RECORDS
ARITHMETICKE:

[illegible]ntayning the ground of [illegible]: In which is taught the generall [illegible] rules, and operations, of the same [illegible] *whole Numbers and Fractions, after* [illegible]ore easie and exact methode then [illegible] heretofore: First written by [illegible] ROBERT RECORD, Doctor in Phisicke.

[illegible]nce corrected and beautified by many Notions, and augmented with most briefe rules of [illegible] and others; necessarie in the trade of Merchandise. Whereunto is added certaine Tables *of the valuation of all Coynes as they are currant* at this present time, with diuers other Tables and their explication. Very profitable and delightfull vnto all Marchants, Gentlemen, and others, as by the contents of this booke doth appeare.

By *Iohn Mellis.*

Also the Art and application of Decimall Arithmeticke: A Table of Board and Timber measure, and the vse thereof. The Extraction and Demonstration of the Square and Cubicke roots, with necessary Questions and Tables for interest after ten in the hundreth is newly thereunto adioyned.

By R. N.

LONDON:
Printed by *T. S.* for *Roger Iackson.* 1615.

186
Douce
R. 466

Francis Douce.

TO THE RIGHT VVorthy and VVorshipfull, Sir HENRY FANSHAW Knight, the Kings Remembrancer in his Highnesse Court of *Exchequer*.

VVOrthy Sir, this now ancient euer-famous Booke of *Arithmeticke*, first made by Master *Robert Record*, hauing beene often imprinted, hath brought no small light to this *Art*, and great *helpe* and *profit* by this *Science*, to our *Country-men*. Master *Record* indeed, at first did set it forth, and gaue it *being*, and to patronize, it had that hopefull and renowmed *King* of this most famous *Kingdome*, *Edward* the sixt: But as the skilfulleſt of all builders amongst men, cannot so perfectly frame, and happily set vp at once, any *house* or *building*, vnto which may not further be added somewhat, that may eyther serue for *necessitie* or *ornament*. So, not the happiest amongst *wits*, in any *faculty*, can doe any thing so exactly, but that the *after-commers* may finde somwhat to adde to that, which before either *was* or *seemed* to be perfect in it selfe. So hath it fared with this Booke; for, since the first imprinting, it hath receiued *increase* and *beautie*, by diuers profitable *Additions*. Amongst the rest, Master *Iohn Mellis* skilfull in this *Art*, in foure seueral *Editions* of this *Booke*, hath more amply adorned it with his praise-worthy *labours*, to make it the more vsefull and commodious to the learners: which Master *Mellis* was a man naturally addicted to these *Studies* of

Arithmeticke, drawing of *Proportions*, *Maps*, *Cards*, *Buildings*, and *Plats*: who hauing first profited himselfe by his diligent studie and industrie in this *Science*, liuing in the Vniuersitie of *Cambridge*, did also by his industrie and labour, profit others, teaching in this Honourable *Citie* and elsewhere, who vsing this booke especially as his *Rule*, he made many both *Children* and *Seruants* to make such progresse in this *Science*, that the *Fathers* and *Masters* of those he instructed, found them both able in this *Art*, and very vsefull in this kinde vnto them.

And now Master *Robert Norton*, with further additions put to the end of this *Booke*, hath augmented it, ioyning thereto these particulars following.

1 The *Art* and application of Decimall *Arithmeticke*.
2 A Table of *Board* and *Timber* measure, exactly calculated, and the vse thereof.
3 The extracting and Demonstration of the *Square* and *Cubicke roots*.
4 Certaine *Questions* and *Tables* for *Interest*, after 10 for 100.

Lastly, the valuation of *Coynes* as they be this present yeere 1614. Truly and plainely performed for the better profit of those that take pleasure, and desire increase of knowledge in this excellent *Art*. And for the further helpe of memory there are added *Rules* and *Tables* of *Brewity* and *Tractise* brought into farre briefer *Methode*, then hitherto at any time hath beene published in our English tongue; which things (as I hope) will be of much *vse* and *commoditie* to my *Countrymen*.

All these therefore with the benefit of this last reuising, I commend to your Worships Patronage, hoping that out of your courteous disposition you will pardon my boldnesse and accept of this knowledgement of my loue, by which I am tied in all respective dueties to your worthinesse, so euer remayning

Your VVorships to be commanded.

R. I.

TO THE MOST mightie Prince, *Edward* the fixth, by the grace of God, King of England, France, *and Ireland, &c.*

ALthough (moſt mighty and Excellent Prince) man hath a wonderful prerogatiue ouer all creatures, euen in his ſhape and outward appearance: yet by the eſpeciall perfection of reaſon, his approches to God himſelfe are made more nearer, and this both Scripture, and all Philoſophicall Treatiſes haue many waies enlarged: ſo that, whoſoeuer laboured moſt to conſider the infinite worke of the minde was enforced to confeſſe, that man could hardly know himſelfe, whereupon aroſe that ſaying: *Magnum miraculum homo: maximum vero ſapiens.*

Who lookes on man, a miracle muſt ſee,
But if a wiſe man, none could greater be.

This brought about that wonderment at the compoſure of mans head, and thoſe infinite diſcourſes of Wit, Memory, Imagination, and ſuch powerfull faculties of the braine, reſem-

bling a rich treasure enclosed in a rich Casket, and yet so vnualuabled for their inestimable prise, that many Philosophers haue bin staggered concerning their owne discription of mans spirit, or if you will; the vnderstanding soule: as *Aristotle de anima*, *Galen in Historia Philosophica*: *Plutarch de Philosophorum placitis*, and *Eusebius* in his fifteenth booke *de Euangelice παρασκευης*: amongst whom, though *Galen* was called a miracle in nature, for his excellent wit and iudgement: yet in searching and exploring the nature and faculties of mans soule, and vnderstanding, he confesseth himselfe ignorant, and proclaimeth it presumption to attempt it. But if we shall a little dilate the matter, howeuer ignorance is enemy to knowledge, what greater proofe of mans excellency can there be, then the feare and obedience by the power of God, of all vnreasonable creatures vnto man; yea. amongst men themselues the base and barbarous people had euer a reuerence and respect toward others of deeper reach, and extraordinary wisedome: Whereupon it came to passe, that the rudenesse of the first age was like a Chaos reducted to vniformitie by the wisdome and gouernment of worthy men, as *Cicero* in his first booke *de Inuentione Rhetorica* thus enlargeth. The time was (saith he) that men wandred abroad in fields vp and downe like beasts, and vsed no better order in feeding, then they: so that by reasons rule they wrought nothing, but most of their doings

doings they atchieued by force: at this time was no iust regard of Religion toward God, there nor dutie to man, no not in the right vse of Marriage, nor did any man know his owne children from other, no man receiued benefit from the iustice of Law: so that through error and ignorance, wilfull lust, like a blinde and headie ruler, abused bodily strength, as a most mortall minister, for the satisfying his desire: Till at last, a certayne man amongst them, of great power and wisedome, considered, that in the mindes of men were both apt instruments, and great occasion to the due accomplishment of most waightie affaires, if they could apply them to vse, and by teaching of rules, frame them to a better trade. This man by perswasion of reason, gathered into one place the people, that were wandring about the fields, & lay lurking in wilde Cotages & Woods, and bringing them into one common societie, did trade them to all such things, as eyther were profitable or honest, although not without repining at the first, because they had not beene so accustomed before: yet at length, by reason and perswasion, they obeyed him more diligently, and so of wilde and cruell, they were reduced to gentlenesse and courtesie.

Thus did *Tully* shew the efficacy of reason: so that we must needs incurre all reproach, in a time of knowledge to be lesse attentiue, and durifull to wisedome and Art, then they were: vnlesse we are so presumptuous to imagine, that

euery man is so sufficiently wise, that he needeth not the instruction of other, wherein self-loue and opinion maketh vs againe more fooles, and lesse excusable, then those barbarous nations dwelling vnder the cloud of ignorance: therefore let euery man humble himselfe to a greater knowledge, and confesse with *Socrates*: *Hoc vnum scio, me nihil scire*: I know this one thing, that I know nothing, whether he meane comparatiue in respect of that he knowes not, or effectiue according to the perfection of knowledge: wherein still some one man had a prerogatiue ouer other; as *Moses* amongst the *Ægyptians* and *Iewes*; *Druis* in the West parts of *Europe*: *Xamolxus* amongst the *Gothes*: *Licurgus* amongst the *Lacedemonians*: *Solon* the *Athenians*, *Zeleucus* the *Locrians*, *Mulmutius Dunwallo* amongst the *Britons*, whose lawes and wisedome continued in force, till eyther better reason altred them, or powerfull authority controlled: but the best course that euer they took, was the drawing out at length the principles of learning and Art, and tying men to some speciall obseruations therein, as *Ouid* noteth:

Pectora mollescunt, asperitasq; fugit.
Stout stomacks doe wax milde,
And fiercenesse is exilde.

Whereupon as *Berrosius* doth testifie, *Sarron* that was the third King ouer the West, erected Schooles of liberall sciences; the fame and vse whereof continued two thousand yeeres, and *Diodorus Siculus* mentioneth, that the wise

men

men of the *Gothes* were therupon called *Saxonides*. Amongſt theſe Arts, Grammer, Logick, and Rhetoricke ſet the tongue on worke, and gaue the ſpeech ſuch a luſtre, that many admirable actions haue beene effected meerely from an Oratory of diſſwaſion & perſwaſion: Geometry and Arithmeticke ſet the hand on worke, whereby the partition of land, the orderly vſe of weights and meaſures, and the properties of diſtributiue and communicatiue iuſtice haue beene enlarged: Aſtronomy ſet the minde on worke, and taught it the rauiſhment of contemplation euen of God himſelfe: but becauſe wee haue thoſe diuine leſſons of Scripture to tie vs to more fearefull & religious obſeruations, then the heathen Philoſophers euer reached vnto; I leaue you to the ſearching of that infallible truth, that ſhall inſtruct you in the abſolute knowledge of God indeed. Muſick ſet the ſenſes on worke, but how? both to make religion more pleaſing, & to excite vs to thirſt after glorious deſignes & an honourable fame, by ſinging Encomions for vertue, and diſgraces for degenerating conditions: whereupon the Muſitions of thoſe times liued in reputation, as you ſtil perceiue by the Bardes of Wales & Ireland. And as theſe Sciences did increaſe, ſo did vertue flouriſh therby: again, as they were ſubiect to vilipending and diminution, vertue loſt both honour and eſtimation: but becauſe my aime onely tended toward the excellency of *Arithmetick*, I leaue the reſt to more worthie diſ-

courſes

courſes, and thus proſecute my firſt purpoſe, that *Arithmeticke* was euer ſo neceſſary in all ages and Common-wealthes, that by it onely iuſt partitions of land were made, the quietneſſe of buying and ſelling contracted, all aſſiſes, weights, and meaſures deuiſed, reckonings and accounts proportioned, and the iuſtification
16 of *meum & tuum* confirmed: yea, as *Ariſtotle* in his Ethicks doth declare, by it were orders of iuſtice limited, & the degrees of eſtates in the Common-wealth eſtabliſhed, and although that particular may be called *Geometricall*: yet doth it appertayne to the Art of *Arithmeticke*: wherefore I may well conclude, that ſeeing it is ſo many waies needfull to the planting of a Comnn-weath, it muſt needs be as neceſſary to the preſeruation of it, for by the ſame meanes is any Common-wealth continued, by which it was erected and eſtabliſhed. And if I ſhall in ſmal matters in appearance, but indeed very waighty, put one example or two, what ſhall we ſay for the Statutes of this Realme, which be the only ſtay of good order in maner now? As touching the meaſuring of ground by length and breadth, there is a good & ancient Statute made by the Art of Arithmeticke, and now it ſhall be to little vſe, if by the ſame Art it be not practiſed and tried,

For the Aſſiſe of bread and drinke, the two moſt common and moſt neceſſary things for the ſuſtentation of man, there was a goodly ordinance in the Law made, which by ignorance

rance hath ſo growen out of knowledg and vſe, that few men do vnderſtand it, and therfore the Statute-bookes are wonderfully corrupted, and the Commons are cruelly oppreſſed: notwithſtanding ſome men haue written, that it is too doubtfull a matter to execute thoſe Aſſiſes by thoſe Statutes, by reaſon they depend of the ſtandard of the Coyne, which is much changed from the ſtate of that time, when thoſe Statutes were made. This ſhall euery man read that liſteth, in the abridgement of ſtatutes, in the title of weights & meaſures, in the ſeuenth number of the Engliſh booke, where he ſhould haue tranſlated a good ordinance, which is ſet forth in the French booke: but no maruell if the Abridgement doth omit it, ſeeing the great book of ſtatutes doth omit the ſame ſtatute, as it hath done diuers other very good lawes. And this is the fruit of ignorance, to reiect all that it vnderſtandeth not, although they vſe ſome clokes for it: but ſuch clokes, as being allowed, might ſerue to repel all good lawes: which God forbid.

Againe, there is an ancient order for Aſſiſe of fire-wood and coales, which was renued not many yeeres paſt, and now how auarice and ignorance doth canuas the ſtatute, it is too pitifull to talke of, and more miſerable to feele.

Furthermore, for the ſtatute of Coynage, and the ſtanderd thereof, if the people vnderſtood rightly the Statute, they would not gather an excuſe for their folly thereby: but, as I ſaid, theſe Statutes by wiſedome, and good knowledge

ledge of Arithmeticke, were made, and by the ſame muſt be continued, And let ignorance no more meddle with the vſe of them, then it did with the making of them. O in how miſerable caſe is that Realme, where the Interpreters of the law, are deſtitute of all good Sciences, which be the keyes of Lawes. How can they eyther make good Lawes, or maintaine them, that lacke true knowledge whereby to iudge them? And happy is that Realme, where the Prince himſelfe is deſirous to vnderſtand equity in all Lawes. Therefore moſt happy are we the ſubiects of your Maieſtie, which may ſee in your highneſſe, ſuch knowledge of diuers Arts, as haue not beene ſeene in any Prince of ſuch yeeres; whereby we are inforced to conceiue this hope: that he which in thoſe yeeres ſeeketh knowledge, when knowledge is leaſt eſteemed, muſt needs when he commeth to more mature yeeres be a moſt prudent Prince and iuſt Gouernour, and a right Iudge, not onely of his ſubiects commonly, but alſo of the miniſters of his Lawes, yea, and of the Lawes themſelues: and to be able to conceiue the true equity & exact vnderſtanding of all his lawes & Statutes, to the comfort of his good ſubiects, and the confuſion and reproch of them which doe labour to obſcure or peruert the equitie of the ſame Lawes and Statutes. How ſome of thoſe Statutes may be applied to vſe, as well in our time, as in any other time, I haue particularly declared in this booke, and ſome other I

haue

haue omitted for iuſt conſiderations, till I may offer them firſt vnto your Maieſtie, to weigh them, as to your highnes ſhall ſeeme good: for many things in them are not to be publiſhed without your Highnes knowledge and approbation, namely, becauſe in them is declared all the rates of all Oyles, for all Standards from one ounce vpward, with other myſteries of Mint-matters, and alſo moſt part of the varieties of Coynes that haue beene currant in this your Maieſties Realme by the ſpace of ſix hundred yeeres laſt paſt, & many of them that were currant in the time that the Romans ruled here.

All which, with the ancient deſcription of England and Ireland, and my ſimple cenſure of the ſame, I haue almoſt compleated to be exhibited to your Highneſſe.

In the meane ſeaſon, moſt humbly beſeeching your Maieſtie to accept this ſimple Treatiſe, not worthy to be preſented to ſo high a Prince, but that my lowly requeſt to your Maieſtie is, that this, amongſt other of my Books may paſſe vnder the protection of your Highneſſe, whom I beſeech God moſt earneſtly and daily, according to my duty, to aduance in all honour and princely Regality, and to increaſe in all knowledge, iuſtice, and godly policie.

Amen.

Your Maieſties moſt obedient Subiect and Seruant

ROBERT RECORD.

To the louing Reader, the Preface of Master ROBERT RECORD.

Wen I consider the pregnancy of naturall wit in our English-men, *and the ability of their bodies, to vndertake any enterprise, I am amased euen with griefe of minde, at the pancitie of worthie Aduenturers, and famous Scholars renowmed for admirable atchiewements: when other Countries haue yet exposed them as Lampes to lighten the world in obscuritie, nor can I yeeld any other reason, then our filthy and obsceane addiction to vanity and pleasures, or disestimation of learning and Art, eyther for that it is laborious, or vnprofitable to wanton, and couetous humours: yet can I not exclude all from noble practises and studious endeauours, but confesse there are some worthy men amongst vs, who to their insupportable charge haue laboured to make learning flourish in our Commun-wealth, onely as the maine hinderance of such proceedings, to the griefe of all worthy spirits, they are eyther derided for their folly, or vilipended for humiliation. But I make no doubt but that vertue hath that power in it selfe to inuse vs, as well as others with her heauenly influence, so that in time* England *shall flourish, with as learned men as other nations. Therefore (gentle Reader) I wish all to meditate better on the same, and when you know any so blessed from God, and so happy in the world that endeuoureth the studie of good Arts, and laboureth the benefit of the Kingdome, and practiseth the suppliment of his Countrey-men, to esteeme him accordingly, and suffer him not to groane vnder the burthen of sorrow and complaint, to see you careleſſe of your owne goods. And although my selfe am farre vnworthy to be ranked in the*

seat

ſeat of learned and iudicious men, yet will I be bold to ſay, that the loue to my Country, and deſire to enlarge the bounds of learning, hath excited me to this painefull induſtrie, wherin how euer the deeper men of knowledge may paſſe it ouer with ſlightneſſe, I am ſure many thouſands of the ignorant and meaner ſort ſhall receiue both profit, inſtruction, pleaſure, and varietie.

As for the praiſe and commendation of the Art it ſelfe, I referre it to the iudgement of the Reader, & neceſsity of all people and vocations, ſpeaking in one word with the old prouerbe, The good Wine needs no Iuie buſh: but yet if there be any ſo Cinicall and ſequeſtred, that ſetteth lightly by the witty deuice and knowledge of Numbring, let him beſides many infinite precepts, vnderſtand this one example, that whereas the Poet hath exemplified, how many bruite beaſts excell man in ſeuerall properties, man hath a ſpeciall difference ouer them all in this Art of Numeration, whereby his reaſon and ſingular wiſedome is apparant: For ſaith he,

The Foxe in craftie wit exceedeth moſt men,
A Dogge in ſmelling hath no man his Peere,
To foreſight of weather if you looke then,
Many beaſts excell man, this is cleare.
The wittineſſe of Elephants doth letters attayne,
But what cunning doth there in the Bee remayne?
The Emmet fore-ſeeing the hardneſſe of Winter,
Prouideth victuals in the time of Summer.
The Nightingale, the Linet, the Thruſh, the Larke,
In muſicall harmony paſſe many a Clarke.
The Hedgehog of Aſtronomy ſeemeth to know,
And ſtoppeth the caue where the winde doth blow.
The Spider in weauing ſuch Art doth ſhow,
No man can him mend or follow I trow.
When a houſe doth fall, the Mice full quicke
Flee thence before, Can man doe the like?

But when you come to Number, and make vſe of Arithmetick, neuer was Beaſt found ſo cunning, that could know or diſcerne one thing from many, or many from a few, as in a familiar example of a Bitch or a Henne may appeare, from whom if you

take

take their young ones, though by naturall instinct, they may seeme to resist you: yet if you leaue them some, they neuer misse the rest. Nay if they haue but one they are appeased, wherein by taking it away, and bringing it againe, you shall see some difference.

But to returne to my first purpose, I haue written this booke in the forme of a Dialogue, wherein the Scholar may aske euery doubt orderly, and the Master answere to his question plainly: and although captious wit may except against the method I haue vsed, and obiect that many excellent bookes haue beene alreadie published of Arithmeticke, *whereby this may seeme superfluous, yet when they well consider, how simply I went to worke, meerely for the instruction of the ignorant, and plainely for the loue to learning and my Country, they cannot chuse but accept of my good will: and truly consider how neuer booke in English was published in this kinde so full of examples, rules, and easie additions: and although I haue as it were shaded it vnder the couert of his Princely maiesty, yet such is his clemency, and admirable loue to all good practises, with the choysest pregnancy to learning that euer I knew, or read of, that I am sure he will be content, yea much desirous to vnderstand, how his louing and painefull subiects shall make vse of it, and apply the same to their profit, which if they doe, and that I perceiue the same, it shall induce me to set forth those further instructions concerning* Geometry *and* Cosmography, *which I haue already promised, and am sure, hath not hitherto in our English tongue beene published. And thus I commit you all to the true fountaine of perfect numbers, which wrought the whole World by Number and Measure, he is* Trinitie in Vnitie *and* Vnitie in Trinitie, *to whom be all praise, honour, and glory.*

Amen.

Heere followeth a Table of all the Contents of this Booke.

The Contents of the first Dialogue, containing the Declaration of the profit of Arithmeticke.

- Numeration, with an easie & large table.
- Addition. } with diuers examples,
- Subtraction. } and all their proues,
- Multiplication. } and some new formes
- Diuision. } of worching, &c.
- Reduction with diuers Declarations of coynes, Waights and Measures, of sundry formes newly added, with a new Table, containing most part of the Gold Coynes throughout Christendome, with the true waight and valuation of them now in currant money English, &c.

Progression both Arithmeticall and Geometricall, with diuers sundrie questions touching the same.

The Golden Rule of three: and the Backer rule of three: with diuers questions thereunto belonging, newly added and augmented.

The double Rule of Proportion.

The rule of three, composed of fiue numbers.

The rule of Fellowship both with time and without time.

Unto all these are added their proofes.

The second Dialogue containeth.

The first fiue kindes of Arithmeticke, wrought by Counters.

The common kindes of casting Counters, after the Merchants fashion, and Auditors also.

The Contents of the second part, touching Fractions.

What a fraction is.
Numeration in fractions
The order of working { Multiplication { fractions
Diuision. { fractions
{ with diuers familiar questions proposed for the perfect vnderstanding, and proofe of eyther of them.

Reduction of {
diuers fractions into one denomination in the varieties.
Fractions of Fractions.
Improper Fractions.
Fractions to the smallest denomination, with easie rules how to conuert them thereunto.
Fractions in other parts of things, with a table demonstratiue of their proportions.
Fraction, and how it may be turned into any other Fraction, or into what denomination you list.

Againe,

Againe of { Multiplication, Duplation, Diuision, Mediation, Addition, Subtraction } in Fractions.

The Golden Rule, with diuers questions, and their proues.

The Backer rule.

A question of Loane.

The Statute of Assise of Bread and Ale recognised and applied to this time, with new tables thereunto annexed.

The Statute of measuring of ground, with a table thereof, faithfully calculated and corrected.

Questions of societie, with the reasons of the rules and proues of their works.

To finde three numbers in any proportion.

The Rule of Alligation, with diuers questions and the proues of their workes, with many varieties of such solutions.

The Rule of Falshood, or false Position, with diuers questions and their proues.

The Contents of the third Addition to this Booke.

The first Chapter intreateth of Rules of Brauitie and Practise, after a briefer Methode than hitherto hath beene published in

the English tongue.

The second Chapter entreateth of briefe Reduction of diuers Measures, as elles, yards, braces, &c. by rules of Practise.

The third Chapter entreateth of the Rule of three in broken numbers, after the trade of Merchants, something differing from Master Records order, which is comprehended in Rules.

The fourth Chapter entreateth of losse and gaine in the trade of Merchandise.

The fift chapter entreateth of losse and gaine in the trade of Merchandise vpon time, &c. with necessary questions therein wrought by the double Rule of three, or the Rule of three composed.

The first chapter entreateth of Rules of payment, and one of the necessariest rules that appertayneth to buying and selling, &c.

The seauenth Chapter entreateth of buying and selling in the trade of Merchandise, wherein is taken part of readie money, and diuers dates of paiment giuen for the rest, and what is wonne or lost in the one hundred pound forbearance for the twelue moneths, &c.

The eight Chapter entreateth of rate and allowances in the trade of Merchandise, sold by waight, and of their losses and gaines therein, &c.

The ninth chapter entreateth of lengths and breadths

breadths of Arras, and other clothes, with diuers questions thereunto.

The tenth Chapter entreateth of reducing of Pawnes of Geanes into English yards.

The eleuenth Chapter entreateth of Rules of Loan and interest with diuers questions incident thereunto.

The twelfth Chapter entreateth of the making of Factors.

The thirteenth Chapter entreateth of Rules of Barter or Exchange of Merchandise, wherein is taken part ware, and part readie money, with their proofes, and diuers other necessary questions thereunto belonging.

The fourteenth Chapter entreateth of Exchanging of money from one place to another, with diuers necessary questions incident thereunto.

The fifteenth Chapter entreateth of fiue sundrie formes of practises for Reduction of English, Flemish, and French money, and how each of them may easily be brought to other money sterling.

The sixteenth Chapter containeth a briefe note of the ordinary Coynes of most places of Christendome for traffick, and the manner of their exchanging from one Citie or Towne to another, which knowne, the Italians call Parie: whereby they finde the gaine or losse vpon the exchange.

The seauenteenth Chapter contayneth also a declaration of the diuersity of the waights and measures of most places of Christendome for traffiche, proportionated in equalitie one to another, as also vnto our English measure and waight, whereby the ingenious practitioner may easily reduce the waight and measure of each Country into another.

The eightenth Chapter intreateth of diuers sports and pastimes, done by Number.

A Collection of such Tables as are contayned in this Treatise.

A Large Table of Numeration.

A Table of Multiplication.

A Table of all the Gold and siluer Coynes in this Realme, with the most vsuall Gold Coynes throughout Christendome, with their seuerall waights of pence and grains, and what they are worth in current money English.

Certayne Tables or notes of the contents of Ale, Beere, wine, Butter, Sope, Salmon, Eeles, &c. both what such vessels ought to contayne

containe by the Statute, and what those vessels emptie ought to waigh.

A Table of the quantitie of drie measures, as Pecks, Bushels, Quarters, Weyes, &c.

A Table of the proportion of measures, touching length and breadth, to wit, from the inch to the foote, and so to the yard, the elle, with their parts, the Perch, the rod the furlong, the mile, &c.

A Table made by Progression Arithmetical, which containeth a double Table of Multiplication.

A Table or Demonstration of a figure or measure for the perfect understanding of Fractions.

A Table of the contents of the Statute, for the Assise of the waight of Bread. From one shilling the quarter, to 20 shillings, faithfully corrected.

A necessary Table of the Statute of measuring of ground, upon the breadth given, what length it ought to containe: faithfully corrected according to the equity of the Statute, wherein the Authour declareth, how necessary this worthy Art of Arithmeticke is unto Gentlemen, students of the Law, and such other, as are desirous of infallible truth.

Three Tables of the ready reducing of English, French and Flemish money each into others common currant monies.

A briefe Table or collection of the common and vsuall monies of most places of Christendome for trafficke, the manner of their payments or exchanging from one Citie or Towne to another: right necessary for Merchants, and other Occupiers, Trauellers, &c.

Tables of the Waights, Measures, and Customes of most places of Europe for Traffick, whereby, through the ayde of the Rule of three, the ingenious may easily reduce our measure to a perfect valuation of other Countries Measure or Waight, and likewise theirs to ours.

Lastly, a Table demonstrating the true solution of three diuers things hidden, of three seuerall persons in pastime.

Note that this △ Character representeth the Crowne by exchange.

Item, whereas I haue augmented diuers necessary questions and rules vnto the Author, the beginning of those additions are marked with this ¶ note. And where they end, with this * note.

The

The Contents of the new Additions.

1 THE Art and Application of Decimall Arithmeticke.

2 The Table of Board and Timber-measure, exactly calculated, and the vse thereof.

3 The extracting and Demonstrations of the Square & Cubicke rootes.

4 Certayne Questions and Tables for interest after 10 for 100.

5 The valuation of Coynes, this present yeare. 1614.

FINIS.

Before the Introduction of Arithmeticke, it were very good to haue some vnderstanding and knowledge of these Figures and Notes.

j	1	one.	rr.	20	twentie.
ij	2	two	lr.	40	fortie.
iij	3	three.	l.	50	fiftie.
iiij	4	foure	lr.	60	sixtie.
v	5	fiue	lrr.	70	seuentie.
vj	6	sixe	rc.	90	ninetie.
vij	7	seauen.	C.	100	a hundred.
viij	8	eight.	CC.	200	2 hundred.
ix	9	nine.	D.	500	5 hundred.
x	10	ten.	DC.	600	6 hundred.
xj	11	eleuen.	M.	1000	a thousand.
xij	12	twelue.	MD.	1500	a thou. 5. hun.
&c.	&c.	&c.	&c.	&c.	&c.

112 l make C. waight
56 halfe a C. waight.
28 a quarter of an hundred waight.
16 ounces make a pound
A pound Troy which is the standard of England, whereby is onely waged gold, siluer, & bread, is 12 ounces.
12 inches make a foote

3 foote make a yard.
The yard is parted into 4 quarters.
And each quarter into 4 nailes.
5 quarters of a yard make an elle. Which elle is also parted into 4 quarters.
And each quarter into 4 nailes.

A Dialogue betweene the Maſter and the Scholar: teaching the ART and vſe of Arithmeticke with Pen.

The Scholar ſpeaketh.

SIr, ſuch is your authoritie in mine eſtimation, that I am content to conſent to your ſaying, and to receiue it as truth, though I ſée none other reaſon that doth lead me thereunto: whereas elſe in mine owne conceit it appeareth but vaine, to beſtow any time priuately in learning of that thing, that euery childe may, and doth learne at all times and houres, when he doth any thing himſelfe alone, and much more when he taketh or reaſoneth with other.

Maſter. Loe, this is the faſhion and chance of all them that ſéeke to defend their blinde ignorance: that when they thinke they haue made ſtrong reaſon for themſelues, then haue the proued quite contrarie. For if numbring be ſo common (as you grant it to be) that no man can doe any thing alone, and much leſſe talke or bargaine with other, but

he

he shall still haue to doe with number: this proueth not number to be contemptible and vile, but rather right excellent and of high reputation, sith it is the ground of all mens affaires, so that without it, no tale can be told, no communication without it can be continued, no bargaining without it can duely be ended, or no businesse that man hath, iustly completed. These commodities if there were none other, are sufficient to approue the worthinesse of number. But there are other innumerable, farre passing all these, which declare number to exceed all praise. Wherefore in all great workes are Clerkes so much desired? Wherefore are Audites so richly fed? what causeth Geometricians so highly to be enhaunced? Why are Astronomers so greatly aduanced? because that by number such things they finde, which else would farre excell mans minde.

Scholar. Verily sir, if it be so, that these men by numbring, their cunning doe attaine, at whose great workes most men doe wonder, then I see well I was much deceiued, and numbring is a more cunning thing than I take it to be.

Master. If number were so vile a thing as you did esteeme it, then neede it not to be vsed so much in mens communication. Exclude Number, and answere to this question: How many yeares old are you?

Scholar.

Scholar. Pum.

Master. How many dayes in a weeke? How many weekes in a yeere? What lands hath your father? How many men doth he keepe? How long is it since you came from him to me?

Scholar. Pum.

Master. So that if Number want, you answere all by Pummes: How many miles to London?

Scholar. A poake full of Plummes.

Master. Why, thus you may see, what rule number beareth, and that if number be lacking, it maketh men dumbe, so that to most questions, they must answere Pum.

Scholar. This is the cause (sir) that I judged it so vile, because it is so common in talking euery while: For plenty is not daintie, as the common saying is.

Master. No, nor store is no sore: perceiue you this? The more common that the thing is being needefully required, the better is the thing, and the more to be desired. But in numbring, as some of it is light and plaine, so the most part is difficult, and not easie to attayne. The easier part serueth all men in common, and the other part requireth some learning. Wherefore as without numbring a man can doe almost nothing, so with the helpe of it you may attayne to all things.

Scholar. Yea sir: why? then it were best to

learne

learne the Art of numbring, first of all other learning, and then a man nede learne no more, if all other come with it.

Master. Nay, not so: but if it be first learned, then shall a man be able (I meane) to learne, perceiue, & attaine to other Sciences, which without it, he should neuer get.

Scholar. I perceiue by your former words, that Astronomy and Geometry depend much on the helpe of numbring: but that other Sciences, as Musicke, Physicke, Law, Grammar, and such like, haue any helpe of Arithmetike, I perceiue not.

Master I may perceiue your great Clerkelinesse by the ordering of your Sciences: but I will let that passe now, because it toucheth not the matter that I intend, and I will shew you how Arithmetick doth profit in all these, somewhat grossely, according to your small vnderstanding, omitting other reasons more substantiall.

First, (as you reckon them) Musicke hath not onely great helpe of Arithmeticke, but is made, and hath his perfectnesse of it: for all Musicke standeth by number and proportion.

And in Physike, beside the calculation of criticall dayes, with other things, which I omit, how can any man iudge the pulse rightly, that is ignorant of the proportion of Numbers?

And as for the Law, it is playne, that the man

man that is ignorant of Arithmeticke, is neyther mete to be a Iudge, neither an Aduocate, nor yet a Proctor. For how can hee well vnderstand another mans cause, appertaining to distribution of goods, or other debts, or of sums of money, if hee be ignorant of Arithmeticke? This oftentimes causeth right to be hindered, when the iudge eyther delighteth not to heare of a matter that he perteineth not, or cannot iudge it for lacke of vnderstanding. This commeth by ignorance of Arithmeticke.

Now, as for Grammer, me thinketh you should not doubt in what it needeth number, sith you haue learned that Nounes of all sorts, Pronounes, Verbes and Participles are distinct diuerslye by numbers: besides the varieties of Nounes of number, and Aduerbs. And if you take away number from Grammer, then is all the quantitie of Syllables lost. And many other wayes doth number helpe Grammar. Whereby were all kindes of meters found and made? Was it not by Numbers?

But how needefull Arithmeticke is to all parts of Philosophie, they may some se, that doe reade eyther Aristotle, Plato, or any other Philosophers Writings. For all their examples almost, and their probations, depend of Arithmeticke. It is the saying of Aristotle, that he that is ignorant of Arithmeticke, is mete for no Science. And Plato his Master

wrote

wrote a little sentence ouer his Schole-house dore, Let none enter in hither (quoth he) that is ignorant of Geometrie. Saying he would haue all his Scholars expert in Geometrie, much rather he would the same in Arithmeticke, without which Geometry cannot stand.

And how needfull Arithmeticke is to Diuinitie, it appeareth, seeing so many Doctors gather so great mysteries out of number, and so much doe write of it. And if I should goe about to write all the commodities of Arithmeticke in ciuill acts, as in gouernance of common-weales in time of peace, and in due prouision and order of armes, in time of warre. For numbring of the host, summing of their wages, prouisions of victuals, viewing of Artillerie, with other Armour: Beside the cunningest point of all, for casting of ground, for encamping of men, with such other like. And how many waies also Arithmeticke is conducible for all priuate weales, of Lords and all possessioners, of Merchants, and all other occupiers, and generally for all estates of men, besides Auditors, Treasurers, Receiuers, Stewards, Bailiffes, and such like, whose offices without Arithmetick are nothing: If I should (I say) particularly repeat all such commodities of the noble Science of Arithmeticke, it were enough to make a very great booke.

Scholar. No, no sir, you shall not need: For I doubt not, but this that you haue said, were enough

enough to perſwade any man to thinke this Art to be right excellent and good, and ſo neceſſary for man, that (as I thinke now) ſo much as a man lacketh of it, ſo much he lacketh of his ſenſe and wit.

Maſter. What are you ſo farre changed ſinte, by hearing theſe few commodities in generall? By likelihood you would be farre changed if you knew all the commodities particular.

Scholar. I beſeech you Sir, reſerue thoſe commodities that reſt yet behinde vnto their place more conuenient: and if yee will be ſo good as to vtter at this time this excellent treaſure, ſo that I may be ſomewhat inriched thereby, if euer I ſhall be able, I will requite your paine.

Maſter. I am very glad of your requeſt, and will doe it ſpeedily, ſith that to learne it you be ſo readie.

Scholar. And I to your authoritie my wit doe ſubdue, whatſoeuer you ſay, I take it for true.

Maſter. That is too much, and more for no man to be beleeued in all things, without ſhewing of reaſon. Though I might of my Scholar ſome credence require, yet except I ſhew reaſon, I doe it not deſire. But now ſith you are ſo earneſtly ſet this Art to attaine, beſt it is to omit no time, leaſt ſome other paſſion coole this great heate, and then

you leaue off before you se the end.

Scholar. Though many there be so vnconstant of minde, that flitter and turne with euery winde, which often begin, and neuer come to the end, I am none of this sort, as I trust you partly know. For by my good will what I once begin, till I haue it fully ended, I would neuer blin.

Master. So haue I found you hitherto indeed, and I trust you will increase rather then goe backe. For better it were neuer to assay, then to shrinke and flye in the mid-way: But I trust you will not so doe, therefore tell me briefly: What call you the Science that you desire so greatly?

Scholar. Why Sir? you know.

Master. That maketh no matter, I would heare whether you know, and therefore I aske you. For great rebuke it were to haue studied a Science, and yet cannot tell how it is named.

Scholar. Some call it Arsemetike, & some Augrime.

Master. And what doe these names betoken?

Scholar. That if it please you, of you would I learne.

Master. Both names are corruptly written. Arsemetike for Arithmeticke, as the Greekes call it, and Augrim for Algorisme, as the Arabians found it: which both betoken

the

the Science of Numbring. For Arithmos in Greeke is called Number: and of it commeth Arithmeticke, the Art of Numbring. So that Arithmeticke is a Science or Art teaching the manner and vse of Numbring. This Art may be wrought diuerſly, with Penne or with Counters. But I will firſt ſhew you the working with the Penne, and then the other in order.

Scholar. This I will remember. But how many things are to be learned to attaine this Art fully?

Maſter. There are reckoned commonly ſeauen parts or workes of it.

Numeration, Addition, Subtraction, Multiplication, Diuiſion, Progreſſion, and Extraction of rootes: to theſe ſame men adde Duplation, Triplation, and Mediation. But as for theſe three laſt, they are contayned vnder the other ſeauen. For Duplation and Triplation are contayned vnder Multiplication, as it ſhall appeare in their place. And Mediation is contayned vnder Diuiſion, as I will declare in his place alſo.

Scholar. Yet then there remaines the firſt ſeauen kindes of Numbring.

Maſter. So there doth: Howbeit if I ſhall ſpeak exactly of the parts of Numbring, I muſt make but fiue of them: For Progreſſion is a compound operation of Addition, Multiplication, and Diuiſion. And ſo is the Extraction

traction of rootes. But it is no harme to name them as kindes seuerall, seeing they appeare to haue some seuerall working. For it forceth not so much to contend for the number of them as for the due knowledge and practising of them.

Scholar. Then you will that I shall name them as seauen kindes distinct. But now I desire you to instruct me in the vse of each of them.

Master. So I will, but it must be done in order: for you may not learne the last as soone as the first, but you must learne them in that order, as I did rehearse them, if you will learne them speedily and well.

Scholar. Euen as you please. Then to begin, Numeration is the first in order: what shall I doe with it?

Master. First, you must know what the thing is, and then after learne the vse of the same.

Numeration.

Numeration is that Arithmeticall skill, whereby we may duely value, expresse and reade any number or summe propounded: or else in apt figures and places set downe any number knowne or named.

Scholar.

Scholar. Why then me thinketh you put a difference betweene the value and the Figures.

Master. Yea, so doe I: For the value is one thing, and Figures are another thing: and that commeth partly by the diuersitie of Figures, but chiefely of the places wherein they be set. 2

Scholar. Then I must know here three things: the Name, the Figure and the Place. 3

Master. Euen so: but yet adde Order to them as the fourth. And first marke, that there are but ten Figures that are vsed in Arithmeticke: and of those ten, one doth signifie nothing, which is made like an o. and is priuately called a Cypher, though all the other sometime be likewise named. The other nine are called signifying Figures, and be thus figured.

1 2 3 4 5 6 7 8 9.
And this is their value.
i. ij. iij. iiij. v. vi. vij. viij. ix.

But here you must marke, that euery Figure hath two values: One alwaies certaine, that it signifieth properly, which it hath of his forme: and the other vncertaine, which he taketh of his place. 4

A place is called the seate or rome that a Figure standeth in. And loke how many

figures are written in one summe, so many places hath that whole number. And that must be called the first place that is next to the ☞ right hand, and so reckoning by order towards the left hand, so that that place is last, that is next to the left hand As for example: If there stood before you five men in a row, side by side, and you should tell them as they stand in order, beginning with the man that were next to your right hand: then he that were next him should be called the second, and so forth to the furthest from your right hand, which is the first and the last.

Scholar. Sir, I perceive you well: so might I reckon Letters or any other thing. As if I should write eight Letters after this order, a,b,c,d,e,f,g,h, then must I say, h is the first, g the ij. f the iij. e the iiij. d the v. c the vi. b the vij. and a the viij.

Master. That is well done. And after the same sort use hereafter, that what I declare by one example doe you expresse by another: and so shall I perceive whether you understand it or no. And so passe over nothing, till you perceive it well, and be expert therein.

Scholar. Sir, I pray you how many of these places be there in all?

Master. There is no certaine number of them, but they are sometimes more and sometimes fewer, according to the summe that is expressed. For so many as the figures are, so

many

many are the places: and the laſt place is ſo called, not becauſe it is laſt of all other, but it is the laſt of that preſent ſumme, and it may be the middle place in another ſumme.

Scholar. Me ſæmeth I perceiue this very well, as touching the order of reckoning of the places: but as for the number of them, you ſay there is no certaintie. Now there reſteth to declare the value of the figures by the diuerſitie of places, which you called the value vncertaine.

Maſter. But firſt let mée heare whether you know perfectly the certaine value.

Scholar. Yes Sir, as you wrote them ſo I marked them.

Maſter. How write you then fiue?

Scholar. By this figure, 5.

Maſter. And how ſire?

Scholar. Thus, 6.

Maſter, Write theſe three numbers each by it ſelfe, as I ſpeake them, viz. iiij. iij.

Scholar. 7. 4. 3.

Maſter. How write you theſe foure other, ij. i. ix. viij?

Scholar. Thus (I trow) 2. 1. 6. 8.

Maſter. Nay, there you miſſe: looke on mine example againe.

Scholar. Sir, true it is, I was to blame, I take 6. for 9. but I will beware hereafter.

Maſter. Now then take héede, thoſe re-

taine value euery figure representeth when it is alone written without other figures ioyned to him. And also when it is in the first place, though many other doe follow: as for example: This figure 9. is ix. standing now alone.

Scholar. How is he alone, and standeth in the middle of so many Letters?

Master. The Letters are none of his fellowes. For if you were in France in the middle of a thousand Frenchmen, if there were no Englishman with you, you would reckon your selfe to be alone.

Scholar. So it is. Then 9. without moe figures of Arithmeticke, betokeneth ix. whatsoeuer other Letters be about it.

Master. Euen so, and so doth it, if it be in the first place ioyned with other, how many soeuer doe follow, as in this example, 3679. you see 9. in the first place, and doth betoken nine, as if it were alone.

Scholar. I perceiue that, and doth not 7. that standeth in the second place, betoken lxx. and 6. in the third place, betoken vi. and so 3. in the fourth place betoken three?

Master. Their places be as you haue said, but their values are not so. For as in the first place euery figure betokeneth his owne value certaine onely, so in the second place euery figure betokeneth his owne value certaine ten times: as in the example, 7. in the second

place

place is seauen times x. that is, lxx. And in the third place euery figure betokeneth his owne value an hundred times, so that 6. in that place betokeneth vi. C. And in the fourth place euery figure betokeneth his owne value a M. times, as in the aforesaid number. 3. in the 4. place standeth for 3. M. and in the fift place, euery figure standeth for his owne value x M. times, and in the sixt place C M. times, and in the seauenth, place a M M. times, and in the eight place, x M M. so that euery place exceedeth the former x. times.

Scholar. As thus: if I make this number at all aduentures, 91359684. here are eight places. In the first place is 4. and betokeneth but foure: in the second place is 8. and betokeneth ten times 8. that is 80. in the third place is 6. and betokeneth fiue hundred: in the fourth place 9. is nine thousand: and fiue in the fift place is x M. times 5. that is, fiftie M. So 1. in the sixt place is a C M. times 3. that is, C C C M. Then 1. in the seuenth place, a M M. and 9. in the eight place, ten thousand thousand times 9. that is, x M M. But now I cannot easily nor quickly read it in order.

Master. That shall you practise by this meanes. First, put a pricke ouer the fourth figure, and so ouer the seauenth. And (if you haue so many) ouer the tenth, thirteenth, fifteenth, and so forth. Still leauing two figures betweene each two prickes. And those two

6

comes betweene the pricke are called Ternaries.

Then begin at the laſt pricke, and ſee how many figures are betweene him and the end, which cannot paſſe three, reckoning himſelfe for one: then pronounce them as if they were written alone from the reſt, and at the end of their value, ſo many times thouſands, as your numbers hath prickes.

After that, come to the next three figures, and ſound them as if they were a part from the reſt, and adde to their value ſo many times thouſands, as there are pricks betweene them, and the firſt place of your whole number. And ſo doe by every other three figures following, if you haue moe. As in example, 913 59684. this was your number.

Put a pricke ouer 9. in the fourth place, and ouer 1. in the ſeauenth place, and then no more (for your places come not to tenne) as thus: 9'13'59684.

Now goe to the laſt pricke ouer 1. and take it and the figure 9. that followeth it, and value them alone.

Scholar. 91. that is, xci.

Maſter. So it is, then adde for the number of your prickes twice M.

Scholar. That is xci. thouſand thouſand.

Maſter. So it is. Then take the three other figures from one to the next pricke, and value them.

Scholar.

Scholar. 359. that is, CCC.lix.

Master. Now adde for the one pricke, that is betweene them and the first place, 9.

Scholar. CCC.lix. thousand.

Master. Then come to the other three Figures that remaine.

Scholar. 684. that is, vi C lxxxiiij.

Master. Now haue you valued all. And at the end of the last number you shall adde nothing, because there remaineth no pricke nor number after it: yet proue it in another number, as thus, 2308640891053 40.

Scholar. 2308640891053 40. I haue pricked them as you taught me: but I am in doubt, whether I haue done well or no, because of the ciphers: for I remember, you told me that they do signifie nothing, and therefore I doubt whether I should reckon them for a figure in setting of the prickes: and againe, I know not wherefore they serue.

Master. That will I tell you now. Indeede they are of no value themselues, but they serue to make vp number of places, & so make the figure following them to be in a further place, and therefore to signifie the more value, as in this example 90. the Cipher is of no value, but yet he occupieth the first place, and causeth 9. to be in the second place, and so to signifie ten times 9. that is 90. so that two Ciphers thrusteth the figure following them into the third place, and so forth.

Scholar.

Scholar. Then I perceiue in the example aboue, I haue pricked well enough: for though that Cypher that is pricked signifie nothing, yet must he haue the pricke, because he came in the 13. place. Then will I proue to number that sum. First, there is 2,0,9,9,9,9, and then followeth 864. 9.9.9. And what shal I now do? There is a Cypher in the third place and no figure after him, but they that I haue reckoned?

Master. He did serue for them that you haue already reckoned, to make them in a place further than they should be, if hee were away: and therefore now ye shall let him goe. And so doe alwayes when hee occupyeth that place next before any pricke, which is the last of that Ternary, and a Cipher in the last place doth nothing.

Scholar. Then shall I say but 89.9.9.

Master. So, but goe forth.

Scholar. 105. thousand. Now are all my prickes spent, and yet remaine 340. so that I must value them, CCC. xl. onely.

Master. Now can you recken after this sort: and remember, that euery such roome so parted, is called a Ternary or Trinitie: for you haue numbred or valued the summe most truely, and by the ayde of the prickes each Denomination is distinct most plainely.

Scholar. What call you Denomination?

Master.

Master. It is the last value or name added to any summe. As when I say, a hundred two and twentie pounds: Pounds is the Denomination. And likewise in saying 25. men, men is the Denomination, and so of other. But in this place (that I spake of before) the last number of every Ternary is the Denomination of it. As for the first Ternarie, the Denomination is unites, and of the second Ternarie, the Denomination is thousands: And of the third Ternary, thousand thousands or Millions: of the fourth, thousand thousand thousands, or thousand Millions: and so forth.

Scholar. And what shall I call the value of the three Figures that may be pronounced before the Denominator? as in saying: 203000000. that is, two hundred three millions. I perceive by your words, that millions is the Denomination: but what shall I call the CCIII. joyned before the millions?

Master. That is called the Numerator or valuer, and the whole summe that resulteth of them both, is called the summe, value or number.

Scholar. Now is there any thing else to be learned in Numeration: or else have I learned it fully?

Master. I might shew you here who were the first inventors of this Art, and the reason of all

all these things that I haue taught you, but that I will reserue till yee haue learned ouer all the practise of this Art, lest I should trouble you with ouer many things at the first.

But yet this must you marke, that there are three kindes of Numbers, one called Digits: another Articles: and the third mixt numbers.

A Digit is any number vnder ten, as this: 1.2.3.4.5.6.7.8.9.

And 10. with all other that may be diuided into ten parts iust, and nothing remaine, are called Articles, such as are 10. 20. 30. 40. 50. &c. 100. 200. &c. 1000. &c.

And that number is called Mixt, that containeth Articles, & at the least one Article, and a Digit; as 12. 16. 20. 21. 38. 107. 1005. and so forth: and for the more ease of vnderstanding and remembrance, marke this: The Digit number is neuer written with more than one figure, but the Article and the Mixt number are euer written with more than one figure. And thus they differ: that the Article hath euermore this Cypher o in the first place: and the Mixt number hath euer there some Digit.

Scholar. By these last words I perceiue it much better then I did before, and now (I thinke) I will neuer misse to know those three asunder.

Master. If you remember now all that I haue

haue saide, you haue learned sufficiently this first kinde of Arithmeticke, called Numeration. Howbeit, I will exhort you now to remember both this that I haue saide, and all that I shall say, and to exercise your selfe in the practise of it: For rules without practise, are but a light knowledge: and practise it is, that maketh men perfect and prompt in all things.

And as you haue learned to gather and expresse the value of a summe propounded, and set downe before you: so must you practise to marke, note or write downe with apt figures and in due places, any number onely named or recited to you, or of your selfe imagined: as for a proofe. Now note you, or write downe this summe, fiue thousand, two hundred, fifty and seauen.

Scholar. This troubleth me now, whether I should beginne at the first or at the last. For reason (me thinketh) should cause me to begin at the first: and yet if I write it as you speake it, I must begin at the last.

Master. When you know your places perfectly, you may begin where you list. But the more safe for your hand is to begin with the last, that is to say, as I did speake them. Yet for the more surette, a while you may beginne with the first, repeating my words backward thus: Seauen fiftie, two hundred, fiue thousand: or else sounding them all by their digit or

value,

value, as thus: seauen, fiue, two, fiue: for that way is easiest. But then must you looke well whether there be any Cypher in your summe, that he may be set in his place. As if your last value of your sum (as you call it) be aboue 9. then is there a Cipher in the first place. And if it be an hundred or aboue, then is there two Cyphers, one in the first place, and another in the second, and so forth.

But because this thing is such that cannot be set forth without many words, I thinke best here now at the end of Numeration to adde a Table easie and ready for the first exercise of it.

Loe this is the Table.

The left side or hand.

The right side or hand.

The names of Digits, values vncertaine, or values.

The denominations of the place or value vncertaine	Vnites.	Tennes.	Hundreds.	Thousands.	C. Thousands.	M. Thousands.	Millions.	C. of Millions.	M. of Millions.	B. of Millions.
Nine.	9	9	9	9	9	9	9	9	9	9
Eight.	8	8	8	8	8	8	8	8	8	8
Seuen.	7	7	7	7	7	7	7	7	7	7
Sixe.	6	6	6	6	6	6	6	6	6	6
Fiue.	5	5	5	5	5	5	5	5	5	5
Foure.	4	4	4	4	4	4	4	4	4	4
Three.	3	3	3	3	3	3	3	3	3	3
Two.	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1
Cipher.	0	0	0	0	0	0	0	0	0	0
The order of places.	First.	Second.	Third.	Fourth.	Fift.	Sixt.	Seauenth.	Eight.	Ninth.	Tenth.	Eleuenth.

This Table (as yee may see) hath eleuen places, and in each of them are set all the Digits, whose certaine value is written on the right hand of the Table, and the value vncertaine on the left hand, so that by this Table you may learne both how to expresse any number that you list, if that it exceede not

eleuen places) that is to say, ℔C. thousand millions, and so may you by the helpe of it, value all summes proposed vnder the said number.

For example: take the summe that I proposed before, which was fiue thousand, two hundred fifty and seauen. And if you will expresse it, take the first number (as I speake it) which is fiue ℙ. whose valuer or certaine value is v. and his vncertaine value or denomination is ℙ. First, you shall seeke at the right hand of the valuer 5. Then seeke along vnder the title of denomination toward the left hand till you finde thousands, and vnder it, right at the foote of the Table is the number of the place, that is, in the fourth, wherein you must write your digit or valuer 5.

Afterward come to the second part of the number, two hundred, whose valuer is 2. and his denomination C. Seeke two at the right hand of the Table, and goe along vnder the denomination toward the left hand, till you come vnder C: then looke to the Table, and there you shall see the number of the place, that is to say, three, wherein you must set your digit 2.

Then doe so by your other two numbers that remaine, and you shall finde 5. in the second place for your fifty, & 7. in the first place for your seauen. And thus may you doe with other numbers.

Scholar.

Scholar. Master, I thanke you hartily. I perceiue you seeke to instruct me most plainely and briefly, and not to hide your knowledge with subtle words as many doe. For this rule is so plaine that I can desire it no plainer. And though it seeme somwhat long, yet I perceiue it to be a sure way.

Master. So it is, and though it be long, yet it is neyther to long, neyther to plaine for yong learners that lacke practise: for this Table is in stead of a teacher to them that lacke one. But now I trust I haue said enough of Numeration: which after you haue well practised, then may you learne forth.

Scholar. Yet I pray you in one thing to tell me your iudgement. Why doe men reckon the order of the places backeward, from the right hand to the left?

Master. In that thing all men doe agree, that the Chaldees, which first inuented this Art, did set these Figures as they set all their Letters, for they write backeward, as you learne it, and so doe they reade. And that may appeare in all Hebrew, Chaldee, and Arabicke Bookes, for they be not onely written from the right hand to the left, and so must be read, but also the right end of the booke is the beginning of it, whereas the Greekes, Latines, and all nations of Europe, doe write and read from the left hand toward the right: and all their Bookes begin at the left side.

Scholar. That reason doth satisfie me.

Master. It neyther satisfieth me, neyther liketh me well, because I se that the Chaldees and Hebrewes doe not so vse their owne numbers, as at another time I will declare. But this plaine reason may best satisfie you presently: that seeing in pronouncing of numbers we kepe the order of our owne reading, from the left hand to the right: and againe, we do euer name the greater numbers before the smaller: it was reason that the lesser places, containing the lesser numbers, should be set on the right hand, and the greater places containing the greater numbers, to proceede toward the left hand.

Scholar. This reason is to me so plaine, that it seemeth now against reason to make a doubt of that order. So that now for Numeration I am satisfied: hoping that practise shall make me fully ready and expert in it. And in the meane season, I desire to learne the other kindes of Arithmeticke.

Master. That is well said: but what should you next learne? can you tell?

Scholar. I remember you said that Addition was next.

Master. Euen so, and what that is must you first know.

Addition.

Addition.

Addition is the gathering together and bringing of two numbers or more into one summe: as if I haue 160. bookes in the Latine tongue, and 136. in the Greeke tongue, and would know how many they be in all, I must write these two numbers one ouer another, writing the greatest number highest, so that the first figure of the one be vnder the first figure of the other: and the second vnder the second, and so forth in order.

When you haue so done, draw vnder them a right line, then will they stand thus:

Now begin at the first places toward the right hand alwayes, and put together the two first figures of those two numbers, and loke what commeth of them, write vnder them right vnder the line. As in saying 6. and 0. is 6. write 6. vnder, 6. as thus:

160	
136	
6	

And then goe to the second figures, and doe likewise: as saying 3. and 6, is 9. write 9. vnder 6. and 3. as here you see.

160	
136	
96	

And likewise do you with the figures that be in the third place, saying, 1. & 1. be 2. write 2. vnder them, & then will your whole summe appeare thus:

160	
136	
296	

So that now you see, that 160. and 136. doe make in all 296.

Scholar. What: this is very easie to doe: me thinketh I can doe it euen since.

There came through Cheapside 2. droues of cattel: in the first was 848. sheepe, & in the second was 186. other beasts.

Those two summes I must write as you taught me, thus:

	848
	186

Then if I put the two first figures together saying: 6. & 8. they make 14. That must I write vnder 6. and 8. thus:

	848
	186
	14

Master. Not so, and here are you twice deceiued. First, in going about to adde together two summes of sundry things, which you ought not to doe, except you seeke onely the number of them, and care not for the things. For the summe that should result of that addition should be a summe neither of sheepe, nor of other beasts, but a confused summe of both. Howbeit sometimes ye shall haue summes of diuers denominations to be added, of which I will tell you anon: but first I will shew you where you were deceiued in another point, and that was in writing foureteen which came of fiue and eight, vnder fiue and eight: which is impossible. For how can two figures of two places be written vnder one figure, and one place?

Scholar.

Scholar. Truth it is: but yet I did so vnderstand you.

Master. I said indeede, that you should write that vnder them that did result of them both together: which saying is alwayes true, if that summe doe not excede a digit. But if it be a mixt number, then must you write the digit of it vnder your figures as you haue said before: but if it be an article, then write 0. vnder them, and in both sorts you shall keepe the article in your minde, and therefore when you haue added your second figures, which occupie the place of tens, you shall put that one thereto, which you kept in your minde: for though it were ten indeed, yet in that place it is but as one, because that euery one of that place is ten, for that it is the place of tennes: and in like manner, if you haue in the second place so great a number that it amounteth aboue 9. then write the digit and reserue the article in your minde, euer adding it to the next place following: and so of all other places, how many soeuer you haue. And if you haue a mixt number when you haue added your last figures, then write the digit vnder the last figures, and the article in the next place beyond them: so shall your number resulting of Addition, haue one place more then the numbers which you should adde together.

4
7
6
1

Scholar. Now doe I perceiue you, and the reason of this, is (as I vnderstand) because

that no one place can containe above 9. which is the greatest figure that is, and then all tens or articles must be put to the next place following: for every place (as I may say) exceedeth the other place next before him by 10.

Now (if it please you) I will returne to my example of cattell. But I remember you said I might not adde summes of sundry things together, and that I might see by reason.

Master. Truth it is, if you seeke the due summe of any thing, but if you onely seeke a bare summe, and haue no respect to the thing, then were it better to name the summe onely without any thing: as in saying, 848. without naming sheepe or any thing else. And likewise 186. naming nothing.

Now let me see: how can you adde those two summes?

Scholar. I must first set them so, that the two first figures stand one ouer another, and the other each one ouer his fellow of the same place, then shall I draw a line vnder them both. And so likewise of other figures, setting alwayes the greatest number highest, thus, as followeth.

Then must I adde 6. to 8. which maketh 14. that is mixt number, therfore must I take the digit which is 4. and write it vnder 6. and 8. keeping the article 1. in my minde thus:

848	
186	
4	

Next that, I doe come to the second figures,

figures, adding them together, saying, 8. and 4. make 12. to the which I put the one reserved in my minde, & that maketh 13. of which number I write the digit 3. vnder 8. and 4. and kæpe the article in my minde, thus:

	848
	186
	34

Then come I to the third figures, saying 1. and 8. make 9. and 1. in my minde maketh 10. Sir, shall I write the Cypher vnder 1. and 8?

Master. Yea.

Scholar. Then of 10. I write the Cypher vnder 1. and 8. and kæpe the article in my minde.

Master. What nædeth that, sæing there followeth no more figures?

Scholar. Sir, I had forgotten, but I will remember better hereafter. Then sæing I am come to the last figures, I must write the Cypher vnder them, and the article in a further place after the Cypher, thus:

	848
	186
	1034

Master. So now you sæ, that of 848. & 186. added together, there amounteth 1034.

Scholar. Now I thinke I am perfect in Addition.

Master. That will I proue by this example. There are two armies of Souldiers: in the one are 106800. and in the other, 9400. How many are there in both armies, say you?

Scholar.

Scholar. First, I set them one ouer another, beginning with the first numbers of the right hand thus:

106800
9400

But the nether number wil not match the ouer number.

Master. That forceth not.

Scholar. Then doe I adde 0 to 0, and there remaineth 0, that muſt I write vnder the first place thus:

106800
9400
0

Master. Well ſaid.

Scholar. Then likewiſe in the ſecond place I adde 0 to 0, and there ariſeth 0, which I write vnder the ſecond place thus:

106800
9400
00

Then I come to the 3. place, ſaying, 4 & 8 make 12. of which I write the digit 2. & keepe the article 1. in my minde, thus:

106800
9400
200

Then I adde 9. to 6. which maketh 15. to that I adde the article 1. that was in my minde, and it is 16. I write 6. vnder 6. & 9. and keepe one in minde thus:

106800
9400
6200

Master. Why doe you not write both Figures, ſeeing you are come to the laſt couple of numbers?

Scholar. Nay, reaſon theweth me, that I muſt adde that article that is in my minde vnto the nert figure of the ouer ſumme, though there

there be no more in the nether summe.

Master. That is well considered: then doe so.

Scholar. Then say I, 0. in the ouer summe, and 1. in the minde, maketh 1. that write I vnder 0. Then followeth there yet one more in the ouer summe, which hath none to be added to it, for there is none in the nether summe, nor yet in my minde: therefore I thinke I must write that euen as it is.

Master. Yea.

Scholar. Then doth my whole summe appeare thus.

	106800
	9400
	116200

Master. If you marke this, you haue learned perfectly the common Addition of all sums which are of one denomination: so that yee obserue this also, that in Addition you must haue two numbers at the least: or else how can you say that you doe adde? And euer let the greatest number be written highest: for that is the best way, though it be not necessary.

And forget not this, that (if you haue many numbers to adde together) you shall haue oftentimes an article of a greater value than 10. sometimes 20. sometimes 30. sometimes more, yea (peraduenture) 100. Therefore, as you did with the Article 10. so doe with them, reserving them in your minde, and adding to the number next following, so many, as their value

valuer or value certaine is: that is to say, 2. for 20. 3 for 30. 5 for 50. 10 for 100. 12. for 120. and so forth of other like. So that, if the article be 100. then must you set downe the 0. and kéepe 10. in minde, to be carried to the next row of figures or place, if any such happen to come. For your better understanding take this example for all.

12
13

I would adde these fiue summes	4889
into one, which I set after this	4599
manner: then doe I begin and ga-	2290
ther the summe of the first row	3699
of figures, which commeth to	2299
107. (for I take 9. there 5. times,	4099
and that is 90.) then 9. and 8. is	1099
17. that is in all 107. of which	3298
summe I write the 7. vnder the	299
first row of figures, and then	699
for that 100. is 5. tens, I kéepe 5.	499
in minde, which ten I must adde	899
vnto the next row of figures,	389

which are in the second place: which second row of figures (when they are added together with that 5. that I had in my minde) make in all 125. of which sum I write the digit 5. vnder the second row, and then (for that 120. containeth fiue tens,) I kéepe fiue in minde, to be added to the third place or row of figures: which being added together, make in all 60. the Cypher 0. I set downe vnder the row of figures in the third place.

And

And the figure 6. I keepe in minde to be added to the row of figures in the fourth place: which (when they are added together) make 29. The figure or digit 9. I set downe vnder the fourth place. And, because it is my last worke, I set downe the 2. also that I haue in my minde to the 9. in the fift place: those summes doe make in all 29057.

¶ But (for your more ease in worke) when you haue an addition of so many summes to be added together, you were best part that summe into two or three parts, and worke them seuerall, and so put their additions together, and this were the best thing you could doe when ouer many summes fall to be added.

4889
4599
2290
3699
2299
4099
1099
3298
299
699
899
499
389
29057

Scholar. This seemeth somewhat hard, by the reason of so many numbers together.

Howbeit, I thinke (if I doe often proue, euen with this small example, either by working of it alone, or else by parting it as you said euen now) that I shall be able to doe so shortly with any other summe.

Master. So shall you. For it is often practise that maketh a man quicke and ripe in all things. But because, as well in great summes as in small, there may chance to be some

errour. I will teach you how you shall proue whether you haue done well or no.

Scholar. That were a great help and ease.

The proofe of Addition.

Master. Begin first with the highest number, and then to all the other orderly, and adde them together, not hauing regard to their places, but as though they were all vnites: and still (as your number increaseth aboue 9) cast away 9. Then goe forth, euer casting away 9. as often as it amounteth thereto: and so doe till you haue gone ouer all the numbers that you intended first to adde: and whatsoeuer remaineth after such additions & castings away of 9. write it in some void place by the end of a line, for the better remembrance: and thus is the first part of your worke proued. Then secondly, put together the figures that result of the addition vnder the line, still casting away 9. also. And then that that remaineth write at the other end of that line: and if those two figures be like, then haue you well done: but if they be vnlike, then haue you missed. As for example, in this present summe: The first figure of the ouer line is 9. let him goe: then 8. and 8. is 16. take away nine, there resteth 7. and adde that 7. to 4. that followeth, and it maketh 11. from which if you take 9. there resteth 2. Then come to the next row, whose first and second numbers are 9. therefore ouer-passe them both, and take the fiue to the 2. which did remaine in the first row, that maketh

maketh 7. put thereto the 4. following, and that maketh 11. thence take 9. and there remaineth 2. Next unto that, goe to the third line, whose two first numbers you may let passe because they are nines: then take the two figures of 2. which (with the other two that remayned in the second row) make 6. Then goe to the fourth row, whose two first numbers let goe, and take the 6. to the 6. that remayned, and that maketh 12. take away 9. and there resteth 3. which with the 3. that is next, maketh 6. And so goe through all the other nimbers, and you shall finde that there remaineth 5. after you have cast away 9. as often as you can finde it: therefore write 5. at the end of the line in a void place, thus:

5———

Then, gather all the Figures of the totall summe, which is under the lowest line, and cast away 9. as often as you can finde it: as thus: 7. and 5. make 12. take away 9. there resteth 3. to that if you adde the 2. that is last (for you may omit the 9.) then both it make 5. which 5. you must write at the other end of the line that you made in the void place, thus:

5———5

And then you see that those two figures be like. Whereby you may know that you have done well. And so you may prove in any other.

Scholar. (If it please you) I will prove in another summe.

Master.

Master. With a good will.

Scholar. Then will I take one of your former examples, which was this.

First in the highest line 8. and 6. make 14. then 9. taken away, there remaine 5. to which I adde the 1 that followeth, & that maketh 6. Then come I to the second line, where I finde first 4. which with 6. maketh 10. from that I take 9. and there resteth 1. the next figure is 9. and therefore I let him alone, & finde I one remayning which I set at the end of a line, thus:

	106800
	9400
	116300

1

Then I come to the totall summe, and there I finde that all the figures put together, make tenne, from which I take nine, and there resteth 1. also, which I put at the other end of the line, thus: 1 ——— 1

And because they be like, I know that I have well added.

Master. So, you know now both how to adde two summes or more together, and also how to proue whether you have done well, or no: and now I will teach you how to adde summes of diuers denominations together: which thing can neuer be but when the one denomination is such that it containeth the other certaine times. And yet you shall adde them to the other, not after this sort (as you did them that were of one denomination) but

16

after

after such a sort as I will now shew you, that is to say:

If you haue a summe of diuers denominations, then looke that you set euery denomination by himselfe, with some note or figure of his denomination, as they are wont to be written. Then write your other sums so vnder that first, that euery one be set vnder the other of the same denominations: as for example: if your denominations be pounds, shillings and pence, write pounds vnder pounds, shillings vnder shillings, and pence vnder pence: and not shillings vnder pence, nor pence vnder pounds.

Scholar. Now that you haue spoken it, me thinketh, it needeth not to warne me of it: for it were against reason so to confound summes: but yet, if you had not spoken of it, peraduenture I should haue beene deceiued in it.

Master. If you doe say it is so plaine, I will speake no more of it, but with an example make the matter to appeare euidently.

First, one man oweth me 22 l', 6 s, 8 d. Another oweth me 45 l', 16 s, 6 d. And another oweth me 4 l'. 3 s. I would know what this is all together? Therfore must I first set downe my greatest sum, and then the other, euery one vnder his Denomination agreeing to the greatest summe, as here you see with a line vnder them.

	l'	s	d
	22	6	8
	45	16	6
	4	3	

Then must I begin at the smallest numbers (which must alwaies be set next to the right hand) and adde them together: and (if the sum of them will make 1, or 2, or 3, of the next denomination) then must I keepe it in my mind till I come to that place, and vnder that first place must I note the residue (if there remaine any of the same denomination:) but, if there remaine none, then neede I to write vnder it nothing. And this is all that you must marke in this Addition: for all other things are like to the other maner of addition before mentioned. Therefore, the cheifest point of this Addition is, to know the values of common coines and rated summes. As how many shillings be in a pound: how many pence in a shilling: of which (and of other like things) I will instruct you hereafter in teaching of reduction. But now I may not disturbe your wit from the thing that we are about.

Therefore let vs returne to that former example which I proposed of three Debtors: which summes when I had set orderly, they stode thus with a line vnder them.

	l'	s	d
	22	6	8
	45	16	6
	4	3	2

Then to adde them into one summe, I must begin at the right hand where the smallest denomination is, and adde them together, first saying: 6. and 8. make 14. Now, seeing these 14. are pence, which containe one shil-

ling

ling and 2. pence: the 2. pence I set downe vnder the like of pence: and the one shilling I keepe in my minde to carry to the next row being the place of shillings.

l'	s	d
22	6	8
45	16	6
4	3	
	6	2

Then doe I adde the shillings together, saying: 1. in mind and 3. make 4. and 6. make 10. and 6. make 16. and 1. in the second place which standeth for 10. make 20. which is 1. pound 0. s. The 0. s. I set downe vnder the place of shillings, as appeareth in the example. And the 1. pound I keepe to carry to the pounds.

l'	s	d
22	6	8
45	16	6
4	3	
2	0	2

Then come I to the pounds, adding them all together, saying, 1 that I keepe and 4 make 5. and 5. make 10. and 2. make 12. The figure or digit 2. I set downe right vnder that place or rolo of pounds where I gather them. And the article 1. I keepe to carry to the next place, saying: 1. in minde & 4. is 5. and. 2. is 7. which 7. I set downe directly vnder that rolo also. And then appeareth my whole summe thus.

l'	s	d
22	6	8
45	16	6
4	3	
72	0	2

And thus must you doe with any such like summes whatsoeuer, whether they be money,

weight or measure, which (if you practise your selfe well therein, by setting downe of diuers sums) you shall be well acquainted with the feat of Addition.

But now, can you tell how to proue this Addition, or such other like of diuers denominations, and to trie whether you haue well done or no?

Scholar. I would I could.

Proofe of Addition of diuers denominations.

Master. That shall you doe by this meanes. You must make a crosse which shall haue so many lines as you haue sundry denominations in your Addition: As, if you haue but two denominations, then you may make it thus: that the ouer part and the nether part may serue for one denomination. And if you haue three denominations (as pounds, shillings, and pence) then must you make three lines, thus: The vpright line may serue for pounds, and the highest thwart line for shillings, and the lowest for pence: as for example by the summe which we last wrought.

l	s	d
22	6	8
45	16	6
4	3	0
72	6	2

For

For the proofe of the which, because it contayneth three denominations, I must make a crosse of three lines, as in the example before. Then I reckon, first, at the right hand, the pence: 6. and 8. make 14. from which I take 12. for the next Denomination, that is to say, a shilling, and there resteth 2. which I must write at one end of the nether thwart line.

After that, I gather the summe of the shillings, 3. 16. 6. which maketh 25. to whom I put 1. that I toke of the pence, & that maketh 26. from those I take 20. the quantitie of the next greater Denomination, that is to say, a pound, and there resteth 6. which I write at the end of the highest thwart line.

Thirdly I adde together the pounds 4. 5. and 2. which make 11. to them I adde the one that came of the shillings, and they make 12. from whence I cast nine, and there resteth 3. That three I ioyne to the 4. in the next place, and they make 7. then the two more make nine which I cast away, and so there is left 0. which 0. I set at the vpper end of the Crosse also: (And so should there be set also any number that were left vnder 9.) And thus is my first part of my worke proued.

That done, I come to the totall summe vnder the line, and examine it, beginning at the pence, where I finde but two and cannot take nine from him: therefore, I set him at the other end of the neather thwart line: then I

come to the shillings, where I finde onely 6. which (because it is lesse than nine, I set it at the other end of the line of shillings, that is, the ouer-most thwart line.

Last of all, of the 72.l'. I take eight times 9. which is 72. and there remayneth 0. which I write vnder the vpright line: eyther else I may reckon them simply without any respect of their valuation or place: in saying, 2. and 7. make 9, which I take away, and so resteth 0. Then I consider euery number, comparing it to the number that is against it: and, because I finde them to be euery one like his match, I know that I have well done.

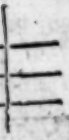
Scholar. This cypher I perceiue doth serue for these three denominations, pounds, shillings, pence. But, what if I had l'. s. d. ob. and q?

Master. These lines (as I haue said) doe serue for three denominations, such as they be, as here three doe serue for pounds, shillings and pence: but, if you haue no pounds in your summe, then may they serue for shillings, pence, and halfe pences: yea, for d. ob. and q. or in weight, for C. q. and l: or, in measure, for Elles, quarters, and nailes, if you haue no greater Denomination: so that you remember that the vpright line serueth for the greatest Denomination, and the highest thwart line for the next, and the lowest for the least.

And so, if you haue foure Denominations,

you

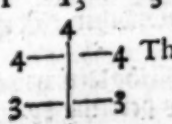
you muſt make your croſſe with ſo many lines. And (if that your ſumme be of more denominations) make ſo many lines in your croſſe. And thus will I make an end of Addition, ſauing that here (for the better vnderſtanding of this Rule) I haue ſet you downe certain examples both of mony, weight and meaſures, with their workes and proues.



Examples of Addition.

l'	s	d
23	10	4
45	6	8
37	2	9
15	13	6
121	13	3

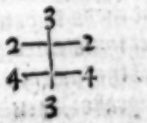
l'	s	d
130	17	10
28	6	8
13	13	4
120	0	0
292	17	10



The Prooſes.

C	q;	l
24	1	3
12	2	2
7	3	4
13	0	13
57	2	22

yards.	q;	nayles.
17	3	3
35	2	1
26	1	3
54	2	0
134	1	3



Subtraction.

Scholar.

When haue I learned the two first kindes of Arithmeticke: now (as I remember) doth follow Subtraction, whose name (mee thinketh,) doth sound contrary to Addition.

Master. So it is indeede: for, as Addition increaseth one grosse summe, by bringing many into one: so contrariwise Subtraction diminisheth a grosse summe by withdrawing of other from it. So that Subtraction or Rebating, is nothing else but an Art to withdraw and abate one summe from another, that the remayner may appeare.

Scholar. What doe you call the Remayner?

Master. That you may perceiue by the name.

Scholar. So me thinketh: but yet it is good to aske the truth of all such things, least in trusting to mine owne coniecture I be deceiued.

Master. So it is the surest way. And (as I see cause) I will still declare things vnto you so plainely, that you shall not neede to doubt. Howbeit, if I doe ouerpasse it sometimes (as the manner of men is to forget the smal knowledge of them to whom they speake:) then doe you

you put me in remembrance your selfe, and that way is surest.

And, as for this word that you last alledge, take you this discription: The Remayner is a summe left after due subtraction made, which declareth the excesse or difference of the other two numbers: as if I would abate or subtract 14. out of 18. there should remaine 4. which is called the Remayner, and is the difference betweene those two numbers 14. and 18.

Scholar. I perceiue then what Subtraction is: Now resteth to know the order to worke it.

Master. That shall you doe by this meanes: First, you must consider, that if you should goe about to rebate, you must haue two sundry sums proposed: the first, which is your grosse summe (or summe totall:) and it must be set highest: and then, the rebatement (or summe to be withdrawne) which must be set vnder the first, (whether it be in one parcell or in many) and that in such sort, that the first figures be one iust ouer another, and so the second, and third, and all other following, as you did in Addition: then shall you draw vnder them a line, and so are your summes duely set to beginne your working.

Then begin you at the right hand (as you did in Addition) and withdraw the neather number out of the higher, and if there remaine

any

any thing, write that right vnder them beneath the line: and if there remaine nothing (by reason that the two figures were equall) then write vnder them a Cypher of nought. And so doe you with all the other figures, euermore abating the lower out of the higher, and write vnder them the Remayner still, till you come to the end. And so wil there appeare vnder the line what remayneth of your grosse summe, after you haue reduced the other summe from it, as in this example.

I receiued of your Father 48 s. of which I haue laid out for you 36. s. now would I know what doth remaine? And therefore I set my numbers thus in order. First, I write the greatest summe, and vnder him the lesser, so that the figures at the right side be euen one vnder another, and so the other, thus.

	s
	48
	36

Then doe I rebate 6. out of 8. and there resteth two, which I write vnder them right beneath the line, thus:

	48
	36
	2

Then I go to the second figures, and doe rebate 3. out of 4, where there remayneth 1. which I write vnder them right, and then the whole summe and operation appeareth thus.

	48
	36
	12

Whereby

Whereby it appeareth, that if I withdraw 36 out of 48. there remayneth 12.

Scholar. Now will I proue in a greater summe, and I will subtract 2367924. out of 3468946. those sums I set in order thus:

3468946	
2367924	

Then doe I beginne at the right side, and deduct 4. out of 6. and there resteth 2. which I write vnder them. Then goe I to the second figures, and withdraw 2. out of 4. and there remayneth 2. which I set vnder them also: then I take 9. out of 9. and there resteth 0. which I write vnder them (for you say that if the figures be equall, so that nothing doe remaine, I must write this Cypher 0. vnder them.)

Maister. It was well remembred: now goe forth.

Scholar. Then I come to the fourth place, and draw 7. out of 8. and there remayneth 1. which I write vnder them also. Then, in the fift place I take 6. out of 6. and there resteth nought (for it I write vnder them the cipher 0) Then, in the first place 3. rebated from 4. there remayneth 1. which I write vnder them: and likewise in the seauenth and the last place, 2. taken from 3. there is left one, Which I write vnder them: so haue I done my whole working, and my summes doe appeare thus. Whereby

3468946	
2367924	
1101022	

I

I see, that (if I doe rebate 2367924. out of 3468946.) there remaineth 1101022.

Master. This is well done. And that you may be sure to perceiue fully the Art of Subtraction, let mee see how you can subtract 52984732. out of 82500003456?

Scholar. First, I set downe the greatest summe, and after that, I write vnder it the lesser number, beginning at the right side: and then my figures will stand thus.

82500003456	
52984732	

Then take I 2. from 6. and the rest is foure, which I write vnder them: then doe I withdraw 3. from 5. and there remaynes 2. which I write vnder them. Then take I 7. out of 4. but that I cannot, what shall I now doe?

Master. Marke well what I shall tell you now, how you shall doe in this case, and in all other the like: if any figure of the neather summe be greater than the figure of the summe that is ouer him (so that it cannot be taken out of the figure ouer him) then must you put 10. to the ouer figure, and then consider how much it is, and out of that whole summe withdraw the neather figure, and write the rest vnder them. Can you remember this?

Scholar. Yes, that I trust I shall. Now then in mine example where I should haue taken 7. out of 4. and could not, I put 10. to

that

that 4. which maketh 14. from it I take away 7. and there resteth 7. also, which I write vnder them.

Master. So haue you done well, but now must you marke another thing also: that (whensoeuer you doe so put ten to any figure of the ouer-number) you must adde one still to the figure or place that followeth next in the neather line: as in this example there followeth 4. to which you must put one, and make him 5. and then goe on as I haue taught you.

Scholar. Then shall I say, 4. and 1. (which I must put to him for the 10. that I added to 4. before) make 5. which I should take out of 3. but that cannot be: therefore I must put to it also 10. and then it will be 13. from which I take 5. and there resteth 8. to be written vnder them: and because of that 10. added to the 3. I must adde 1. to 8. that followeth in the neather line, & that maketh 9. which I should take out of 0. and cannot: therefore, I put thereto 10. and that maketh 10. from 10. I take 9. and there remayneth 1. which I write vnder them.

Then doe I adde one likewise to the next figure beneath, which is 9. and that maketh 10. that 10. should I take out of the figure aboue, but I cannot: for it is 0. therefore I put 10 to it, and so take I 10. out of 10. and there resteth 0. to be written vnder them.

Then

Then come I to the nert figure which is 2. and to him I doe adde 1. which maketh 3. that 3. I cannot take out of nought: therefore of that nought I make 10. and thence doe take 3. so there remayneth 7. to be written vnder them: likewise doe I put 1. to 5. that followeth, and then it is 6. that would I take out of 5. and cannot, therefore I adde 10. to that 5. and make it 15. from which I rebate 6. and there remaineth 9. which I write vnder them. Now haue I spent all the neather figures, and what shall I doe more?

Master. You should haue added one to the nert figure following (if there had beene any) because you added 10. to the last figure before of the ouer line: but seeing there is no figure following, you must adde that one to the place
9 following, and then deduct that one from the number aboue.

Scholar. Then shall I say, because I borrowed ten to the ouer 5. I must put 1. in the nert place beneath, that is vnder 2. then must I subtract that 1. from 2. & there resteth one, to be written vnder that in the ninth place. Now I haue no more to subtract, for there is not any figure remaining beneath, neither yet any vnit to be added, because I borrowed not 10. to the figure last before: and yet is there 8
10 remayning in the ouer line, which I thinke (by reason) should be set at the end of the figures in the lowest row, which is vnder

the

the line, for because their was nothing taken from it.

Master. That is well considered, and reason teacheth so indéede.

Scholar. But, Sir, I beséech you, shall I alwayes when any number is remayneth alone as this eight did) write him vnder the line straight against his owne place?

Master. Yea, what else? Whether they be one or many: and this well remembred, you haue sufficiently learned Subtraction. Howbeit, because of certaine things that might deceiue you, if you did not take good héede to your working, I will propose to you another example of many numbers to be subtracted, as thus.

I receiued of a friend of mine to kéepe 2869. crownes, of which at one time I deliuered him againe 500. at another time 368. at another time 440. at another time 80. and another time 64. Now would I know, how many doe rest behinde? Therefore first I set downe my grosse summe and a line vnder it: and vnderneath it I set all the parcels thus, and vnder them a double line.

2869. *Crownes receiued.*	
500	
368	*Deliuered.*
440	
80	
64	

Then first I begin at the first place, & gather together the sum

of

of all those lines (saue the outermost) in their first figures: and so I doe with all the figures of the second place, and so forth, as I did in Addition, saue that I leaue out the highest row of numbers (as the line warneth mee) and that sum so gathered betweene the double line, is the sum deliuered in all: which sum I doe afterwards subtract out of the highest row of numbers, and the remayner doe I set vnder the neathermost line: as for example.

I set the summes as before: then doe I gather the first figures of al the places deliuered, together: where I finde but 4. and 8. that make 12. (for three Cyphers increase no summe in Addition, as you learned before:) of the 12. therefore, doe I write the Digit 2. betweene the double line, and keepe the Article in my minde, till I come to the second place, where I finde 6. 8. 4. 6. that maketh 24. to them I put the article in my minde, and it is 25. of which I write 5. vnder the second place and keepe the digit 2. in my minde for the third place, where I finde 4. 3. 5. that makes 12. to the which I adde the 2. in my minde: and it maketh 14. thereof I write the 4. vnder the third

2869	*Crownes receiued.*
500	
368	
440	*Deliuered.*
80	
64	
1452	*Deliuered in all.*
1417	*Rest behinde.*

third place: & because there remaines no moe figures to be added, I write the digit in the fourth place, as you see in the example: and so it appeareth, I haue deliuered in all, a thousand foure hundred fifty two Crownes.

Then come I to the subtracting of this summe betweene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I proceede to subtract it from the ouer-most summe, saying, 2. from 9. remayne 7. to be written vnder them beneath the lowest line. Then in the second place I take 5. from 6. and there resteth 1. to be written vnder them. Then in the third place 4. from 8. resteth 4. Last of all, in the fourth place, 1. from 2. remayneth 1. And thus I see that after those 5. summes are subtracted from 2869. the Remayner is 1417.

Master. This is a very sure and easie way for a learner: therefore I counsell you to practise it well.

Scholar. This I perceiue: but is there no shorter way and more spedy?

Master. Yea, when you are a while exercised in it: for you may (as fast as you can gather the numbers together) withdraw them out of the highest summe. But, if in quantitie those numbers added together exceede the highest summe or vpper number, then shall you (as before hath beene taught you) imagine to borrow 10. 20. or 30. moe, as neede shall

require, and put them to the vpper number, to helpe to further the abatement, restoring or restoring the Articles that you borrowed to the nert place againe: and so still goe forward till you haue ended your worke: as for example: In the last summe proposed, I gather first in the first place 4 and 8. that maketh twelue, which 12. I should deduct or take out of 9. in the vpper number aboue the line, but I cannot: therefore I adde vnto 9. an article of 10. and that maketh the vpper number 19. from whence I take 12. then there resteth 7. Then for the Article 10. I adde to the nert place of money deliuered: saying, 1. that I bring, and 6. make 7. and 8. make 15. and 4. make 19. and 6. make 25. which 25. I should take out of 6. in the vpper number, but I cannot. Therefore I adde 2. tennes of 20. vnto 6. in the vpper number, and that maketh 26. then 25. out of 26. resteth 1. then the two tennes which I borrowed, or haue in minde, I adde to the nert row or summe deliuered, saying: 2. that I bring, & 4. make 6. and 3. make 9. and 5 make 14. then 14. out of 8. I cannot take, but 14. out of 18 resteth 4. Now because there are no more places to be added, the one that I borrowed or haue in my minde, I rebate from 2. in the vpper line, and there remayneth 1. which I set downe in the remayner line, and so my summe appeareth (as before) to be 1417. Crownes. Loe thus haue you now a shorter way.

Scholar.

Scholar. I like both wayes well, and I perceiue both well: yet, as in the one the working seemeth somewhat long, so in the other it leaneth very much (mee seemeth) to remembrance, and therefore may cause errour quickly, except a man haue a quicke and an exercised remembrance. But yet for the sharpning of my wit by your patience (if you will giue me leaue) I will try what I can doe in such a like summe, to worke it the shortest way: whereupon I would subtract out of 40301964. these three parcels.

	40301964
20003428	20003428
10002432	10002432
10101401	10101401

Therefore I set them first in due order:

then I gather the parcels of the first place, which are 8. 2. 1. that is 11. which I should take or deduct out of 4. which is ouer him, but I cannot: therefore I adde an article or one ten to 4. which maketh 14. then 11. out of 14. therefore resteth 3. to be written vnder the first place betweene the two liners.

Then come I to the second place, saying, 1. that I borrowed to haue in minde, and 6. make 7. and 3. make 10. and 2. make 12. which I cannot take from 6. therefore I adde 10. to 6. which maketh 16. and then 12. from 16. resteth 4. which I write vnder

the second place betweene the two lines.

Then come I to the third place, saying: 1. that I borrowed or haue in minde, & 4. make 5. and 4. is 9. and 4. make 13. which I should take out of 9. that is ouer them, but I cannot: therefore I adde 10. to 9. which maks 19. then 13. out of 19. rest 6.

Then come I to the 4. place, saying: 1. in minde & 1. is 2. and 2. make 4. and 3. make 7. which because it cannot be taken from 1. I take it from 11. and there rests 4.

After that, I come to the fift place, where are onely three Ciphers, which make nothing: vnto which I adde 1. in minde, then should I take that (that is to say) 1. from the figure ouer them, which is also a Cipher: therefore I must say thus: I cannot take 1. from 0. but 1. from 10. remaineth 9. so must I write 9. vnder them. Then in the sixt place I finde but 1. which 1 in mind I take out of 3. ouer him, & the remainer is 2. that must be written betweene the two lines in the sixt place. So I goe to the seauenth, where I finde onely Ciphers, and in the grosse summe ouer them a Cipher also: therefore must I write the remainer (which is nothing) with a Cipher also. Then in the eight and last place I gather 1. 1. 2. that maketh 4. which if I take out of that 4. that is ouer them, there will nothing remaine. And that must be noted with a Cipher betwixt the two lines (as I haue often said) and so haue I ended my worke, and

the

the figures stand thus.

	40301964
	20003428
	10002432
	10101461
	00194643

But Sir, I remember you taught me that Cyphers should not come in the last place, for because they serue onely to increase the value of other figures which follow them and serue not those figures that goe before them: and now in my example I haue set two Cyphers in the two last places.

Master. I commend you for your remembrance. And truth it is, you should not haue set them here, but onely because that I would make you plainely to perceiue the Art of Subtraction. Therefore, seeing that you doe now perceiue it, whensoeuer you would write downe a Cypher, looke whether any other figures be yet behinde: and if not, then let goe the Cipher also, for it needeth not to write him in the latter places, where no other figure doth follow, except it be (as I did now suffer you) to teach the vse of Subtraction the playner.

Therefore your figures must stand thus when the worke is ended.

40301964.	*Charge.*
20003428	*Discharge.*
10002432	
10101461	

Scholar. Sir, I doe think with that that you taught me before, and by these two summes that you taught

taught me last also, that now I could subtract any summe.

Master. So may you, if you haue marked what I haue taught you. But, because this thing (as all other) must be learned surely by often practise, I will propound here two examples to you: wherein if you often exercise your selfe, you shall be ripe and perfect to subtract any other summe lightly, for in them is contayned all the obseruances of whole numbers. And because you shall perceiue somewhat both how to doe it, and also whether it be well done when you haue proued to doe it, therefore haue I written vnder them both the Remayners.

30696. *Lent.*	30606. *Debt.*
10249	10354
10249 *Repaid.*	10249 *Paid.*
163	163
20661. *Paid in all.*	9840. *Rest.*

9840. *Rest to pay.*

Scholar. Sir, I thanke you. But I thinke I might the better doe it, if you did shew me the working of it.

Master. Yea, but you must proue your selfe to doe some things without any aide, or else you shall not be able to doe any more than you are taught: And that were rather to

learne

learne by rote (as they call it) than by reason. And againe, there is nothing in this example, or any other of whole number) but I haue taught you the rules of them already, saoing I haue not yet shewed you how you shall proue this worke.

Scholar. Sir, I pray you then shew it me also.

Master. For the performance whereof if you marke well what I said in Addition, you may easily perceiue what may be done for the proofe of Subtraction, and that is onely performed by the aide of Addition, thus: Draw vnder the lowest number, which is your Remainer) a line, and then adde the summe paid in all (which before was made by the particulars) and the remainer together. And if these twaine being added together in one summe doe make the contents of the vpper number aboue the line, then is this Subtraction well wrought or else not.

For the tryall whereof I will propone one of our examples which are done. which was of 2869. crownes receiued to keepe of a friend.

Crownes 2869. receiued.

	500
Of which (at sundry	363
times) I deliuered	440
these particular sums	80
	64

which particulars in one Addition make, so resteth to my friend.

1452. *delivered in all.*
1417. *rest to pay.*
2869. *Proofe.*

Now put or adde this remainer, and the summe delivered together, saying, 7. and 2. make 9. and 1. and 5. make 6. & 4. and 4. is 8. and 1. and 1. is 2. which in one summe amounteth to 2869. and is equall with the summe 2869. above the line: whereby I know the subtraction is well wrought: and this is the order and proofe hereof, and of all such like workes of Subtraction.

Scholar. Then I trust by practise to attaine the use of it. And is this all that I shall learne of Subtraction?

Master. Yea, saving that (as you have seene in Addition) there are numbers of diuers denominations, in which the working is not much vnlike: yet (without some instructions be giuen of it) it might seeme to a learner more difficult than indeede it is. Therefore I will briefly shew you the vse of it onely by one example or two.

A certaine man owed to me 14 l. 12 s. 8 d. of which he paid me at one time 4 l. 6 s. 8 d: at another time 3 l: at another 2 l. 3 s. 4 d: and last of all, 6 s. 8 d.

Now, would I know what remaineth vnpaid yet? therefore I set my summes thus,

every

every one in their due place: As pounds vnder pounds, shillings vnder shillings, pence vnder pence.

Scholar. Sir, I pray you why doe you write 2. l. for the common speech vseth rather to say, 40 s?

l	s	d
14	12	8
4	6	8
3	0	0
2	3	4

Master. We must here vse the Denomination that is greatest in any summe, so that we may not write according as we vse to speake, saying, 16 d. 18 d. or likewise, 7. groats, 8. groats, 24 s. 40 s. 48 s. and such other: but we must write every Denomination that is in any summe by it selfe.

Namely, shillings and pounds. So must we write for the last summes now named, 1 s. 4 d. 1 s. 6 d. 2 s. 4 d. 2 s. 8 d. 1 l. 4 s. 2 l. 8 s. and so forth of other like.

Scholar. So that we may not write in Arithmeticke, pence, when the summe amounteth to shillings, nor shillings when the summe maketh pounds. Now, (if it please you) end your example.

Master. When my summes are so set as I shewed, then (according to the rules of Addition) I gather all the particular summes which be paid me into one totall summe, directly to be set vnder them betweene the two lines, not medling with the 14 l. 12 s. 8 d. as the line warneth me: therefore must I first begin with the

the smallest denomination, saying, 8. 4. 8. is 20. pence, which maketh one shilling and 8. pence: the 8. d. I set downe vnder the place of pence, and the one shilling I keepe in mind to carry to the next denominations of shillings. Then come I to the shillings, and say, one that I bring or haue in minde, and 6 is 7. and 3. is 10. and 6. makes 16. which, because it containeth not one pound, I set directly vnder the place of shillings. Then come I to the pounds, whose parcels are 2. 3. 4. that is in all 9. that 9. doe I set downe directly vnder the pounds: And so the totall or whole Addition of all the particulars paid, amounteth to 9 l. 16 s. 8 d.

	l	s	d	
	14	12	8	
	4	9	8	
	3	1	0	
	2	3	4	
		6	8	
	9	16	8	
	4	16	0	Rest.

Now, for the worke of Subtraction, I must rebate that totall summe of Addition out of the higher number, that is to say, from the 14 l. 12 s. 8 d.

Therefore to performe the worke, I say 8 d. out of 8 d. remaineth or resteth nothing: therefore, in the place of the rest or remaine, right vnder that denomination I set down 0. Then comming to the shillings where I finde 16. which should be taken out of 12. but I cannot: therefore I imagine to borrow 1. of the next denomi-

denomination, that is, of the 9 l'. and put that one pound so borrowed vnto 12 s. that maketh 32 s.

Now 16 s. out of 32 s. resteth 16 s which 16 s. I set downe directly vnder the place of the resk.

Lastly, comming to the pounds saying, one pound in minde that I borrowed, and 9. make 10. then 10. out of 14. there resteth 4.

So doth my whole rest remaine appeare to be 4 l'. 16 s. 0 d.

This I account the easiest way for a young beginner to practise, though it be something long.

Scholar. Is there any shorter way for this worke also?

Master. Yes, as in this last example I will also shew you, for you may adde together the particular summes as they are set in order, beginning with the pence, saying, 8. 4 8 make 20 d. which 20 d. you should take out of the 8 d. aboue the line, but you cannot, therefore shall you borrow 1. of the next denomination, that is to say, one of the shillings, and put it to the 8 d. that maketh 20. now 20. pence out of 20 d. rest 0. which Cipher I set downe directly vnder them.

	l'	s	d
	14	12	8
	4	6	8
	3	0	0
	2	3	4
	0	6	8
	4	16	0

Then one shilling that I borrowed or had

in

in minde, and 6. make 7. and 3. make 10. and 6. make 16. then 16. out of 12. I cannot take, therefore of the next denomination I doe borrow one pound, and put it to 12 s. which maketh 32 s. then 16 s. out of 32 s. resteth 16 s.

Lastly, I come to the pounds, saying: 1 l. in minde, or that I borrowed, and 2. make 3. and 3. is 6. and 4. is 10. then 10. out of 14. there resteth 4.

So doth my remayner or rest appeare as before to be 4 l. 16 s. 0 d.

Scholar. This doe I perceiue very well, and if there be none other thing to be learned in Subtraction, then may I come to Multiplication, for that you reckoned to be next in order.

Master. Wee haue done indeede with the Art of Subtraction, as touching the working. But yet before we goe to Multiplication, I will instruct you how to examine your worke whether it be well done or not. For the performance whereof, if you marke what I said right now in the last manner of proofe, you may easily perceiue what is to be done for this proofe, which is onely made (as before was taught you) by the aid of Addition, thus:

Draw vnder the lowest number (which is your Remayner) a line, and then adde the summe paid in all, and the Remayner together. And if these twaine added together in one summe make the contents of the vpper

number aboue the line, which in this example is 14 l. 12 s. 8 d. then is the subtraction well wrought, or else not.

As for example, in our first summe, which stood thus.

Where, in the title of pence I finde 8. and 0. the 8. I set downe directly vnder him in that place of pence.

Then in the place of shillings I finde 16. and 16. which make 32. shillings, wherein is contained 1 l. and 12 s. the 12 s. I set downe directly vnder them in their due place of shillings, and one pound I keepe.

Then comming to the pounds saying 1. that I keepe, and 4. is 5. and 9. is 14. which 14. in due order I set downe directly vnder them as this figure sheweth. Which is also 14 l. 12 s. 8 d. agreeing with the vpper number aboue. So I finde the worke is good, and the subtraction well wrought.

Now for the proofe of the latter Subtraction which was reckoned for the shorter worke: Draw vnder the remayner also a line, then adde that remainer and all the other numbers that ye did subtract before together, and write that that amounteth vnder the lowest line, and

	l.	s	d	
	14	12	8	
	4	6	8	
	3	0	0	
	2	3	4	
		6	8	
	9	16	8	* Paid in all.
	4	16	0 rest.	
	14	12	8	

and if the summe that commeth thereof be equall to the highest number aboue, then is the Subtraction well wrought, or else not.

As for example also in the last sums which stood thus.

	l'	s	d
	14	12	8
	4	6	8
	3	0	0
	2	3	4
	0	6	8
	4	16	0
	14	12	8

First, in the title of pence I adde 8. 4. 8. that maketh 20 d. which containeth one shilling and 8. pence, The 8. I set downe vnder the lowest line in the row of title of pence, and that one shilling I keepe to carry to the next denomination or place of shillings.

Then returning to the shillings, saying: one in minde, or that I keepe, and 16. make 17. and 6. make 23 and 3. make 26. and 6. make 32 shillings, which amounteth to one pound, 12 s, the 12 s I set downe vnder the title of shillings, and 1 pound I keepe or haue in minde to cary to the next denomination or place of pounds. Then come I to the pounds, saying, one that I bring and 4 make 5, and 2 make 7, and 3 is 10, and 4 make 14, then doe I write 14 vnder the pounds, and so haue I ended the Addition, and I see that the lowest line is like vnto the vppermost line in number, wherefore I know that I haue well done.

And thus now haue I taught you the Art

of

of Subtraction, and the meanes to proue whether it be well wrought or not.

Many other workes and proues might be shewed of Subtraction, but one of these, whether of them you please, are most apte, and best allowed of any other worke or proue, whether it be of l, s, d, or any other grosse summe whatsoeuer.

Scholar. Sir, I thanke you most heartily, for now I vnderstand well both the feate of Subtraction and his proue.

Master. Therefore here will I make an end of Subtraction, and will instruct you in Multiplication.

Multiplication.

Multiplication is an operation that two summes produce the third, which third summe so many times shall containe the first, as there are vnites in the second. And it serueth in the stead of many Additions. As for example. When I would know how many are 30 times 48: if I should adde 48 30 times, it would be a long worke. Therefore was this worke of Multiplication deuised, which shall doe that at once that Addition should doe at many times.

Scholar.

Scholar. I perceiue the commoditie of it partly, but I shall not see the full profit of it, till I know the whole vse of it. Therefore Sir, I beseech you, teach me the working of it.

Master. So I iudge it best, but because that great summes cannot be multiplyed, but by the multiplication of digits, therefore I thinke it best to shew you first the way of multiplying them: As when I say, 8. times 8. or 8. times 9. &c. And as for the small digits vnder 5. it were but folly to teach any rule, seeing they are so easie that euery childe can doe it: But for the multiplication of the greater digits, thus shall you doe.

2 First, set your Digits one ouer the other right, then from the vppermost downe-ward, and from the neathermost vpward, draw straight lines, so that they make a crosse, commonly called Saint Andrewes Crosse, as you see here. Then looke how many each of them lacketh of 10. and write that against each of them at the end of the lines, and that is called the difference: as if I would know how many are 7. times 8. I must write those Digits thus.

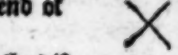
Digit difference.

8
7

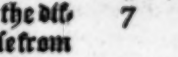
Then doe I looke how much 8. doth differ from 10 and I finde it to be 2. that 2

doe

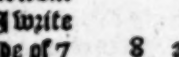
doe I write at the right hand of 8, at the end of the line thus.



After that I take the difference of 7 likewise from 10, that is 3, and I write that at the right side of 7 as you see in this example.

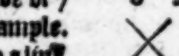


Then do I draw a line vnder them, as in Addition thus.



Last of all, I multiply the two differences, saying, 2 times 3 make 6, that must I euer set vnder the differences, beneath the line: then must I take one of the differences (which I will, for all is like) from the other digit (not from his owne) as the lines of the Crosse warne me, and that that is left must I write vnder the digits. As in this example if I take 2 from 7, or 3 from 8, there remayneth 5: that 5 must I write vnder the digits, and then there appeareth the multiplication of 7 times 8, to be 56. And so likewise of any other Digits, if they be aboue 5, for if they be vnder 5, then will their differences be greater than themselues, so that they cannot be taken out of them. And againe, such little summes euery childe can multiplie, as to say: 2 times 3,

Digit difference.



or 4 times 5. and such like.

Scholar. Truth it is. And seeing mee seemeth that I vnderstand the multiplying of the greater digits, I will proue by an example how I can doe it. I would know how many are nine times 6.

Master. It is all one in value to say nine times 6, or 6 times 9: but yet the order is best to put the lesse summe first, saying, 6 times 9, and so of all other summes.

Scholar. Then would I know how many are 6 times 9, therefore I set the digits thus, and make the crosse thus.

9
6

Then do I set their differences from 10 at the right side, the difference of 9, which is 1, against it, and the difference of 6, which is 4, against it also, as in this example.

9 1
6 4

And vnder them I draw a line. Then doe I multiply the digits together, saying: one time 4 maketh 4, that 4 doe I write vnder the differences thus.

9 1
6 4
4

Then take I one of the differences from the other digit, as one from 6 or else 4 from 9, and each waies there resteth 5, which I doe write vnder the digits,

Abraham Bourne

digits thus. And so appeareth the multiplication of 6 times 9 to be 54. Thus I see the feat of this manner of multiplication of digits.

Master. Now might you goe straight to the multiplication of greater numbers, saue that both for your ease and suretie in working I will draw you here a Table, whereby shall appeare the multiplication of all the Digits, and this is it that followeth.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the product in any multiplication of Digits, seeke your first or last Digit in the greater figures, and from it go right forth toward the right hand, till you come vnder the number of your second Digit, which is in the highest rowe: and then the number that is in the meeting of the rowes of little squares [illegible] come

come directly from both your propounded digits) is the multiplication that amounteth of them. As if I would know by this Table the multiplication of 7 times 9, seeke first 7 in the greater figures, and then goe right forth toward the right hand, till you come vnder 9 of the highest row, in which place where you so come vnder the other digit (as here for example you come vnder 9) is alwaies contained the ofcome or product, which you seeke: and that place we terme to be in the common angle in respect of the two numbers so taken on the outsides, as here in that common angle, where the rowes of little squares (directly proceeding from 7 & 9 doe meete, you haue 63, which 63 is the summe of the multiplication of 9 by 7.

Scholar. This is very good and ready. And so may I find the multiplication of any digits. But now how shall I do in greater summes?

Master. When you would multiply any summe by another, you shall marke that it
5 is the meetest order to set the greatest number highest, which is the place of the number that must be multiplied: and likewise the lesser number vnder it, for that is the place of the Multiplyer or Multiplicator, that is to say, the number by which multiplication is made, and is in English alwayes put before this word, Times: in such speaking when I say, 20 times 70. And the number that followeth this word Times, is

that

that which must be multiplied.

Therefore when I would multiply one number by another, I must write the greatest highest, and the lesser under it, as in Addition. And under them must I draw a line. As for example: If I would multiply 264 by 29, I must set them thus.

264
29

¶ Of which numbers thus set downe to be multiplied, may be formed a question, as thus: There are 29 men, and each man hath 264 Lambes. The question is, how many Lambes they have in all.

To the performance whereof, I must multiply every figure of the higher row, by every figure of the nether row: and that that amounteth, I must set under the line, as thus. First I doe multiply 4 by 9, saying: 9 times 4 (or 4 times 9 which is all one) & that maketh 36, as the Table before of Digits doth declare, of that 36 I must write the 6 that is the Digit, under the 9, and the Article 3 I keepe in minde to carry to the next place.

264
29
6

Then come I to the second figure of the higher row, which is 6, and say: 9 times 6 make 54, and with the 3 in my mind make 57, the 7 I set downe under the 2, and 5 I keepe in minde.

264
29
76

After that I come to the next figure, which is 2, and do multiply it by 9, and that maketh 18, and with 5 that I haue in minde maketh 23: wherefore because it is the last worke of the multiplier, I set it down in order as you see:

264
29
2376

And so haue I ended the first figure of the multiplier. Wherefore I giue it now a fine dash with my pen.

264
29
2376

Then begin I with the next figure, and multiply it into all the higher figures as thus:

First, 2 times 4 make 8, that 8 doe I write vnder the second place: for euermore the Digit or first figure of Multiplication that amounteth of the first figure of the higher number, must be set vnder the multiplier of it, and the other in their order toward the left hand.

Scholar. I vnderstand you thus, that the Digit of the summe amounting of the multiplication of the first figure of the higher row, by the first figure of the lower row or multiplier, must be set vnder the first place: and that that amounteth of the same first figure by the second multiplier must be set vnder the second place, and so of the other, if there be more multipliers.

Master. So meane I indeed: and if there amount but a Digit, then must it be set vnder the Multiplier.

And

And now to goe forth: I multiply by the same 2, the second figure of the higher row, which is 6, saying: two times 6, make 12: whereof I write the digit 2 under the third place, and the Article 1, I keepe in minde.

Then doe I multiply the last figure of the higher summe by that same 2, saying: two times 2 is 4, and with the 1 that I haue in minde maketh 5, which 5 I write vnder the fourth place. And so haue I ended the whole multiplication: wherefore I also giue the 2 a dash with my pen, thus: and so I doe euer as soone as I haue dispatched any Digit, by which I multiply: and the summes stand thus.

Then must I draw a line vnder all those summes that amount of the multiplication, and must adde all them into one summe, as in the example you may see.

264	
29	
2376	
28	
264	
29	
2376	
528	
264	
29	
2376	
528	
7656	

Where in the first place I finde but 6, and therefore write I it vnder the line. Then in the second place 8 and 7 make 15, whereof I write 5, and keepe one in my minde, and so forth as you learned in Addition. And so appeareth the whole summe to be 7656, which amounteth of the Multiplication of 264.

by 29, and that is the iust number of Lambes the 29 men had.

Scholar. If there be no more to be obserued in it, then can I doe it, I suppose, as by this example I shall proue.

¶ There is a peece of ground which containeth 1365 yards in length: and 236 yards in breadth, I would know how many yards square there is in all this peece of ground, which numbers I set downe with the greater aboue, and the lesser vnder, as you see.

	1365
	236

Then doe I multiply 5 by 6, saying: 6 times 5 make 30: of which I write the cipher in the first place, and the Article 3 I doe keepe in minde to carry to the next place.

	1365
	236

Then doe I by the same 6 multiply the second figure of the higher summe, which is 6, saying, 6 times 6 make 36, and 3 in my mind make 30, of which I write the 9 vnder the second place, & the Article 3 I keepe in minde.

	1365
	236
	90

Then doe I multiply the third figure, which is 3, by the same 6, and that maketh 18: and with 3 in my minde make 21. Then I I set downe, and keepe in minde.

	1365
	236
	190

Then come I to the last figure of the higher summe,

summe, and multiply it by 6, saying: 6 times 1 make 6, and 2 in my mind make 8, that 8 doe I write vnder the fourth place. And so haue I ended the first multiplier, and dash him sleightly with my pen.

Then beginne I with the second multiplier, and say first, 3 times 5, that maketh 15, of which I set the 5 vnder the second place, because that the multiplier is there, and the Article 1 I kepe in minde.

Then come I to the second figure, that is 6, and multiply it by 3, which maketh 18, and with one in minde maketh 19, the 9 I set downe vnder the third pace, and 1 I kepe in minde.

Then come I to the third figure, which is 3, and multiply it by 3, saying: three times three make 9, and with 1 in my minde make 10, the Cipher I set vnder the fourth place, and the Article I kepe in minde.

And then comming to the last figure 1, I multiply it by 3: and it maketh 3, and with the one in minde it maketh 4: which 4 I set in the fit place, and then haue I ended

1365	
236	
8190	
1365	
236	
8190	
1365	
236	
8190	
95	
1365	
236	
8190	
095	
1365	
236	
8190	
4095	

ended two of the multipliers, & the summes stand as you may see in the latter end of the page going before. And then I giue 3 his oath.

Then come I to the third multiplier, and multiply it into every figure of the higher sum. and first I say: 2 times 5 make 10, of which I set the cipher vnder the multiplyer in the third place, and the Article 1 I kæpe in minde.

1365
236
8190
4095
0

And so multiplying the second figure 6, by that same 2, there amounteth 12: & with one in my minde make 13, whereof I write the Digit 3 vnder the fourth place, and the Article 1 I kæpe in minde.

1365
236
8190
4095
30

Then doe I multiply the said 2 by the third figure of the higher summe, which is 3, and that maketh 6, and with the one in minde make 7, which 7 I set downe vnder the fift place, as appeareth by the example.

1365
236
8190
4095
730

Then come I to the last place, and multiply that 1 by 2, and there amounteth 2, which I set in the first place, and then doth the summe stand thus.

1365
236
8190
4095
2730

And so haue I ended the whole

whole Multiplication.

But now (as you taught me) to know what this whole summe is, I must adde all those parcels together, and then vnder the line will appeare, as you may see, the grosse or totall summe, that is, 322140. Whereby I know there is so many yards square in that peece of ground.

	1365
	236
8	8190
	4095
	2730
	322140

Master. This is well done, and call it workeman.

Scholar. Then, me thinketh I could call it well done, when I know, whether I had well done or no.

Master. It is to be proued by 9, as Addition was, but the surest proofe is by diuision, and therefore I will reserue that proofe by diuision, till you haue learned the Art of diuision. And anone I will shew you how it is commonly proued.

¶ But first, for your further instruction in this exercise of Multiplication, I will with one example more try your cunning, and so make an end: And the question is this. I would know how many dayes it is since the Natiuity of our Lord and Sauiour Iesus Christ vnto this present yeere 1590. Which to performe, you must multiply by the dayes in one whole yeere, which are 365.

9

Scholar. Now, for that you haue giuen me so

so much light into the question, you shall see I will handsomely finish the worke, for according to your former instructions I set them downe with a line vnder them, thus.

	1590
	365

Then say I, 5. times 0. is 0, the 0. I set downe vnder the first place, as here appeareth. Then say I 5. times 9. make 45, the digit 5, I set downe in the second place vnder 9, and the article 4. I keepe in minde to be added to the next multiplication. Then saying fiue times 5 make 25, and 4. in minde makes 29, the 9. I set down in the third place, and 2. I keepe in minde. Then comming to the last figure, I say once fiue is 5, or 5 times 1 is 5, and 2 in minde make 7, that 7 doe I set downe vnder the fourth place. And thus haue I ended my first Multiplier, and therefore I giue it a dash with my pen.

	1590
	365
	7950
	9540

Then come I to the second Multiplier, which is 6, and doe likewise multiply it into the vpper number, saying, 6 times 0 is 0: the 0 I set downe in the second place right vnder his multiplier. Then say I, 6 times 9 make 54, the 4 I set downe vnder the third place, and 5 I keepe in minde. Then say I, 6 times 5 make 30, and 5 I keepe in minde make 35, the digit 5 I set downe in the fourth place, and 3 I keepe

kéepe in minde. Then say I, 6 times 1 is 6: or once 6 is 6, and 3 in minde make 9, which I set downe next, and so haue I ended two multipliers: wherefore I daily the 6 with my penne.

Then begin I to multiply the third multiplier into the ouer number, saying, 3 times 0 is 0: the 0 I set downe in the third place right vnder his multiplier. Then say I, 3 times 9 make 27, the digit 7 I set downe in order next, and two I kéepe in mind, then say I, three times 5 is 15, and 2 in minde is 17: the 7 I set downe, and 1 I kéepe. Lastly I say, once 3 is 3, and 1 I kéepe is 4, which I set downe orderly next: And so haue I ended the multiplication, and my figures stand thus.

1590
365
7950
9540
4770

Master. I commend you for your diligence, the worke is very perfectly done, which parcels if you now adde together into one summe, it will be 580350, which is the grosse or totall summe of that multiplication, and declareth the number of daies since our Lord and Saniour his incarnation, vnto the end of 1590 yéres, beside 397 dayes and twelue houres for leape yéres.

Scholar. This is marvellous, me thinketh, that such great matters may so easily bée atchieued by this Art, which heretofore I euer thought had béene impossible, as infi-

nite

nite sorts of people are of that minde.

Master. Truth it is, that knowledge hath no greater enemy than ignorance, for this is one of the least of temne thousand things that may be done by this Art, as hereafter you shall be able to iustifie.

Scholar. This manner of Multiplication I perceiue, if there be no more in it.

Master. Yes, there are other formes and helps for ease and shorter labour of the worke of multiplication: but I will remit them till you haue a little tasted Diuision, where also the like helpe into Diuision may be vsed: and so therefore vnder one example for both, will I shew you both ease in Multiplication, and also in Diuision.

But sith the other formes and workings doe nothing differ from these works in effect, but onely in setting of the numbers, I will ouerpasse them till a more meet place and time. And now will I instruct you in Diuision, so that you thinke your selfe sufficiently to perceiue what I haue taught you.

Scholar. Yes, Sir, I thanke you, but I do not perceiue how to examine my worke, to trie whether I haue well done, or no: therefore as you promised me ere-while, I pray you first shew me how I shall proue it.

Master. That is commonly vsed by the proofe of 9, as you learned before in Addition, saiuing that it differeth from that forme in diuers

diuers respects: as for example.

First, you must make a crosse after this manner.

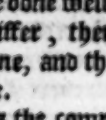
Then must you examine your sum that should be multiplied, and loke what remayneth after casting away of 9, that set you at the one side of the crosse, then examine the multiplier, and whatsoeuer remayneth in it after casting away 9 so often as you can, write that at the other side of the crosse: then must you multiply those two numbers together, and loke what amounteth thereof, if it be vnder 9, write at the higher part of the crosse: but, if it be aboue 9, then take thence 9 as often as ye can, and write the rest at the head of the crosse: as first for example, we will proue the example you put forth of the peece of ground that contayned 1365 yards in length, and 236 yards in breadth.

Therefore first, I cast away all the nines from the summe to be multiplied, saying: 5 and 6 make 11 cast away 9, rest 2: then 3 and 2 make 5, and 1 is 6: that 6 I write at the one side of the crosse thus.

Then doe I examine the multiplier, which is 236 wherein, when the 9 is cast out, there remayneth 2, that 2 therefore I set at the other side of the crosse.

Then doe I multiply 6 by 2, and it maketh

maketh 12, from which 12 I withdraw 9, then resteth 3, that 3 doe I set at the head of the crosse. Then doe I examine the grosse summe amounting of the multiplication, which is 322140, where I finde 9 once, and 3 remayning, that 3 I set at the foote of the crosse, and then I see it to agree with the other 3 at the top of the crosse, & so know I that I haue done well: for if they two did differ, then were my worke vaine, and the multiplication false.



This is the common proofe, but the most certaine proofe is by Diuision, of which I will anone instruct you.

Scholar. Sir, what is the chiefe vse of Multiplication?

Master. The vse of it is greater than you can yet vnderstand: howbeit, these plaine commodities it hath, that if you would resolue any great and whole value into many small and lesse portions: as if you would change pounds into shillings, pence or any other greater or smaller parcels, by Multiplication, ye shall doe it spedily and easily. Also if you should neede to adde one summe to it selfe, or to any other oftentimes, you shall doe it by Multiplication much more spedily, readily, easily and surely, than by often and sundrie Additions. Take you these commodities grossely shewed for an answere at this time,

and

and hereafter I will more abundantly make you to perceiue the vse of it.

Diuision.

Scholar.

WEll Sir, then in Diuision I pray you to instruct mee. But mee thinketh by the name of it, that it should bee all one with Multiplication: for I call that Diuision, when any thing is parted into diuers and many parts.

Master. You take it as it is taken commonly: howbeit, if you marke well, you shall perceiue that it is quite contrarie to Multiplication, and doth not part one thing or fewe things into many, but contrarie wayes, it bringeth many parcels into few, but yet so, that these few taken together, are equall in value to the other many: for by Diuision, pence are turned into shillings, and shillings into pounds: as for example of 120 shillings it maketh 6 pounds, so are 120 turned into 6, which is a smaller number: but then if you consider the Denominations, you shall see that they are such, that

one of the latter is equall to 20 of the first, and so in value the summes are one, though in number they doe farre differ, and the latter summe is the lesser, and so it is alwayes in Diuision: howbeit, yet in the working, the summe is parted by another, and thereof doth it take the name.

Scholar. I thinke I shall better vnderstand the reason of the name when I know the vse of the worke, therefore now would I gladly learne that.

Diuision what it is.

Master. Diuision is distributing of a greater summe by the vnites of a lesser. Or Diuision is an Arithmaticall producing of a third number, in respect of two propounded numbers, which third number shall so often containe an vnite, as the greater of the two propounded numbers can containe the lesser. So that euen as Multiplication did seeme to serue instead of many Additions, so Diuision may seeme to be in place of many Subtractions: Because that third number briefely expresseth how many times the lesser of your two propounded numbers may be subtracted from the greater: as in practise will more plainely appeare. Therefore (as you may perceiue) vnto Diuision are required three numbers: the first, which should be diuided, and that must (generally) be the greater: and the second, by which the other must be diuided, and that is (generally) the lesser, and is called

the

the Diuisor. And the third, which answereth to the question (How many times?) and therefore is called the quotient.

The first must be first written, and the second so set vnder it, that the last figure of the lower number be right vnder the last of the higher, contrariwise to the worke of the other kindes of Arithmeticke: for in them the two first figures were set euer meete one vnder the other: but in Diuision the last figures must be set meete, except it chance so that the last figure of the diuisor be greater than the last of the higher number, for then you shall set the last of the diuisor vnder the last, saue one of the higher number, as for example. A generall rule for placing the figure. An exception.

If you should diuide 365 (which are the summe of the dayes of a yeare) by 28, which are the dayes of a common moneth, then should you them thus.

365
28

But if you would diuide those 365 dayes, by 52, which is the number of weekes in one yeare, then should you set them thus.

365
52

Likewise, if I would diuide the same 365 by 4, which is the summe of the quarters of the yeare, then must I set them thus.

365
4

Scholar. Sir, this doe I vnderstand, but how now should I doe to diuide the one by the other?

Master. You must begin with the last figure next the left hand, and see how many times the last figure of the Diuisor may be taken out of the last figure of the other number, and that shall you note within a crooked line toward your right hand. As for example.

I would diuide 365 by 28,
then set I those two summes thus.

365 (
28

And I loke how many times I may finde 2 (which is the last figure of the Diuisor) in 3, (which is the last of the number to be diuided) and considering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I set 1, and that is called the Quotient number, as I told you. Then because that when 2 is taken out of 3, there remaineth 1, I must write that one ouer 3, and deface or cancell the 3 and the 2, then will the figures stand thus.

Quotient number.

1
365 (1
28

Then must I goe to the next figure of the Diuisor, and take it likewise so many times out of the figures that be ouer it, and loke what doth remayne, that I must write ouer them, and cancell them, as in this example.

Therefore now doe I take once 8 out of 16, and there remayneth 8, which I must set ouer the 6, and cancell or crosse out the 16, and the 8 of the Diuisor: and then will

will the figures stand thus.

And so haue I once wrought. 28

Scholar. So I perceiue that 365 (1
you take the neather figure, not 28
onely out of the other that is
right ouer him, but out of that with the other also that remayneth before, and are written toward the left hand.

Master. So must you doe: for you must so take the Diuisor out of the ouer number, that there remayne not ouer it so great a summe as it selfe is, for then were your worke in vaine.

But yet againe here must you marke, that when you seeke how many times the last figure of the Diuisor may be found in the number ouer him, that you looke also whether you may as often finde all the figures following in those that are aboue them (considering all the remaynders, if there be any) if not, take your Quotient lesse by one, and then proue againe and so still till you finde a meete quotient: and by that meete Quotient must you alwayes multiply your Diuisor, and the product set vnder your Diuisor, so that the first figure stand vnder the first figure of your Diuisor, and the second vnder the second, and so forth: and then subtract that product from the number to be diuided, that standeth directly ouer it, as you haue seene me doe.

When you haue thus wrought once, then

must you begin again, and write your Diuisor anew, nearer toward the right hand by one place, as in this example, you shall set 2 vnder 8, and 8 vnder 5 thus.

```
18
365 (1
288
 2
```

Then (as before) seeke how many times you may take your diuisor out of the number ouer him now.

Scholar. That may I doe here 4 times.

Master Truth it is, that you may finde 2 foure times in 8: but then marke whether you can finde the figure following so many times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholar. No, neyther yet once.

Master. Therefore take 2 out of 8 once lesse.

Scholar. That is three times.

Mark how to consider this kinde of Remayner.

Master. Well, then 3 times 2 make 6: if I take 6 out of 8, there remayneth 2: which 2 with the 5 following make 25, in which summe I finde 8 three times also: and therefore I take 3 as a true quotient, and write it within the crooked line of the quotient before the one thus.

```
18
365 (13
288
 2
```

Then say I, 3 times 2 make 6, then 6 out of 8 resteth 2, therefore I cancell the 8, and write ouer it the 2 that doth remayne, thus

Then

Then do I take 8 as many times out of 25 saying: 3 times 8 make 24, and if I take 24, out of 25, there remayneth 1, so then I cancell 25 and 8, and ouer the 5 I set 1, thus.

Or, you might (after you found 3 to be a fit Quotient) straightway haue multiplied the whole Diuisor 28, by that 3 at once: which giueth 84, which being set vnder 28, and duely subtracted from 85, of the number diuided, giueth 1, the remayner of the whole diuision, as before you had. Work which way you list, here you see also the forme.

```
  2
 181
 365 (13
 288
  2
```

And now haue I done with the diuiding, for I cannot finde my Diuisor 28 no more in the ouer summe.

Sholar. No, except you would part the 1 that remayneth into 28 parts.

Master. That is well said, and so must wee doe in such cases, when there remayneth any thing: but I will let that passe now, and will make you perfect in Diuision of whole numbers, and will hereafter teach you particularly of broken numbers, called Fractions.

Now if you doe perceiue the order of Diuision, then do you diuide this summe 136280 by 452.

Scholar. First, I set downe the number

that shonld be diuided, then doe I set the Diuisor vnder it, so that the last figure of it bee right vnder the last figure of the ouer number. Then will it bee thus.

	136280
	452

Master. Can you take the last of your Diuisor (which is 4) out of 1, which is the last of the ouer number?

Scholar. I had forgotten, because the last of the Diuisor cannot be taken out of the last of the ouer number, in so much as it is the greater, therfore must I set the Diuisor one place more forward, toward the right hand thus.

	136280
	452

And then must I loke how often I may finde the last figure of the Diuisor (that is 4) in 13, which thing I may doe 3 times, therefore doe I say: 3 times 4 is 12: which I take out of 13, and there remayneth 1. Then doe I make at the right hand of my summes a crooked line, and write before it my Quotient 3, and I cancell 13 and 4, and ouer the 3 I set the 1 that remayneth, and then the figures stand thus.

	1
	136280(3
	442

Then do I multiply the same Quotient into euery figure of the Diuisor, and withdraw the summe that amounteth out of the numbers

bers ouer them, as first I say: 3 times 5 make 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remayneth thus.

XI
XZ6280 (3
452

Thus doe I say likewise, 3 times 2 make 6, which I take out of 12, and there resteth 6, therefore I cancell the 12 and the 2, and ouer the 2 I write the 6 that remayneth thus.

XI6
XZ6280 (3
452

Then should I set forward the diuisor into the next place toward the right hand thus.

XI6
XZ6280 (3
4522
45

Master. But you may see that ouer the 4 is no figure, therefore I must set the Diuisor yet forwarder by another place.

And marke, whensoeuer it chanceth so, that you should set forward the Diuisor, and that it cannot stand there, because there is no number ouer the last place, or if there be any it is lesser than the last figure of the Diuisor, then must you remoue the Diuisor yet once againe: and because that his first place of remouing serued not to subtract him so much as once, therefore you shall write in the quotient a cipher 0. And if you should by chance neede to doe so oftentimes, for euery time write a

cipher

cipher in the Quotient. The reason of this will I shew you hereafter.

Scholar. Then must I set my summes thus.

And, because I remoued the Diuisor, so that I ouerskipped one place, I must write a Cipher in the Quotient: and then must I seeke a new Quotient, as in this example I must say. How many times 4 is there in 6? (and sith it can be but once) therfore doe I write 1 in the Quotient: and then say I, 1 times 4 taken out of 6, remayneth 2, I cancell the 6 and the 4, and write 2 ouer them thus.

Then say I againe once 5 out of 28, remayneth 23. I let the 2 stand as it did, and ouer that 8 I set 3, cancelling the 8, and the 5 vnder it, thus.

```
 ~~1~~~~1~~~~6~~
 ~~1~~~~3~~~~6~~~~2~~80(3
 ~~4~~~~5~~~~3~~52
 ~~4~~

 2
 ~~1~~~~1~~~~6~~
 ~~1~~~~3~~~~6~~~~2~~80(301
 ~~4~~~~5~~~~2~~52
 ~~4~~

 ~~2~~
 ~~1~~~~1~~~~6~~
 ~~1~~~~3~~~~6~~~~2~~80(301
 ~~4~~~~5~~~~2~~52
 ~~4~~

 ~~2~~
 ~~1~~~~1~~~~6~~3
 ~~1~~~~3~~~~6~~~~2~~80(301
 ~~4~~~~5~~~~2~~52
 ~~4~~
```

Masser. You might as well haue said, once 5 out of 8, and so remayneth 3, but now goe forward.

Scholar.

Scholar. Then once 2 out of 0 cannot be: What ſhall I now doe?

Maſter. Borrow of the next number that is behinde (for there is 230) and doe as you learned in Subtraction in a like caſe.

Scholar. Then muſt I borrow 1 of the 3 comming behinde next, and make that 0 to be 10, and then take I 2 out of 10, and there reſteth 8. And becauſe I borrowed one of the 3, I muſt cancell the 3, and write 2 ouer it: then doth the figure ſtand thus:

22
136380 (301
452
455
4

Maſter. Now haue you done, and yet remayneth 228, and your quotient ſheweth you that if you diuide 136280 by 452, you ſhall finde your Diuiſor in your greater number 301, that is CCC times and once, and 228 remayning.

And in the other example (where I diuided 3695 by 28 the Quotient was 13 and 1 remayned, whereby I knew that in a yere (which containeth 365 dayes) there are 13 moneths, reckoning 28 dayes (or 4 weekes) iuſt to a moneth, and 1 day more.

Scholar. Why then doe we call a yere but twelue moneths?

Maſter. Of that at a more conuenient time will I fully inſtruct you: but now it is not conuenient to intangle your minde with

other

ther things, then doe directly pertaine to your matter. Therefore if you remember what you haue heard, you haue learned a short manner of Diuision, which I would haue you often to practise, so that you may be perfect in it, and hereafter I will shew you certaine other proper points touching it.

Scholar. Then I pray you yet tell me how I shall examine and trie my worke, whether I haue done well or no, that though no man be by me to tell me, yet I may perceiue it my selfe.

Master. Some men (yea and commonly most) doe trie it by the rule of 9, as in all the other kindes, saue that their order is: First, they cast away 9 as often as they can out of the Diuisor, and that remayneth they set at one side of a crosse, as in our first example the Diuisor was 28, from which you may take 9 three times, and one remayneth: which they set by a crosse, thus:

Then doe they likewise examine the Quotient (which in our example is 13) and from thence they cast away 9 as often as they can, and the remaynder they set at the other side of the crosse, and then multiply they together those two remayners: and to it that amounteth they adde the remayner of the Diuision, if there were any: from that whole summe they withdraw 9 as often as they can,

and

and the reſt they ſet at the head of the croſſe, as in our example the quotient is 13, from which take 9, and there remayneth onely 4, and therefore muſt you ſet 4 at the other ſide of the croſſe, thus:

Then multiply 4 by 1, and it yeldeth but 4: thereto adde the remayner of the Diuiſion (which was 1) and it will be 5, which ſumme doth not amount to 9, and therefore muſt be ſet wholly at the head of the croſſe, as you ſée héere.

And this number on the head of the croſſe is the firſt proofe, to which if you finde another like in the number that was diuided, then haue you done well.

Therefore now ſhall you likewiſe examine the whole ſumme that was diuided, and take away 9 as often as you can, and that that remayneth, ſet at the foot of the croſſe: and if it be equall to that in the head of the croſſe, then haue you well done, elſe not.

As in our example the whole ſumme was 365, which maketh 14 from that take 9, and there reſteth 5, which ſet at the foote of the croſſe, thus:

And you ſhall ſée that they agrée: therefore haue you well done.

Now will I likewiſe examine our ſecond example,

example, where the Diuisor was 452, which maketh 11 from thence I take 9, and the 2 that remayneth I set at the right side of the crosse, thus:

Then examine I the Quotient, which was 301, where I finde but onely 4, that doe I set at the other side of the crosse, thus:

Then doe I multiply 4 by 2, and it maketh 8: to that doe I adde the remayner of the diuision, (which was 228, and maketh 12) and they two make 20: wherein I finde twice 9, and 2 remayning: that 2 must I set at the head of the crosse, thus:

Then doe I examine the whole number to be diuided, which was 136280, where I finde twice 9 and 2 remayning, which I set at the foote of the crosse, thus:

And because that it doth agree with the figure at the head of the crosse, I know that the diuision was well wrought.

Againe of the proofe of diuision

Master. This is the common proofe. Howbeit, the more certaine working is by the contrary kinde: as, to proue Diuision by Multiplication, thus:

Multiply the Quotient by the Diuisor, and if

if the ſumme that amounteth be equall to the ſumme that ſhould be diuided, then haue you well diuided: elſe not.

Howbeit, this muſt you marke, that if there remayned any thing after the Diuiſion, that muſt you adde to the ſumme that amounteth of the Multiplication. As in our firſt example our quotient was 13, and the Diuiſor was 28: Now multiply the one by the other, and the ſumme will be 364: to that if you adde the one that remayned after the diuiſion, then will it be 365, which was the ſumme that ſhould be diuided: and therefore I know that I haue well done.

Scholar. Now will I proue the ſame in the ſecond example, whoſe Diuiſor was 452 and the Quotient 301: theſe doe I multiply together, and there amounteth 136052: to which if I adde the 228 that remayned, then will it be 136280, which was the whole ſumme to be diuided: and therefore I perceiue that I haue well done.

Maſter. This is the ſureſt way, to examine Diuiſion by Multiplication: and contrariwiſe the ſureſt proofe of multiplication is by Diuiſion.

And therefore (according to my promiſe) now will I ſhew how you may proue Multiplication by Diuiſion.

A proofe of Multiplication by diuiſiõ.

When you haue ended Multiplication, and would know whether you haue well done

or not, set the crosse summe that amounteth of the multiplication ouermost, and diuide it by the multiplier: and if the Quotient be the same number that should be multiplied, then haue you well wrought, else not, as in that example where we multiplied 264 by 29, the crosse summe was 7656.

Now if you will know whether that multiplication be true, you shall diuide that 7656 by the multiplier 29, and you shall perceiue that the quotient will be 264, and that is a token that you haue well wrought.

Scholar. By your patience I will proue that, and first set downe the grosse summe and the multiplier, not after the rule of Multiplication, but after the rule of Diuision, for now that number is become

the Diuisor, that was before	7656
the Multiplier, I shall set	29

them therefore thus:

Then shall I seeke how many times 2 in 7: that may be 3 times, and 1 remayneth: but then may not 9 be found so often in 16, therefore must I take a lesser Quotient, that is to say 2: then say I, twice 2 maketh 4, which I take out of 7, and there remayneth 3, then doe I cancell 7 and 2, and

ouer 7 I write 3, and in	3
the Quotient I set 2: so	7656 (2
the figures stand thus:	29

Then say I forth, two times 9 make 18, which

which I abate out of 36, and there resteth 18:
then cancell I 3, and ouer
him set 1, and likewise I 1
cancell 6 and 9, and ouer 38
them I set 8, so that thus 7656 (2
stand the figures: 89

Then doe I set forward the Diuisor by one place, and seeke a new Quotient, that is to say, how many times 2 are in 18, which I finde to be 9 times: but then can I not finde 9 so many times in 5, therefore I take a lesser Quotient, as to say, 8: but yet that is too great: for if I take 8 times 2 out of 18, there remayneth but 2, and I cannot finde 8 times 9 in 25: therefore yet I take a lesser Quotient, that is 7, which is also too great, for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 25, therefore yet I seek a lesser quotient
as to say, 6: then say I, 6 times 16
2, makes 12, that I take out of 38
18, and there remayneth 6: so 7656 (26
I cancell 18, and the 2, and 899
write 6 ouer 8 thus:

Then say I forth, 6 times 9
maketh 54, that take I out of 1
65, and there remayneth 11 16
and the figures stand thus: 381

Then must I set forth the 7656 (26
diuisor againe, and seeke a new 399
quotient, which will be 4: for 8

I though

though I may finde 2 in 11 fiue times, and 1 remayne, yet I cannot finde 9 so often in 6: therefore I set the figures thus:

And the 4 in the Quotient I multiply into the figures of the Diuisor, saying, 4 times 2 maketh 8, which I take out of 11, and there resteth 3: therefore I cancell the 11 and the 2, and set 3 ouer the first place of 11, thus:

And then doe I say forth: 4 times 9 maketh 36, which I take from 36, and there remaineth nothing: so that the quotient of this diuision also (where 7656 is diuided by 29) is 264. Which both declare, that if 264 be multiplied by 29, the sum will be 7656. And thus I perceiue now how both Multiplication is proued by Diuision, and Diuision also by Multiplication.

Master. Now haue I ended the fiue common kindes of Arithmetick. For (as touching Mediation, Duplation, Triplation, and such other) they are no seuerall kindes of Arithmeticke, but are contayned vnder the other. For Mediation is contayned vnder Diuision, and is nothing else but diuiding by 2: and so are Duplation and Triplation contayned vnder Multiplication. For Duplation is nothing else but multiplying by 2: and Triplation is multiplying

tiplying by 3: of which I will only propose an example for the rules you haue heard already.

If you would mediate or diuide into 2 this summe 4531010, you shall set 2 for the Diuisor, and worke as you learned before, as thus:

```
4531010 (
2
```

An example of Mediation.

Then I finde 2 in 4 two times: therefore my Quotient must be 2: so I cancell 4 and 2, and remoue the Diuisor forward thus, as the worke requireth, and as before in Diuision hath beene declared.

```
4531010 (2
 2
```

Which mediation or diuision by 2 being finished, you shall haue for your Quotient 2265505, which is the halfe of 4531010, as you may trie by Duplation: for, double that Quotient, or multiply it by 2, and the same number will amount.

Duplation.

I will no longer tarrie about these, seeing they are but members of the other kinds. But here now (according to my promise) I will teach you certain easie formes both of Multiplicatione of diuision. And first of Multiplication.

If you would therefore multiply any summe by 10, you shall need to doe no more but adde a cipher before his first place: as for example, 36 multiplied by 10 make 360.

Easy formes of Multiplication.

Likewise if you would multiply any summe by 100, put two ciphers at his beginning. So if you would multiply any summe by a thousand, adde three ciphers to the beginning of it.

Scholar. This doe I well perceiue, and also the reason of it.

Master. I will omit all reasons till our next meeting, when I shall tell you the reason of all other parts of Arithmetick also: and as to our matter now, loke, as I haue told you, that you both remember it, and also often practise it.

And now haue you learned how to multiply eallily by 10,100,1000: and of like manner may you doe with any other of that sort.

But now if you will multiply by 20,30,40, and so forth, or by 200, 300, and such like, where there is one cipher in the first place, or many orderly in the first places, you shall take away those ciphers, and multiply the summe onely by the other figure, or figures (if they be many) and then at the beginning of the summe that amounteth you shall set so many ciphers as you toke away.

Example of 2873, which I would multiply by 300. First I omit the 2 ciphers from the multiplier, and I multiply the summe by onely three that is left, and it amounteth to 8619: before which I put the two ciphers that I before omitted or toke away, and then is it 861900. And that is the summe that amounteth when 2873 is multiplied by 300.

Scholar. And if there were two or more figures beside the ciphers, I must onely take away the ciphers, and multiply by the other

figures,

figures, as I learned before: as if I would multiply 93648 by 25000, I ſhould take away the three ciphers, and multiply the ſame by 25, and then at the beginning of that totall ſumme ſhould I adde the three ciphers againe.

Maſter. Euen ſo: but and if it chance the number that ſhould be multiplied, or both the ſummes, as well the number that ſhould be multiplied, as the multiplier, to haue ciphers in their firſt places, euermore omit the ciphers, and worke by the reſt. But remember to reſtore as many ciphers to the amounting ſumme as you bated before, as in this example: 30200 ſhall be multiplied by 206, I ſhall onely take away the two ciphers from the greater number, and then multiply 302 by 206, and afterward adde the two ciphers againe. But if I would multiply the ſame 30200 by 2060, I ſhall not onely take away the two ciphers from the number that ſhould be multiplied, but alſo I may take away the one cipher from the multiplier, and then muſt I adde 3 ciphers to the ſumme that amounteth: but take héd that you take away no cipher that commeth after any ſignifying figure, as in this laſt example, you may not take away that in the fourth place of the higher number, neyther that in the third place of the multiplier: howbeit, yet thus you may doe: If one cipher or more come in the midſt

of your summes, you may multiplie by the other figures, and ouerskip them: but so, that you giue euerie figure his due place: as thus: I will multiplie 3026 by 2004: therefore I set them thus:

3026
2004
12104
6052

And thus I doe multiplie them: First, foure times 6 make 24: I set the 4 vnder the first place, and keepe the 2 still in my minde. Then say I againe, 4 times 2 maketh 8, and the 2 that is in my minde maketh 10: I set downe the cipher 0, & keepe the article 1 in my minde: Then 4 times 0 is 0, and the one in my minde maketh 1: I set downe the figure 1, and say againe, 4 times 3 is 12: I set downe 2, and keeping the 1 still in my minde (hauing no more places of the vpper number to multiply it with all) I put it downe next 2 in the fift place.

But now when I come to the next place (being a cipher 0) I let it go, because it multiplieth nothing: and likewise the second cipher.

But then, when I doe come to the 2, and multiplie into the 6 of the ouer number, you must take heede (according as I taught you in Multiplication) that the first number amounting of the Multiplication be set right vnder the Multiplier, & the other orderly toward the left hand, according as you may see in this example:

3026
2004
12104
6052

which

which being finished, with the addition thereof gathered together, will stand as this example sheweth.

Which is indeede wrought so much the sooner and shorter by ouerskipping of the 2 ciphers: which otherwise (if the same example were wrought at length) it would haue had 2 workings more, as by the same example here also set downe doth appeare.

	3026
	2004
	12104
6052	
6064104	

Scholar. Sir I thanke you: for I see great ease in this way of multiplication: and (if you can shew me such like in Diuision) you shall greatlie further me.

Master. Yes, I will teach you some easie waies in Diuision also, and first this: If you would diuide any summe by 10, you shall only with your pen make a square line betweene the first figure of your summe and the second, and then haue you done: for the whole number that followeth the line standeth for the Quotient, and the figure that is before the line, is the remayner: as for example, 3648 diuided by 10 will stand thus:

364 (8

Where 364 is the Quotient, and betokeneth that so many times are 10 in 3648, and the 8 after the line is the remayner, which cannot be diuided into 10, but by breaking it

into fractions, wherewith I will not meddle yet.

And so likewise if you would diuide any summe by 100 with your pen, you shall cut away the two first figures, and if you would diuide by 1000, you must cut away the three first figures, and so of any other diuisor, whose last figure is 1, and the other ciphers, looke how many ciphers the diuisor hath, and so many figures at the beginning shall you cut away with the square line, and they stand alwaies for the remayner, because they are lesse then the diuisor, and cannot be diuided by it, and the other figures that are behinde the line stand for the Quottient.

But now if your diuisor haue any other figure in his last place then 1, and in all his other places haue ciphers, looke how many ciphers they be, cut away so many of the first figures of the number that should be diuided, and diuide the rest that followeth the line by that figure that is in the last place, as if it were the whole diuisor.

Example of 64284, which I would diuide by 300: here must I cut away the two first figures (for so many ciphers my diuisor hath) and must diuide the rest by 3, which is the figure in the last place of the diuisor: First therefore I part away the two first figures, and the summe standeth thus:

642 (48

Then

Then doe I diuide 642 by 3, and the Quotient is 214: for in 6 I find twice 3, and in 4 once, and 1 remaining, which 1 with the 2 next before doth make 12: wherein I finde 3 foure times: and this is a ready way to turne shillings into pounds: for fith one pound doth contayne 20 shillings, I muſt diuide the whole number of shillings by 20. Therefore eaſily to doe it, I ſee it that my diuiſor hath one cipher, and therefore I cut away one figure from the beginning of the whole ſumme of shillings, and then I doe mediate or diuide by 2 the other figures of ſumme that followeth.

Scholar. I will put an example.

If you would diuide 64287 shillings by 20: that is to ſay, if I would turne ſo many shillings into pounds, I muſt cut away the firſt figure, that is 7, and diuide the reſt, that is 6428 by 2, ſo ſhall the quotient be 3214, whereby I know that 64287 shillings make 3214 pounds, and 7 shillings remayning.

Maſter. Now proue by Multiplication whether you haue done well or no.

Scholar. The quotient is 3214, which I doe multiply by the Diuiſor 2, and it doth amount to 6428.

Maſter. Hereby you may perceiue not only that you haue well done, but alſo how by Diuiſion you may turne ſhillings eaſily into pounds: and contrariwiſe by multiplication you may turne pounds into ſhillings.

But

But here shall you se amongst diuers men diuers formes of such diuision: but if you marke what I haue told you, you shall perceiue easily all the waies. For some men doe not cut away so many of the first figures of the summe that they would diuide, as there are ciphers in the first places of the Diuisor: but they set all their ciphers orderly vnder the first places of the number that they would diuide; and then with the other figure or figures (if there be many) they diuide the rest of their summe. Example. If they would diuide 725931 by 3400, they doe set their summes thus:

Another manner of the abridgement.

725931	
3400	

And then doe they diuide orderly till they come to the ciphers: for there they stay and end their worke, as in this example. They seeke how often 3 may be found in 7, which is two times, and one remayning: therefore they set 2 in the quotient, and cancell 3 and 7, and ouer 7 they set the 1 that remayned, thus:

1	
725931	(2
34 00	

Then doe I goe forth saying, two times 4 maketh 8, which they take out of 12, and there remaineth 4, thus:

24	
725931	(2
34 00	

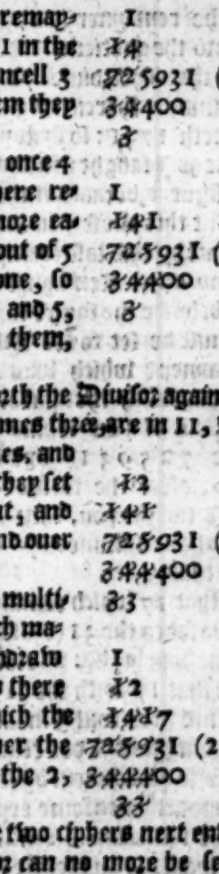
Then renew they the diuisor forward, and seeke how often 3 may be found in 4, which is

is but once, and 1 remayneth, then set they 1 in the Quotient, and cancell 3 and 4, and ouer them they set that 1, thus.

Then take they once 4 out of 15, and there resteth 11. Or else more easily: Take once 4 out of 5 and there resteth one, so they cancell the 4 and 5, and set one ouer them, thus.

Then set they forth the Diuisor againe, and seeke how many times three are in 11, which they find three times, and 2 remayning: so they set 3 in the Quotient, and cancell 11 and 3, and ouer them set 2, thus.

Then doe they multiply 4 by 3, which maketh 12, that withdraw they out of 29, and there resteth 17, of which the 7 must be set ouer the 9, and the 1 ouer the 2, thus

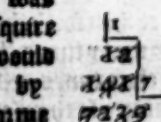


And now are the two ciphers next ensuing, so that the Diuisor can no more be set forward, and therefore is the diuision ended, and

and the remayner is 1731.

Now the quotient, which is 213, doth declare, that if you diuide 725931, by 3400, you shall finde it therein 213 times, and there remayneth 1731: so shall you finde it, if you worke as I taught you, by cutting away the 2 first figures, because of the 2 ciphers.

But this must you marke (as you may perceiue by this last example,) that if there be left any other Remayner in the summe that was behinde the squire line, that the Remayner must be set to the latter end of the first Remayner, which was cut away with the squire line: as if you would diuide 725931 by 3400, after the same that I taught you, then would your summes appeare thus,



So that 17 which remayneth after the line, must be set to the 31 (that was cut away with the line) in higher places, as you se here: where that 17 with the 31, doe make 1731.

¶ And here will I make an end of Diuision, (sauing that I doe request you to exercise your selfe well herein by many summes, till you haue attayned some expertnesse therein.) For the reasons and conclusions thereof are so many, and so anaileable for all sorts of men whatsoeuer, that if I should speake of the infinite

finite vles thereof, I should rather lack words then matter. And therefore recommending it to your iudgement hereafter, vpon your further trauell into the Art, I will here end this Treatise, representing vnto you one example or simple question of Diuision and Multiplication in stead of many, which is this.

There are 4 brasse peces: The first of them at a shot spendeth 9 pounds of powder, the second spendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed against the battery of a hold, and there is allowed by the Maker gunner 700 pounds of powder to be spent by these 4 peces in this assault. The question is two-fold: The first how many shot each pece shall iustly make about with this 700 pounds of powder? And lastly, how many pounds of powder ought iustly to be allowed to each pece for his true proportion?

Scholer. Why sir, you make me smile, to beare me in hand, that these two demaunds may be simply resolued by Multiplication and Diuision.

Master. Truly that they may: and that may you by and by worke your selfe with a little labour: First adde together their quantities of powder, that is 9 pounds, 5 pounds, 4 pounds, and 2 pounds, all which make 20: Diuide the 700 pounds of powder by that 20, & your Quotient giueth 35, as here appeareth,

which

which sheweth for most certainty that they shall make iust 35 shottes about.

1
700
200 (35
2

Scholar. Sir, all this haue I done, and I see it is so, but whether it be true or not, I cannot tell.

Master. To trie the truth of the same, multiply the first peece that spends 9 pounds by 35, and you shall see his allowance, which is 315 pounds of powder. Multiply also the second peece that spends 5 pounds by 35, and you shall finde 175 pounds his allowance: then 4 by 35, and you shall finde 140 pounds his allowance. Lastly, multiply 2 by 35, and you shall finde 70 pounds his allowance. All which foure particular summes you shall adde together by Addition, as here appeareth, and it maketh iust 700 pounds, and so is the question truely absolued

315
175
140
70
700

Scholar. Truly sir, these excellent conclusions doe wonderfully more and more make me in loue with the Art.

Master. It is an Art that the further you trauell, the more you thirst to go on forward. Such a fountaine that the more you draw the more it springs. And to speake absolutely in one word (excepting the studie of Diuinitie, which is the saluation of our soules) there is no

no ſtudy in the world comparable to this for delight in wonderfull and godly exercise: For the skill hereof is well knowne immediately to haue ſtolued from the wiſedome of God into the heart of man, whom hee hath created the chiefe image and inſtrument of his praiſe and glory.

Scholar. The deſire of knowledge doth greatly encourage me to be ſtudious herein: and therefore I pray you ceaſe not to inſtruct me further in the vſe hereof.

Maſter. With a good will. And now therefore for the further vſe of these two later, that is, Multiplication and Diuiſion, I will briefely ſhew you the feate of Reduction.

Reduction.

REduction is, by which all ſummes of groſſe denomination may be turned into ſummes of more ſubtile denomination. And contrariwiſe, all ſummes of ſubtile denomination, may bee brought to ſummes of groſſer denomination.

Scholar. What call you groſſe denomination and ſubtile denomination.

Maſter. That I call a groſſe denomination which

which doth containe vnder it many other subtiller or smaller: As a pound (in respect to shillings) is a grosse denomination, for it is greater then shillings, and containeth many of them. Any shillings (in comparison to pounds) are a subtile denomination, for because they are lesser then pounds, and many of them are contayned in one of the other: and so likewise of other things, whatsoeuer thing is compared to other, if it be a greater, and containeth many of them, it is a grosser denomination: but if it be lesser (so that many of them are in the other (then are they called subtile denominations: whereby you may perceiue that one denomination may be called a grosse denomination, and also a subtile (that is to say, a great and a small) in diuers comparisons. For shillings compared to pounds, are a subtile or small denomination: but compared to pence, they are a grosse or great denomination.

Scholar. Now I vnderstand the name, I pray you teach me the vse.

Master. The vse is easily learned, if you remember what you haue learned before. For if you will reduce any summe of a grosse denomination into a summe of a smaller or subtiller denomination, you must consider how many of that subtiller denomination do make one of the grosser denomination, and by that number or Numerator doe you multiply the other sum: as if you would reduce 20 pounds

into

into shillings, you must consider that in a pound are included 20 shillings, therefore multiply the one 20 by the other 20, and there will amount 400, whereby you may know that in 20 pounds are contayned 400 shillings. Likewise, if you would reduce 30 shillings into pence, considering that in a shilling are 12 pence, you must multiply 30 by 12, and it will be 360: whereby you finde, that in 30 shillings are contayned 360 pence. And thus may you reduce any grosse denomination into a more subtiller, by multiplication, if you know how many of the lesser doe make the greater: of which thing I will anone giue you a briefe table for the most accustomed kindes of money, weights, measures, & time, and such like: whereby you may know how often each subtile denomination is contayned in the grosser, when you shall neede it for the foresaid kinde of Reduction. And also the same shall serue you, if you would reduce any summe of a subtiller denomination, into a summe of a grosser denomination. For in such Reduction you must consider (as in the other forme) how many of the smaller doe make the greater, and by that number must you diuide the other summe, and the Quotient will declare how many of the greater denomination are comprehended in that summe, as for example. If you would know how many shillings are contayned in 3240 pence, consider that 12 pence

doe make 1 s̄: you must diuide that 3240 by 12, & your quotient will be 270, whereby you know that so many shillings are in 3240 pence. But, and if you would know further how many pounds are in these 270 shillings, seeing that euery pound contayneth 20 shillings, diuide that 270 by 20, and it will be 13, and 10 remayning, whereby you may know that in 3240 pence, (or 270 shillings) are 13 pounds and 10 shillings. For euermore the remayner must be named by the name or denomination of that summe that was diuided, which in this place were shillings. And thus may you doe with any other kindes of denominations.

¶ Wherefore to the intent you may haue certaine light or knowledge in most common coynes, weights, and measures (which is the most chiefe and principallest thing in trafficke to be known) I haue in each reduction as they come in order, set downe certayne instructions incident thereunto. And first I haue hereunto added this Table, wherein is comprehended not only our currant and common coynes, but also ỹ most part of the vsual coyns of Christendome, with their iust weights and value currant in this Realme of England, intending at the latter end of my Addition to this booke, to write of the ordinary money vsed in diuers places, and their common values currant for trafficke, with the manner of their exchanges from place to place, &c.

A Table of the names and now valuation of the most vsuall Gold-coynes throughout Christendome, with their seuerall weight of Pence and Graines: and what they are worth of currant English money this present yeere. 1614.

The names and titles of the Gold.	The weight Pence.	Grains	The value. Shil.	Pence.
Great Soueraigne.	10	0	33	0
Double Souer. K.H.	8	1	22	0
Double Sou. of Q.E	7	7	22	0
Royall.	4	23	16	6
Halfe Royall.	2	11.d	8	3
Old Noble.	4	6	14	8
Halfe Noble.	2	3	7	4
Angell.	3	8	11	0
Halfe Angell.	1	16	5	6
Salute.	2	5	6	11 ob.
2 parts of Salute.	1	11	4	7
George Noble.	3	0	9	9 ob.
Halfe George Nob.	1	12	4	11 q.
First crowne K.H.	2	9	6	11 ob
Base crowne K.H.	2	0	5	6
Souer. K.H. best.	3	14	11	8 ob. q
Soueraigne K.H.	4	0	11	0

The names and titles of Gold.	The weight Pence.	Grains	The value. Shil.	Pence.
Edward Souer.	3	15.d	11	0
Elizabeth Souer.	3	15.d	11	0
Elizabeth Crowne.	1	19.d	5	6
Halfe Crowne.	0	19.d	2	9
Vnite.	6	12	22	0
Double Crowne.	3	6	11	0
Brittaine Crowne.	1	15	5	6
Thistle Crowne.	1	7	4	4 ob.q
Halfe Crowne.	0	19.d	2	9
Crosse dagger called the 6 p. peece.	3	6.d	11	0
Halfe crosse dagger.	1	15	5	6
Rose Royall.	8	21	33	0
Spurre Royall.	4	10.d	16	6
The Angell.	2	23	11	0
Halfe Angell.	1	11.d	5	6

¶ All the seuerall peces of Gold heretofore mentioned are set downe according to their valuation by the Kings Maiesties Proclamation for Gold, Dated the 23. of November, 1611.

The

The names and titles of Gold.	The waight Pence	Grains	The value. Shil.	Pence.
Vnicorne of Scot.	2	10	6	0
Scottish Crowne.	2	5	6	0
French Noble.	4	16	13	4
All sorts of French Crownes.	2	5	6	0
Old French crowne.	2	5	6	0
Flanders Rider.	2	8	6	6
Gelders Rider.	2	2	3	6
Philips Royall.	3	10	10	0
Philips Crowne.	2	5	5	0
Collen Gilden.	2	2	4	8
New Andr. Gild.	2	3	5	0
Flaunders Noble.	4	10	12	0
Halfe Flaun Noble.	2	6	5	0
Flem. Angell best.	3	6	9	0
Fland Royall orkey	3	10	10	0
Carolus Gilden.	1	21	3	6
Flaunders Royall.	2	6	5	0
Saxon Gilden.	2	2	4	8
Flaunders Crowne.	2	5	6	0
Philips Gilden.	2	3	4	2
Halfe Phil. Gilden.	1	1	2	1
Golden Lyon.	2	16	7	8
3 parts of gold. Ly.	0	21	2	5

The names and titles of the Gold.	The waight Pence	Grains	The value. Shil.	Pence.
$\frac{3}{4}$ parts of gold. Li.	1	19	4	11
Dauds Gilden.	2	2	4	0
Horne Gilden.	1	12	4	11
Old Andre. Gilden.	2	3	4	10
Cruſa. long crotle.	2	6	6	0
Cruſa. ſhort crotle.	2	6	6	2
Mil rayes.	4	20	13	4
Halfe Mil rayes.	2	10	6	8
Portigue 1 ounce.	2	16	68	0
Golden Caſtilio.	2	23	8	10
Ducket of Aragon.	2	6	6	6
Hungarie Ducket.	2	7	6	4
Double Piſtolet.	4	9	11	8
Single Piſtolet.	2	4. d	5	10
Ducket of Floren.	2	5	6	4
Double Ducket.	4	11	13	0
Single Ducket.	2	6	6	6
Doub.duc.of Rome.	4	13	21	8

The prices of Gold which the bringers in of forraine Gold shall receiue at the Mint according to the Kings Maiesties Proclamation, Dated the 14 of May in 1612.

For an ounce of French crowns being 22 Carrots fine.	3 l. 6 s.
For euery ounce of Spanish Pistolets being 21 Carrots, 3 grains and a halfe fine.	3 l. 6 s.
For Duckets of Spaine, being 23 Carrots, 1 graine fine at least the ounce.	3 l. 8 s. 8 d.
For Milreas Crusado long crosse. Crusado short crosse, the ounce.	3 l. 6, 2 d.
For Hungary Duckets being 23 Carrots, 1 gr. fine at least ỹ ounce.	3 l. 9 s.
For ỹ Checkeen of Venice being 23 Carrots, 1 gr. fine at least ỹ ounce.	3 l. 10 s.
For Barbary gold being 23 Carrots, 1 gr. fine, at ỹ least the ounce.	3 l. 9 s.

¶ And if the said Barbary gold be of lesse fineneſſe, a-batement to be made according to the rate.

For Soultains being 23 Carrots, 1 gr. fine, at least the ounce.	3 l. 8 s. 8 d.
For all other gold being 22 Carrots fine the ounce.	3 l. 6 s.

¶ And being finer, a greater price according to that rate, and being courser a lesse, so that the bringer in supply the lesse fine, with the more fine in such sort, that in the totall it make good the said rate of 22 Carrots fine.

The prises of Siluer which the bringers in of forraine Siluer shall receiue at the Mint, according to the Kings Maiesties aforesaid Proclamation.

For the Ounce of Spanish Siluer money of Ciuill.	5 s.
For the Ounce of Mexico mony	4 s. 10 d.
For Ingots of Siluer, being 11 Ounces, 2 d. weight fine, according to the Standard of England, the ounce	5 s.

¶ And for other Siluer of more fineneſſe, a better price according to that rate, and for courſer a leſſe: ſo that the bringer in ſupply the leſſe fine with the more fine, in ſuch ſort that in the totall it make good the ſaid rate of 11 ounces, 2 pence weight fine, according to the Standard of England.

The

Of Siluer Coynes currant in this Realme.

The Edward Crowne of 5 s.
The Edward halfe Crowne of 2 s, 6 d.
The Edward Shilling, halfe shilling, and the three pence.
Philip and Maries Shilling, & halfe Shilling.
The Mary Groat, and Mary two pence.
Queene Elizabeths Shilling, 6 d, 4 d, 3 d, 2 d, 1 d, three farthings, and halfe peny.

It is to be vnderstood (gentle Reader) that whereas the weight is called by the name of a peny, it is not meant a peny of siluer money, but a peny of Goldsmiths weights, which is 24 Barly cornes drie. And 20 of those pence make an ounce: and 12 of those ounces make a pound Troy. So that if a man haue not the weight wherewith to weigh any peece that may come to his hand, he may doe it with the Barly graines, or cornes being drie, and taken out of the middle of the eare.

Here would I now expresse the values of sundry other coynes of diuers countries, but for three causes I now refraine. The first and chiefest is, because they are not currant by the statutes of this Realme. Another cause is, by reason they are so vncertayne, that they be neuer long at one rate. And againe, they are so different in so many places, that it were matter

matter enough for a great booke, to speake sufficiently of them all. Howbeit yet because you shall not be altogether ignorant of them, I will shew you the values of some that are most in vse, and first of France.

The most common money are Deniers, Soulx, and Frankes: 12 Deniers make 1 s, 20 Soulx make 1 Franke: so that as you see these three kindes are like in the rate to pence, shillings, and pounds with vs, but that this is the difference, that their Denier is but the ninth part of our penie, and so their Soulx, (commonly called Solzes) goe 9 to our shillings, and 9 of their Frankes to an English pound of money. So that 3 of their Frankes make a Noble. And by those three may you practise how to reduce French money into English money, according as I haue set forth heere following.

2160 Deniers make 240 d, or 20 s.
3240 Deniers make 360 d, or 30 s.
8352 Deniers make 929 d, or 2 l, 17 s, 4 d.
2160 Soulx make 240 shillings. And so of other in like rate. As for the rest of their coynes I omit them till hereafter that you haue some vnderstanding in broken numbers.

But now as for the coynes of Flaunders, they be so changeable, that you must know them from time to time: else you cannot reduce them into our money certainly. But yet that you may haue an example of their money

to

to exercise you withall, you shall take those that be most common: as Stiuers both single and double, Groats Flemish, Carolus, and Gildens. A Flemish Groat is a little above 3 Farthings English. A single Stiuer is 1 d, ob, q, halfe farthing. The double Stiuer is 3 d, farthing. The double Stiuer Carolus is 4 d, ob, halfe farthing. Then there is also the Carolus Gilden, which is worth 20 Stiuers. And the Flemish Noble is worth 3 Carolus Gildens and 12 Stiuers.

So that if you would conuert Flemish money, or any other kinde of money whatsoeuer it be, into sterling, you must reduce it first into the smallest part of English money, that is in that coyne, as for example. If I would reduce 368 double Stiuers into English money, (considering that a double Stiuer containeth 3 d farthing,) you shall first loke how many farthings be in the double Stiuer, and you shall finde them 13: therefore multiply the summe of the Stiuers by 13, and then haue you their value in farthings, which is 4784. Now, if you diuide that by 4, then there will appeare the number of pence: but better it were to diuide it by 48 (for so many farthings are in 1 shilling) and then will the Quotient declare the summe of the shillings.

Likewise if you would reduce any summe of single Stiuers into English money, you must multiply the summe first by 13, and then haue

you

you reduced them into a certaine summe, that is to wit, halfe farthings, which summe if you diuide by 8, then will amount the summe of pence: or if you diuide it by 96, the summe of shillings will appeare.

But marke this in all Diuision: when ye doe reduce to bring one denomination into another, if there be any Remayner after the Diuision, that must be named by the denomination of the grosse summe that was diuided: as for example. I would bring 254 farthings into pence, therefore I doe diuide that 254 by 4 (for so many farthings make a peny) and the quotient is 63, which is the summe of the pence, and then remayneth yet 2, which are farthings still, as one may proue by diuiding. And this must be marked in all Diuision, namely, when it is done for Reduction.

¶ Touching Danske money, they haue their Souir, whereof 20 is a Liuer: which is 2 s̄ sterling. They haue also their Grath, whereof 80 make a Gilden, which is 4 s̄ sterling. They haue also Dollors, and their common or old Dollor is 35 Grath. New Dollors they haue, which be diuers, some valued at 24 Grath, some at 26, and some at 30. And thus much I thought good to adde to the Author, touching Danske money.

Concerning Spanish money, whereof the most common are Cornados, Paruetdes, Ryals and Duckets: fire Cornados make a Paruetde,

Maruelde, 4 Maruelds make 1 Riall, and 11 Rials make one Ducket, so the Ducket contayneth 374 Maruelds, which is about 5 s, 10 d, sterling. Therefore if you would conuert 124 l, 5 s sterling into Duckets, consider that pence is the least value or denomination named in this question: therefore reduce 124 l, 5 s into pence, and it maketh 29820 pence: which if you diuide by the pence that a Ducket is worth, (which is 70.) you shall haue for your Quotient 426 Duckets, your desire.

Thus much haue I said of money: Now will I shew you in like sort, the distinction of weights, after the Statutes of England, where the least portion of weight is commonly a Graine (meaning a Graine of Corne, or Wheat, dry and gathered out of the middle of the eare.) Of these Grains in times passed, 32 weighed iust 1 peny of Troy, and then was but 20 pence in an ounce. But now are there 46 pence in an ounce, so that there are not fully 14 Graines in 1 peny. But now of ounces after Troy rate (which is the Standard of England,) 12 doe make 1 pound.

But commonly there is vsed another weight called Haberdepoise, in which 16 ounces make a pound. Therefore when you would reduce Ounces into pounds, you must consider whether your weight be Troy weight or Haberdepoise: and if it be Troy weight, you must

must diuide your ounces by 12, to bring them to pounds, but if it be Haberdepoile, you must diuide them by 16. Now againe, there be greater weights which are called an hundred, halfe a hundred, and a quarterne, and also a halfe quarterne, &c.

A hundred weight.

Scholar. Why? so there may be reckoned 20 pound, 40 pound, 200 pound, and such innumerable.

Master. All these are numbers of weight, but they haue not common weights made to their rate, as the other haue. And againe these that I did name are not iust in number as they seeme by their name, for an hundred is not iust 100, but is 112 pound. And so the halfe hundred is 56: the quarter 28, and the halfe quarter 14. And these be the common weights vsed in most things that are sold by weight.

Woll weights. Todde. Stone.

Howbeit there are in some things other names, as in woll, 28 pound is not called a quarterne but a Todde: and 14 pound is not named halfe a quarterne but a Stone, and the 7 pound halfe a stone. Other names because they differ in many places, and agree in few, I let them passe.

Sacke.

But a Sacke of Woll by the Statutes, is limited to be 26 stone.

Cheese-weights.

¶ Now in Cheese, though it be sold by the hundreth, and by the stone in some places, yet the very weights of it are Cloues and

Weyes

Weyes. So that a Clove containeth 8 pound and a Wey 32 Cloues, which is 256 pound, that is, twelue score and sixteene pound: and so much weigheth the Wey of Suffolke-Cheese, and the like weight is or should be the barrell of Suffolke Butter.

The Wey of Esser Cheese containeth sixe score, and sixteene pound: and so much is also the Barrell of Esser Butter.

Now of weights are made other measures both for graine and liquor. ℥ or a pound in weight maketh a pinte in measure, so that 8 pound or 8 pintes doe make a gallon: halfe a gallon is named a pottle: a halfe pottle is called a quart, which containeth two pintes. Now abone a Gallon the next measure is a Firken: then the tertian, a Kilderkin, or halfe Barrell: and a Barrell. And by these measures are sold commonly Ale, Beere, Wine, and oyle, Butter and Sope: Salmon, Herrings, and Eeles. Measures for liquor. A Pinte. Gallon. Pottle. Quart. Firken. Tertian. Kilderkin. Barrell.

But as these be vnlike things, so the measures of their Ale are, as followeth. Ale measures.

Of Ale { the Firkin / ỳ Kilderkin / the Barrell } containeth { 8 / 16 / 32 } gallõs

Of beer { the Firkin / ỳ Kilderkin / the Barrel } containeth { 9 / 18 / 36 } gallõs.

Sope

Sope measures. Sope measures, both Firkin, Kilderkin, and Barrell should bee equall to Ale measure. Moreouer the Statutes doe limite the weight of euery of those three vessels being emptie.

A Barrell	to	26	pounds
Halfe a Barrell	weigh	13	
A Firken	empty	6½	

Herrings. Herrings also be sold by the same measures that Ale and Sope be sold by.

Herrings also are sold by the tale, 120 to the hundred, ten thousand to the Last.

Salmon and Eeles. Salmon and Eeles haue a greater measure.

the Butte		84	gallons
Salmon & Eeles the Barrell	holdeth	42	
halfe Bar.		21	
the Firken		10½	

Howbeit, some Statutes did limite Eele vessels equall with Herring vessels.

Wine measures. Now as for wine vessels they are seldome smaller then Hogsheds, which are of 63 gallons: euery Hogshed is two Barrels: yet there are many other wine vessels, but of them all see this Table, and marke the measures one by another.

Of

Of wine and oyle.	the Kondlet	holdeth	18	Gallons.
	the Barrell		31	
	the Hogshed		63	
	the Tertian		84	
	the Pipe		126	
	the Tunne		252	

But you shall marke that there be other kindes of Tertians: for there be Tertians (that is to say) Thirdels of Pipes, of Hogsheds, and of Barrels, as well of other thighs as of wine. Tertians.

Also Palmeseyes, and Backe, ec. the halfe Tun is not called a Pipe, but rather a Butte. Butte.

And thus much haue I thought meet to tell you at this time.

Scholar. And is that alwayes true?

Master. I haue told you how it should be, but how it is, I may not say: how they do differ daily from their iust measure, that Gagiers can tell you better then I. But I will let this passe now, and speake briefely of the other measure.

And as of weights there did spring the liquid measures, (whereof I spake last) so of the same springeth dyle measures, as Pecks, Bushels, Quarters, and such like, whereby are measured corne and like graines, also Salt, Lime, Coales, and other like. And this is the order and quantitie of them. Dry measures.

A Pecke in the measure of two Gallons. A pecke.

A Bushell, A Bushell containeth foure Pecks.
Quarter. A Quarter holdeth eight Bushels.
Wey. A Wey containeth sixe Quarters.

These are the common names and measures, but in diuers places there be diuers sorts.

Strikes. The Bushell in many places is two Bushels: but then is that Bushell there called a Strike, and in some places halfe a quarter is called a Cornocke. But these diuersities are too many to tell you briefly them all, and againe, sith they are against the law and statutes, I count them vnmete to be vsed.

Measure to meate length, breadth, and thicknesse. But now remayneth yet another kinde of measures, whereby men meate length and breadth, and chicknesse, and those are, an Inch, a foot, and such other: whose names and quantities this table sheweth.

An inch. 3 Grains of Barly in length make an inch.
Foote. 12 Inches make a foote.
Yard. 3 Foote make a yard.
Elle. 3 Foote and 9 inches make an Elle.
Perch. 5 Yards and a halfe, make a Perch.
1 Perch in breadth and 40 in length, doe make a rodde of Land, which some call a rod, some a yard land, and some a Farthendele.
4 Farthendels make halfe an acre of ground.
4 Farthendels make an acre.

Acre. ¶ Note, 40 rods in length do make a Furlong: 8 furlongs make a mile, which containeth 320 perches.

So

So that an English mile, grounded vpon the statute, is in length 1760 yards, 5280 fote and 63360 inches.

Somewhat greater than the Italian mile of 1000 paces, and 5 fote to a pace.

Here might I tell you many things else touching measure, and also how to reduce strange measures to our measures, but because it cannot bee well done without the knowledge of Fractions, which as yet you haue not learned, I will let them passe till an other time that I haue taught you the knowledge of broken numbers.

The parts of time.
A day.
An houre.
Weeke.
Moneth.
Yeere.

Scholar. But yet sir of the parts of time, I pray you tell me somewhat.

Master. You know that a naturall day hath 24 houres, and euery houre hath 60 minutes. It needeth not to tell you, that 7 daies make a weeke, and 4 weekes make a common moneth, and 13 moneths make a yeere, lacking one day, and certayne houres, and minutes: But of that I shall instruct you hereafter.

Here will I make an end of Reduction for this time, which though it be counted no kinde seuerall of Arithmeticke, yet you see it is no lesse needfull to be knowne, or easier to be done, than any of the other.

Scholar. Marry sir, it seemeth vnto me much harder then any other sort, for it requireth the knowledge of so many things: but

now ſir when you ſee time, I am readie to learne forth: for as much of Reduction as you haue taught me, I remember, but and if I do at any time forget, I ſhall haue recourſe to the tables which you ſet forth for me.

Maſter. So doe you, for it will not be remembred without exercise. But in as much as you vnderſtand ſo much as we haue intreated of, I will now inſtruct you in Progreſſion.

Progreſsion.

Although vntill this day the moſt part of writers haue defined Progreſſion as a compendious kinde of Addition, yet truely it is not ſo: for Progreſſion (as the very nature of the word doth informe any man) is a going forward and proceeding in numbers, & that regularly and orderly, whoſe place is aptly choſen to be very neare, or rather next after the expoſition of the 4 principal parts of Arithmeticke, for in it after a moſt eaſie manner, are all the 4 former parts exerciſed and practiſed: and not onely Addition, as cuſtomably is done. Which cuſtome hath bin the cauſe, why it hath ſo ſpecially beene named a kinde of Addition, and de fined to be a quicke

and

and briefe Addition of diuers summes, proceeding by some certayne and reasonable order.

You shall also vnderstand, that there are infinite kindes of Progressions, but for you (as yet) two are sufficient to be exercised in: of which the one I call Arithmeticall, and the other Geometricall.

Arithmeticall Progression is a rehearsing or placing downe of many numbers, number after number, in such sort that betweene euerie two next numbers rehearsed or placed downe, the difference, diuersitie, or exces[illegible] be equall and alike. Arithmeticall Progression.

Scholar. Sir, I thanke you for that you haue both opened vnto me what progression is, truely, and also why it is here placed. But I pray you with an example make plaine your definition.

Master. Examples cannot want, seeing all reasonable creatures naturally vse the order of one kinde of Arithmeticall Progression, (which therefore is also named natural) whensoeuer they distinctly doe count or number any multitude by one, saying: 1, 2, 3, 4, 5, 6, whereby the proceeding from the number to number, and euery one surmounting and exceeding his fellow next before by a like quantitie, (which here is 1) declareth the same to be Arithmeticall Progression. And for the more plainnesse, I set it downe in this manner.

maketh 16, take halfe of it, & multiply by the 7 which you noted for the number of the same places, and the summe that amounteth, is the summe of all those figures added together, as in this example: 8 multiplied by 7 maketh 56, and that is the summe of all those figures.

Scholar. That will I worke by another example. I would know how much this sum is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tell the places and there are 9, that I note. Then I put the first number 5, and the last 29 together, and they make 34, I take the halfe of it, that is, 17, and multiply by 6, and it maketh 153. That you say is the summe of all the numbers.

Master. So shall you finde it if you trie it.

Scholar. How shall I trie it?

Master. By your common addition, for if you adde all the parcels together, you shall see the same summe amount, if you did worke well. And that manner of Addition trieth all kindes of summing any Progression.

Scholar. Then can I summe a Progression, if the numbers of the parcels be odde. But what if they be euen: as in this example, 1, 2, 3, 4, 5, 6, 7, 8?

Master. When the number of the parcels is euen, then note that also as you did before, and likewise adde the first summe to the last, and by the halfe of the number of the places doe you multiply it: as in your example, the parcels

parcels are 8, that I note: then adding the first summe to the last, there amounteth 9, that doe I multiply by halfe of the parcels, that is, by 4, and maketh 39, which is the summe of the parcels.

But if you will take one rule for these both, doe thus; multiply the halfe of the one, by the other whole, and the summe will amount all one. For sometime it chanceth that the number of the parcels be odde, so that their halfe cannot be taken: and sometime it chanceth the Addition of the first number and the last, do bring forth an odde number, so that the halfe of it cannot be taken: but they will never be both odde. A generall rule.

Scholar. Then I perceive this, if there be no more belonging to it.

Master. This is enough for Progression Arithmeticall: howbeit there is another manner of Progression called Geometricall, when the numbers increase by a like proportion, that is, if the second number containeth the first, 2, 3, or 4 times, and so forth: then the third containeth the second so many times also: and so the fourth the third, and the fift the fourth, wherefore I set these three examples. Progression Geometricall.

3, 6, 12, 24, 48,	27
1, 3, 9, 20, 81,	
2, 10, 50, 170.	

Here in the first example you see, that every number containeth the other (that goeth next

The common excesse. 1 1 1 1 1

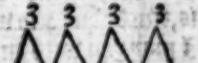
The Progression. 1 2 3 4 5 6

Scholar. This is most evident. And I thinke that I am able to tell you now of any Progression Arithmeticall propounded, what is that common excesse or difference whereby it proceedeth, if this order be kept in it.

Master. What say you of 3, 6, 9, 12, 15?

Scholar. They exceede each other by 3. And that may I set downe in such evident order, as you did your example of naturall Progression in this wise.

The common excesse. 3 3 3 3



3 6 9 12 15

Master. And doe you not also now perceive, that the whole Table of Multiplication may be made by the order of Progression Arithmeticall? eyther if you will begin at the first number of any of them on the left hand, and so proceede right ouerthwart: or at any of the first numbers of the vpper row, and goe directly downeward.

Scholar. I pray you let me consider the thing a little, and I will answere you.

By

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it now very well, for the common excesse or difference betweene any two next, is continually as much as the first number of euery row, either from the left hand ouerthwart taken, or from any of the uppermost ouerthwart rowes downeward.

Master. Now then if of any such Progression you would speedily know the totall summe, much quicklier then by common rules of Addition: first tell how many numbers there are (which numbers here we call places or parcels) and if they be odde, write their sum downe by it selfe, as in this example, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7 as you may see, therefore set downe 7 in a place alone, then adde together the first number and the last, as in this example: adde 2 to 14, and that

To know the totall summe of an Arithmeticall progression.

maketh 16, take halfe of it, & multiply by the 7 which you noted for the number of the same places, and the summe that amounteth, is the summe of all those figures added together, as in this example: 8 multiplied by 7 maketh 56, and that is the summe of all those figures.

Scholar. That will I worke by another example. I would know how much this sum is, 5,8,11,14,17,20,23,26,29. I tell the places and there are 9, that I note. Then I put the first number 5, and the last 29 together, and they make 34, I take the halfe of it, that is, 17, and multiply by 6, and it maketh 153. That you say is the summe of all the numbers.

Master. So shall you finde it if you trieit.

Scholar. How shall I trie it?

Master. By your common addition, for if you adde all the parcels together, you shall see the same summe amount, if you did worke well. And that manner of Addition trieth all kindes of summing any Progression.

Scholar. Then can I summe a Progression, if the numbers of the parcels be odde. But what if they be euen: as in this example, 1,2,3,4,5,6,7,8?

Master. When the number of the parcels is euen, then note that also as you did before, and likewise adde the first summe to the last, and by the halfe of the number of the places doe you multiply it: as in your example, the parcels

parcels are 8, that I note: then adding the first summe to the last, there amounteth 9, that doe I multiply by halfe of the parcels, that is, by 4, and maketh 39, which is the summe of the parcels.

But if you will take one rule for these both, doe thus; multiply the halfe of the one, by the other whole, and the summe will amount all one. For sometime it chanceth that the number of the parcels be odde, so that their halfe cannot be taken: and sometime it chanceth the Addition of the first number and the last, do bring forth an odde number, so that the halfe of it cannot be taken: but they will never be both odde. A generall rule.

Scholar. Then I perceive this, if there be no more belonging to it.

Master. This is enough for Progression Arithmeticall: howbeit there is another manner of Progression called Geometricall, when the numbers increase by a like proportion, that is, if the second number containeth the first, 2, 3, or 4 times, and so forth: then the third containeth the second so many times also: and so the fourth the third, and the fift the fourth, wherefore I set these three examples. Progression Geometricall.

3, 6, 12, 24, 48,	27
1, 3, 9, 20, 81,	
2, 10, 50, 170.	

Here in the first example you see, that every number containeth the other (that goeth next

next before him) two times: and in the second example three times, and in the third example five times. Now if you will know how to find easily the summe of any such numbers, doe thus: Consider by what number they be multiplied, whether by 2, 3, 4, 5, or any other, and by the same number doe you multiply the last summe in the Progression.

Scholar. I pray you worke it by this example, 2, 8, 32, 128, 512, 2048, which I have framed by proceeding from 2, and continually multiply by 4.

Master. Then must I multiply the last summe (which is 2098) by 4 also, and it will be 8192. Now must I abate from this summe the first number of Progression, which here is 2, then resteth 8190, which summe I must divide by 1 lesse then was the number that I multiplied by. Seeing then I multiplied by 4, I must divide by 3, so dividing 8190 by 3, the Quotient will be 2730, which is the summe of all the Progression. And now to prove whether you can doe the same. I give you these numbers to adde by this rule, 3, 15, 75, 375, 1875, 9375, 46875.

Scholar. I cannot well tell by what number this Progression doth increase.

Master. In any such doubt doe thus: Divide the second number by the first, and the Quotient will shew you the number that engendreth

gendzeth the Progression.

Scholar. Then is that number in this example 5, for so many times is 3 in 15.

Master. So is it. Now worke as I taught you.

Scholar. The last number is 46875, which I multiply by 5, and it yeldeth 234375, from which I abate the first number of the Progression, that is 3, and there resteth 234372, which I divide by 4, for that is one lesse than 5, and the quotient is 58593, which is the whole summe of the Progression.

Master. If you remember well this, you haue learned the Art of Progression both Arithmeticall, and also Geometricall, which you may proue eyther by subtracting of each number alone from the summe, and so will there nothing remayne: or else by adding together of all the parcels, for so will the same summe amount.

¶ And now for the vse and better vnderstanding of both these rules, I will propone vnto you certayne pleasant and necessarie questions Arithmeticall and Geometricall, and to the performance of their workings, such necessarie rules and documents, as are requisite for the vnderstanding of them, or any such like.

A certayne Mercer sold 20 yards of Velvet to be paid in 12 wéekes, by Arithmeticall proportion: that is to wit, to receiue the first

wéeke

weeke 6 shillings, the second weeke 12, shillings, the third weeke 18 shillings, & so forth, encreasing the number of weekes by 6 shillings, till the twelfth and last weeke were expired. The question is how many pounds he had for 20 yards of Velvet.

To the performance of this question and such other the like, I set forth the 12 payments in such sort, as for example here appeareth in the margent.

6
12
18
24
30
36
42
48
54
60
66
72

Then touching the adding together of these summes, without the ayde of Addition, according to the rules I taught you in Progressi-on Arithmetical. I note the number of the places, which are 12, then adding the last number of the Progression, which is 72, and the first number together make 78, and multiplying 78 by halfe the number of the places which is 6, amounteth to 468 shillings, and in pounds maketh 23 l, 8 s. And so much hath the Mercer for his 20 yards of Velvet, which is nie about 23 s, 5 d a yard.

Scholar. I understand this worke very well: but is there any proofe for the justifying hereof as you have of other workes?

Master. The worke of it selfe (being so perfectly wrought) that in your proceeding and going forward from number to number, each number exceeding his fellow by an equall or like quantitie, is all that is demaunded for justifying of the same: yet notwithstanding because

cause your request is reasonable, I will propone an example for the proofe hereof.

A certayne man is bound to pay for 20 yards of Velvet, the summe of 23 ℔, 8 s̄, and it is to be paid weekely in 12 weeks or termes by Arithmeticall Progression. The question is therefore to know with what number the same Progression is to be begun and continued in such equall proportion Arithmeticall, that in 12 weekes the same may iustly be accomplished. The proof of the last question.

For the solution whereof, and of all such other like, reduce 23 ℔, 8 s̄, all into shillings, which maketh 468 s̄. A generall rule.

Then adde 1 vnto 12, the number of the termes, it maketh 13, which 13 you shall multiply by halfe the number of the termes, which is 6, it maketh 78, then diuide 468 by 78, and you shall finde 6 in the quotient which is the true number that shall begin and continue the said Progression. That is to say, the first weeke 6 s̄, the second 12 s̄, the third weeke 6 s̄ more, which is 18 s̄, and so euery weeke as they rise, 6 s̄ more than the weeke before, as is manifest in the question aforesaid.

A Farme is to be sold to be paid by the weekes in a yeere, the first weeke to pay 4 s̄, the second weeke 8 s̄, the third weeke 12 s̄, and so forth, increasing each number by 4, till the number of 52, which are the number of weekes in a yeere, be expired. The question is

what

what the price of the farme commeth to.

Scholar. I doubt not, but by that you haue alreadie taught me, to end this question very well, wherefore I set forth the Progression with his excesse 52 times.

Master. Nay stay a while. And here for your further ease, (to abridge you of great labour that appeareth to fall out in this question, and so may doe in any other the like, if a question were proponed of 100 or 200 places or moe, and that this question nor any other the like can be ended, vnlesse you know absolutely what the last number of the Progression at the 52 place is or ought to be) I will giue you a generall rule how to know the last number of any Progression Arithmeticall, as we las if you had ordinarily proceeded by continuall addition, till you had come to the last worke, which is this.

A generall rule.

Multiply the excesse by a number lesse by one than the number of the places, and thereto put the first number of the Progression, and you shall haue your desire.

Scholar. This rule is well worth the noting: for if I vnderstand you aright I consider that my excesse is 4, which I multiply by 51, which is one lesse than the number of the places, and it maketh 204: whereunto I adde the first number of the Progression, which is 4, and then it is 208: which you say is, or should be the last number of the Progression.

Master.

Master. This is a most approued truth if there were neuer so many places.

Scholar. This rule is so easie, that I were much to blame, if I doe not remember it. For by the benefit hereof I haue such an ease and light into this excellent Art, that my first entrance doth seeme to passe a great many mens further studie and longer continuance.

Master. Many moe considerations could I propound you in these Arithmeticall Progressions: but these are sufficient for a taste, to giue you occasion to thinke that rules of knowledge and Artes are infinite capable of enlargement.

Scholar. Happie were I, if I did but well vnderstand that which is already inuented and written. But these things, in my simple fantasie, offer themselues to be greatly beneficiall vnto the ayde of Progression. Therefore now I will goe forward with your question.

Now considering that the 52 and last place is 208, I adde thereunto the first number of the Progression, which is 4, it maketh 212, which I multiply by halfe the number of the places, which is 26, & it amounteth to 5512 shillings. And so much is the totall summe or addition of this Progression: which maketh 275 l. 12 s. as appeareth here by my tables.

Master. I like well your labour and commend you for your diligence, I will here propone one example more, and therewithall for

this

this time will end progression Arithmeticall.

A certaine man bought 20 elles of Holland to be paid in 17 weekes or termes by progression Arithmeticall. And the first weeke to pay 1 s̄, 8 d, the second weeke 3 s̄, 4 d, the third week 5 s̄, the fourth weeke 6 s̄ 8 d, and so forth each weeke succeeding 20 pence more than the weeke before. The question is, what the summe of his 20 elles commeth to.

Scholar. Because here is mention made both of shillings and pence, I feare there is some harder matter contayned herein, than in the other before: therefore I pray you worke it your selfe, and I will diligently marke your labour.

Master. There is no more to be done in this then in the other before, but because your request is so reasonable, bee attentive vnto me.

First by the generall rules I seeke to finde out the last number of the 17 place, what this Progression ought to be. Therefore heere in my tables multiplying the excesse 20 by 16, which is oneleese then the number of the termes or places, and it commeth to be 320, and thereunto adding the first number of the progression which is 20 pence, all is 340 pence, or 28 s̄, 4 d: for so much ought the last number of the payments to be.

Then finally to know what the whole 17 places amount vnto, I adde the first number of

of the Progression and the last together, both which make 360. Now because 17 is an odde number, whose halfe cannot be taken, I take the halfe of 360, which is 180: and multiplying 180 by 17, commeth to 3060 pence, which maketh as you see by Diuision 12 l, 15 s. And so much is the buyer to pay for his 20 elles of Holland. Which 3060 pence if you diuide by 20, the number of elles that was bought, you shall finde 12 s, 9 d, and so much paid he for an elle one with another.

The Proofe.

A certayne man doth owe 12 l, 15 s, to be paid in 17 weekes or termes by Arithmeticall Progression. The question is, to know with what number he shall begin and continue the Progression in such equall proportion, as the same may be truely paid and satisfied in 17 weekes.

The Answere.

First I reduce 12 l, 15 s all into pence, which as you see heere in my Tables make 3060 pence, that I let stand by a while.

Then I adde 1 to 17, the number of the places or termes, which maketh 18, which I should multiply by halfe the number of the weekes or termes, which is 8 ½, which ½ multi-

tiplied by 18 cannot well be done vnlesse you were acquainted with Fractions or broken numbers, therefore you shall let that passe and multiply 17 by the halfe of 18: which is 9, (for that is all one with the multiplication of $8\frac{1}{2}$: and the multiplication of 9 into 17 maketh as you see 153, with which number you shall diuide the 3060 pence beforesaid: and the quotient bringeth forth 20 pence, which is the first number or payment to begin the progression with all: and so each worke succeeding to the 20d more then the worke before, and thereby in 17 workes shall 12 ℔ 15 s, be paid: as before was sufficiently declared. Thus much for Progression Arithmeticall.

Scholar. Certainly sir, I know not how to render you condigne thanks for these benefits shewed me, which me thinketh are so easie, delightfull and pleasant, that I count my selfe happy to be in your company.

Master. I am glad you delight so well herein, which is an art of wonderful dexterity to all sorts of men of what degree or profession soeuer they be. And now will I propone a question or two of Progression Geometricall.

A Mercer hath 12 yards of Satten which he valueth at 16 s the yard, and selleth the same 12 yards to another man to be paid as followeth. That is to wit, for the first yard to haue one shilling, for the second yard 2 s:

for

for the third yard 4 s, for the fourth yard 8 s, and so forth doubling each number following, till the twelfth and last yard. The question is, who hath made the better bargaine of the buyer or the seller.

	s
First you may set	1
downe 12 the number of	1 1
the yards, as you see here	2 2
in this example. And a-	4 3
gainst each number the	8 4
number of shillings due	16 5
to be paid, as the order	32 6
of Duplation, or the	64 7
Multiplication by two	128 8
teacheth.	256 9
Then resorting to the	512 10
adding up or summing	1024 11
of this progression, where	2048 12

I consider that the increase of this summe proceeded by the multiplication of 2, s therefore after I have drawne a line under the 12, I worke and multiply the last summe by two also, and it yeeldeth 4096: from whence I abate the first number of the Progression which is one, and then resteth 4095: which I should diuide by one lesse than I did multiply by, but seeing it is one, I neede not to diuide it: for 1 (as I haue said before) doth neither multiply nor diuide, therefore I take that summe 4095 for the whole summe of the shillings, which by reduction

amounteth to 204 l, 15 s, and so much hath the Mercer for his 12 yards of satten: which is 17 l, 1 s, 3 d, 1 yard. But I thinke you will buy none so deare.

Scholar. No sir by the grace of God this yere.

Master. Then what say you to this question: If I sold vnto you a horse hauing 4 shoes, and in euerie shoe 6 nayles, with this condition, that you shall pay for the first nayle 1 ob: for the second nayle 2 ob: for the third nayle 4 ob: and so forth, doubling vntill the end of all the nayles. Now I aske you how much would the price of the horse come vnto.

Scholar. First know the number of the nayles, I must multiply 6 by 4, and it maketh 24. Then will I doe thus: I will write the number of the nayles euery one in order from 1 to 24, and against each number of the nayles the summe of halfe pence duely, as the order of Duplation or Multiplication by two teacheth, and as in this next figure following appeareth.

Then doe I resort to the rule of summing by the Progression, where I consider that the increase of this summe proceedeth by the multiplication of two, as the last example did. And therefore multiplying the last summe by two also, and it yeeldeth 16777216, from which I abate the first number which is 1, and then resteth 16777215, which I should

1 2 4 8 16 32 64 128 256 512 1024 2048 4096 8192 16384 32768 65536 131072 262144 524288 1048576 2097152 4194304 8388608

1 diuide by one lesse than I did 2 multiply: but seeing that it 3 is 1, I need not to diuide it, 4 for 1 (as you haue before 5 said) doth neither multiply 6 nor diuide, therfore I do take 7 that number 16777215 8 for the whole summe of the 9 halfe pence, which by Re- 10 duction I find to be 699050 11 shillings and 7 d, halfe pe- 12 nie: that is 34952 pound 13 10 s, 7 d, ob.

14

15 Master. That is well done, 16 but I thinke you will buy no 17 horse of the price.

18 Scholar. No sir, if I bee 19 wise.

20 Master. Well then an- 21 swere me to this questi- 22 on.

23 A Lord deliuered to a brick- 24 layer a certayne number of loades of bricke, whereof he willed him to make twelue walles, of such sort, that the first wall should receiue two thirdes of the whole number: and the second two thirdes of that which was left: and so euery other two thirdes of that that remayned: and so did the Bricke-layer: and when the 12 walles

were made, there remayneth 1 load of bricke.

Now I aske you, how many load went to the wall, and how many load was in the whole?

Scholar. Why sir, it is impossible for me to tell.

Master. Nay it is very easie if you marke it well. Marke well that I said, that euery wall should receiue 2 thirdes of the summe that was left. Now take away two thirdes from any summe, and you must needs grant that that which remayneth is 1 thirde of the summe last before: example of 9. from which if you take 2 thirdes, there will remaine three which is one thirde of 9. Likewise from three bate 2 thirdes, and there will remaine 1.

Scholar. This is true, and now I perceiue that the least wall had but two load of bricke.

Master. And by the same reason may you know how many load euery wall had according as this figure following doth shew, and likewise what the whole summe of bricks was: for if you make 12 summes, multiplying by 3, still from the last remayner, as you may see here on the left side of the table, there will appeare all the remayners of the whole wall: and if you multiply the last of those 12 summes by 2 also, then will that be the sum of the loads which was deliuered to the Bricklayer.

Againe,

Againe, if you doe double euery remayner, as you may see at the right side of this Table, those numbers will shew the summe of loads that went to each wall, whereby you may perceiue that each wall was three times so great as the next lesser.

Scholar. Lo now it appeareth easie enough. Now surely I see that Arithmetick is a right excellent Art.

Master. You will say so when you know more of the vse of it: For this is nothing in comparison to other poynts that may be wrought by it.

Scholar. Then I beseech you cease not to instruct me further in this wonderfull cunning.

The Remayner after euery wall.			Loads due to each wall.
1	12	2	
3	11	6	
9	10	18	
27	9	54	
81	8	162	
243	7	486	
729	6	1458	
2187	5	4374	
6561	4	13122	
19683	3	39366	
59049	2	118098	
177147	1	354394	

Summe of the 531441 loads deliuered.

The Golden Rule.

Master.

BY order of the Science (as men haue taught it,) there should follow next the extraction of rootes of number which because it is somewhat hard for you yet, I will let it passe for a while, and will teach you the feate of the Rule of Proportions, which for his excellencie is called the Golden Rule: Whose vse is, by three numbers knowne to finde out any other vnknowne, which you desire to know, as thus, If you pay for your bord for three moneths sixteene shillings, how much shall you pay for eight moneths?

To know this and all such like questions, you shal consider which two of your numbers be of one denomination, and set those two the one ouer the other, so that the vndermost be it that the question is asked of: as in my question, 3 and 8 be both of one denomination, for they both be moneths: and because 8 is the number that the question is asked of, I set then one ouer the other, and 8 vndermost, thus, with such a 3 / 8 crooked draught of lines. Then doe I set the other

other number which is 16, a- 3 — 16
gainst 3 at the right side of 8 —
the line, thus.

And now to know my question, this must I doe: I must multiply the lowermost on the left side, by that on the right side, and the summe that amounteth, I must divide by the highest on the left side. Or in plainer words, thus, I shall multiply the number of which the question is asked (which is called the third number) by the number of another denomination (which is called the second,) and the summe that amounteth, must I divide by the summe of like denomination, which is called the first. When for the knowledge of this question I multiply 8 into 16, and there amounteth 128, which I divide by 3, and it yeldeth 42 s, and 2 s remayneth, which I turne into pence, and they be 24 pence, of which the third part is 8 pence, so the third part of 128 s, is 42 s, 8 d, which summe I write

at the right hand of the figure 3 16 s.
against 8 thus. 8 42 s. 8 d.

Hereby I know that if three moneths boarding doe come to 16 s, that 8 months boarding will come to 42 s, 8 d: and likewise of any other like question.

But heere must you marke, that the first number and the third be of one denemination, and also the second and the fourth, the

which

which you seeke: or else be of such denominations, that you in working may bring them into one. As if a man should aske me this question.

¶ Twelue weekes iourneying cost me 14 French crownes at 6 s the peece, how many pounds is that in one yeere? Heere you see no two numbers of one denomination, but yet in working you may turne them into like denomination: as thus, turne the one yeere into 52 weekes, and the fourth summe will be French Crownes, by the order of the working: Then to know this question, multiply the third summe 52, by the second 14, and the summe will be 728: that diuide by your first number 12, and the quotient will be 60 Crownes, and 8 Crownes remayning: which if you turne into shillings, they will be 48 s; which if you diuide by your first number 12, the quotient will be 4, which signifieth 4 s: put those 60 French Crownes, which make 18 pounds with the 4 shillings, for the summe that answereth to the question: and it is the iust expences of a yeere: And the sum will be thus.

12 — 14
52 — 60 — 4 s.

And take this euermore for a generall rule touching this whole Art, that the doubtfull or vnknowne number that you would be resolued of, shall allwayes be set in the third place. Note also the first number and the third must

must euer be of one nature and denomination, or else must in working be brought to like denomination, and then of necessity must the other number be in the second place.

Remember also that the place of the first number is the highest on the left side: and the place of the second right against it on the right side: the place of the third number is vnder the first, as by those examples you haue seene.

Scholar. This I trust I can doe.

Master. But and if the question be asked thus: In 8 weekes I spend 40 s, how long will 105 shillings serue me? Here you see that 8 weeks answeres himselfe, and saith 40 shillings. But how long time 105 shillings will serue you know not. Therefore you shall set 105 in the third place, according as I told you euen now. And the first place must alwayes be of the same nature or denomination that the third is of, which here is 40. Then must 8 needs be that other. Now multiply 105 by 8, and it will be 840, which if you diuide by 40, it will yeeld 21, which is the 4 number, and sheweth how many weeks 105 shillings will serue, if you spend 40 s in 8 weekes.

The figure of this question is this: as if you should say: If 40 s serue for 8 weeks, 105 for 21 weekes.

Shillings.	Weekes.
40	8
105	21

serue

Other diuersities there be of worke by this rule, but I had rather that you would learne this one well then at the beginning to trouble your minde with many formes of worke; sith this way can doe as much as all the other, and hereafter you shall learne the other more conueniently:

¶ And for your further aide and instruction, to make you better acquainted with this Golden Rule, I haue here proponed fiue questions and their answeres, which I thinke most conuenient and méete to preferre the desirous to perfect vnderstanding. The first foure are all branches of one question sprung out of the best tree, (for a young learner to talke of) that groweth in this Ground of Artes: for that no manner of question in the Rule of 3 whatsoeuer it be, can be proponed, but it must be comprehended vnder the reason or stile of one of these foure.

The Questions.

If 15 elles of cloath cost 7 l. 10 s: what comes 27 elles to at that price? Answere 13 l. 10 s.

If 27 elles cost 13 l, 10 s: what are 15 elles worth? Answere: 7 l, 10 s.

If 27 elles cost 13 l, 10 s: how many elles shall I haue for 7 l, 10 s? Answere: 15 elles.

If I sell 15 elles for 7 l, 10 s: how many elles

elles are to be deliuered for 13 ℔, 10 s: Answere: 27 elles. If 8 pound of any thing cost 16 s, 6 d: what money is to be receiued for 49 pound? Answ. 5 ℔, 1 s, 0 d.

If 4 ℔ of any thing cost 17 d: what money will 8765 pound of that commodity cost? Answere 155 ℔, 4 s, 3 d, q.

Of all which questions I omit the worke of purpose, that you shall whet your wit thereby at conuenient leasure, to clime each branch, and gather the fruit of them, and doe minde now, before we make an end of this rule, to giue you some instructions of the backer rule of 3, whose order is quite contrary to this that you haue learned.

For in this rule hitherto, euermore loke how much the third number is greater than the first, so much the fourth number is greater than the second. And contrariewayes: loke how much the first summe is greater than the third, (if it doe chance so) so much is the second summe greater than the fourth. But in this rule, there is a contrary order, as this: That the greater the third summe is aboue the first, the lesser the fourth summe is beneath the second: and this rule therefore you may call the Backer rule, as in example.

The backer rule.

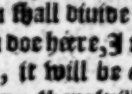
Questions of buying cloath.

If I haue brought 30 yard of cloath of 2 yards breadth, and would haue Cannas of 3 yards broad to line it withall, how many yards should I need?

Scholar.

Scholar. Why, there is none so broad.

Master. I doe not care for that, I doe put this example onely for your easie understanding: For if I should put the example in other measures, it would be harder to understand. But now to the matter: If you would know this question, set your numbers as you did before: but you shall multiply now the first number by the second, and that ariseth thereof, you shall divide by the third: which thing if you doe here, I meane if you multiply 30 by 2, it will be 60: which summe if you divide by 3, there will appeare 20: whereby I know, that if 30 yards of cloath of two yards broad, should be lined with cannas of three yards broad, 20 yards of cannas would suffice, as this figure sheweth



And now because yee found fault with my Example, how say you, perceive you this?

Scholar. Yes sir, I suppose.

Master. Then answere me to this question: how many elles of cannas of elle breadth, will serve to line 20 yards of Say, of three quarters broad?

Scholar. In good faith sir I cannot tell, for I know not how to bring the summes to like denominations.

Master.

Master. Then will I tell you, sith there is mention here of quarters, and againe euerie one of the measures both elles and yards may be parted into quarters, do you part them so both in the breadth and length, and then put forth the question by quarters.

Scholar. Then I shall say thus. How many quarters of canuas of fine quarters broad, will line 80 quarters of three quarters broad?

Master. Now answere to the question.

Scholar. First I will set them downe in their forme thus, for 5 is ioyned with the question, and is therefore the third number: then is 3 the number of the same denomination, I meane because they be both referred to breadth. Now I multiply 80 by 3, and it is 240. which I diuide by 5, and it yeeldeth 48. Then say I that 48 quarters of 5 quarters broad, will suffice to line 80 quarters of 3 quarters broad.

Breadth.	Length.
3	80
5	

Master. Turne the quarters againe into elles and yards.

Scholar. Then I say, that 9 elles and three quarters of a yard of elle broad will serue to line 20 yards of three quarters broad, as this figure sheweth

Breadth.	Length.
3	80
5	48

¶ Master. Now what say you to this question

queſtion: I lent my friend 400 ℔ for 7 moneths, how much money ought hee to lend me againe for 12 moneths, to recompence my courteſie ſhewed him? Can you anſwere to this?

Scholar. Yes ſir, I ſuppoſe, for I will ſet downe my Numbers thus: where I multiply 7 into 400, and it maketh 2800, which I diuide by 12, and it yeeldeth 233 ℔, and there is 4 ℔ remayning of my Diuiſion, what doe I therewith?

Moneths	Pounds
7	400
12	

Maſter. Turne the ſame 4 pound into ſ, and then diuide it by 12 as you did before.

Scholar. Well ſir it ſhall be done, ſo haue I 6 ſ for my quotient, and yet remayneth 8 ſ vpon my diuiſion.

Maſter. You muſt alſo reduce that 8 ſ into pence, which maketh 96, and diuide that alſo by your Diuiſor.

Scholar. So haue I done, and I finde 8 d for my quotient, and nothing is left.

Maſter. This muſt you alwayes doe when any thing remayneth vpon your Diuiſion: whether it bee money, waight, meaſure, or any kinde of thing whatſoeuer. This rule is ſo profitable for all eſtates of men, that for this rule onely (if there were no more but it) all men were bound highly to eſteeme Arithmeticke.

By

By this rule may a Captayne in warre worke many things, as Master Digges in his Stratiaticos doth declare: Onely now in this my simple addition, for a taste and incouragement, I will inlarge the Author with a question or two more, wishing you and euery my countrimen or Gentlemen whatsoeuer, that by nature be any thing giuen to Military affayres, to be familiar and acquainted with this excellent Art, the which hee shall finde not onely at the Sea, but also in the Campe and field-seruice, and abundantly to ayde him, eyther in fortification, paying of soldiers wages, charges of ordinances, powder, shot, munition, and instruments whatsoeuer, as for example.

If it should chance a Captayne which hath 40000 souldiers to bee inclosed with his enemy, that he could haue no fresh purueyance of victuals, and that the victuals which he had would serue that Army but onely three moneths, how many men should he dismisse to make the victuall to suffice the residue eight moneths.

Scholar. As you taught mee, I set the numbers thus, saying: If three moneths suffice 40000, to how many will eight moneths suffice.

Moneths.	Men.
3	40000
8	

To know this, I multiply the first num-

ber 3 into the second 40000, and it yeldeth 120000, which summe I divide by 8, and there will be in the quotient 15000 which if I doe subtract from 40000, the remayner will declare that hee must dismisse 25000 as this figure sheweth.

Moneths	Men.
3	40000
8	15000

¶ Master. Now answere me to this question: If 136 Masons in a moneth be able to build a Fort to preserue the souldiers from the enemy, and such expedition requireth that I would haue the same finished in eight daies, How many workemen say you is there to be appoynted?

Scholar. As you taught me, I set the numbers thus, saying: If 28 dayes require 136 Masons, what number of men by the like proportion will 8 dayes require.

28	136
8	

To know this, I multiply the first number 28 into 106: and it yeldeth mee 3808: which I divide by 8. And my quotient is 476 which is the iust number of Masons that shall supply this worke. And now me thinke these questions are very easie.

Master. Truely if you take delectation herein, you shall finde this Art not onely easie, but wonderfull pleasant and profitable. Now therefore one question more I will propose,

pose, and so leaue off this rule in whole numbers, vntill we come to the vse of it in broken numbers: for had you the vnderstanding of broken numbers perfectly, not onely in this rule, but in all other, the question that in sight or apparance seemeth to bee 100 times more harder to absolue, may thereby be wrought as soone or sooner than this.

Scholar. Your words doe greatly encourage me to be studious to attayne whole numbers: but might I once attayne to be a practitioner in broken numbers, I should thinke my selfe happy.

Master. What say you then to this question: If 48 Joyners in 2 dayes make 200 light horsemens staues, (esteeming they work but 12 houres a day) and such neede requireth that 384 Joyners are set to the finishing of those 200 staues; in what time say you, will they make them vp?

Scholar. I see heere that I must turne my 2 dayes into houres. And so doing I set my numbers thus.

48 Z 24
384

Saying, if 48 men are 24 houres, 384 men will make an end quickly. For it is grounded vpon an old Prouerbe, Many hands make quicke speede.

I multiply 48 into 24, and it amounteth to 1152, which I diuide by 384: and my quo-

tient is three houres, which is my desire.

Note. I take this for a note worthy the marking, eyther in the rule of three, forward or backward, when the two numbers are multiplied together, the Product is of the same nature and denomination, that the second number is of.

The double rule. Master. Well, sith you perceiue now the vse of this rule, I will shew other which ensue of the same, and first the double rule wich is so called, because there is in it double working, by which thing onely it differeth from this.

Scholar. Then by an example I shall vnderstand it well enough.

Of carriage. Master. So shall you, and let this be the example: If the carriage of 1 C weight (that is to wit 112 pound) 30 miles, doe cost 12 d, how much will the carriage of 5 C weight cost, being carried 100 miles?

Scholar. I pray you shew me the working of it.

Master. You must make two workings of it: the first thus. If 1 C weight cost 12 d, how much will 5 C. weight cost? Set your figures thus.

C weight.	Pence.
1	12
5	Z

And multiply 5 by 12, and thereof amounteth 60, which if you diuide by one, the quotient will be still 60, that is the price of fiue C weight

weight for 30 miles.

Then begin the second worke, saying, If 30 miles cost 60 d, how much will 100 miles cost? Set your figures thus.

Miles.	Pence.
30	60
100	—

Then multiply 100 by 60, whereof amounteth 6000, which being diuided by 30, will yeeld 200 d. Then you may say, that so many pence shall cost the carriage of 500 pound waight 100 miles, after the rate of 12 pence for the 100, carried 30 miles. Question of proui-sion on touching an as-mie.

Scholar. Now I perceiue it also.

¶ Master. These and such other like questions of the double Rule of three, are to be answered much soner, at one onely working by the Rule of three compoſed of 5 numbers, which anone I will shew you; and then when you haue the vse thereof, you may vse which way you thinke good.

Scholar. Sir, I thanke you much for your courtesie, and I long now till this rule be ended, that I may see how I shall behaue my selfe with that new rule of fiue numbers: for that I haue euer since you taught me hitherto, in the Golden Rule both forward and backward wrought but with three numbers onely.

Master. But yet a while we will goe on forward with this double rule of 3: therefore answere to this question: thirtie bushels of

wheat sowed, yelded in one yere 360; how many will 80 bushels yelde in 7 yeres? I meane sowing euery yere of those 7, still 80 bushels.

Scholar. First I say, that if 30 bushels will yeld 360 in one yere, then 80 bushels will yelde 960 in one yere. Then for the second worke I say: If one yere yelde 960, then 7 yeres will yeld 6720: as these two figures doe shew.

Seede.	Encrease.	Yeere.	Encrease.
30	360	1	960
80	960	7	6720

But now sir, if I set forth 30 bushels of corne to another man for 7 yeres, agreeing so that he shall sow euery yere the whole increase of the corne, and I at the end of those 7 yeres to haue the halfe of the whole increase: I would know how many bushels will there amount to my part, supposing the increase to be after the rate of the last question, for 30 bushels in one yere to yelde 360.

Master. In such a question you must haue so many seuerall workings as there be yeres, as for example: in the first yere 30 bushels yelde 360: then to know the yelding of the second yere, I must say: If 30 yeld 360, how many yeldeth 360? Worke by your rule, and you shall finde 4320. Then say for the third

yere:

yeare. If 30 yeeld 360, how many will 4320 yeeld? you shall haue 51840, and so euery yeare multiplying the whole increase by 360 and diuiding it by 30, the increase of the next yeare will amount, as these 7 figures following do orderly declare: where I haue set 7 letters for the 7 yeares, of which the first is set without art, because that is the increase which you do presuppose: and the last number of each other doth shew the increase of the yeare that it standeth for, which the letters doe declare, so that the increase of the seauenth yeare, is 1074954240 bushels: how many quarters that is, and also how many waies, you may by Reduction some finde.

a

30—360

b

30 Z 360
360 Z 4320

c

30 Z 360
4320 Z 51840

d

30 Z 360
51840 Z 622080

e

30 Z 360
622080 Z 7464960

f

30 Z 360
7464960 Z 89579520

g

30 Z 360
89579520 Z 1074954240

Now with one queſtion more I will proue you. If 6 Mowers doe mow 45 acres in fiue dayes, how many mowers will mowe 3 hundred Acres in 6 dayes?

Scholar. If 45 acres require 6 mowers, then 300 acres require 44. Now againe: if fiue dayes require 40 mowers, then 6 dayes neede but 33 mowers.

Maſter. Why doe you not make mention of the 2 that remayneth in the laſt Diuiſion? for the laſt part of the queſtion is wrought by the Backer rule, where the firſt number 5 is multiplied into the ſecond that is 40, whereof amounteth 200, which if you diuide by the third number 6, the quotient will be 33, as you ſaid: but then will there remayne two, which cannot well be diuided into 6 parts: howbeit you may vnderſtand by the 6 part of 2, the third part of one mans worke, which you muſt put to the 33: or elſe you muſt ſay, that 33 workemen will end all the 300 acres in 6 dayes, ſaue 2 mens worke for one day, or 2 dayes worke for one man. But ſuch broken numbers called Fractions, you ſhall hereafter more better perceiue, when I ſhall wholy inſtruct you of them.

¶ Maſter. Yet one queſtion more of field matters I will propone, and ſo I will make an end of this double Rule of 3.

Scholar. With all my heart ſir I thanke you, and I will diſpatch it as ſone as I can, becauſe

because I would faine see the order of the next rule of 5 numbers.

Master. If a Captayne ouer a band of men did set 300 Pioners a worke, which in eight houres did cast a trench of 200 Rods: I demaund how many labourers will be able with a like trench in three houres to intrench a campe of 3400 Rods.

Scholar. I thinke I am now in the Backhouse ditch: for I know not well which way to goe about it. And besides that truely I thinke I shall neuer come to preferment that way, my growth is so small.

Master. You know not how God may raise you hereafter by knowledge and seruice into the fauour of your Prince, for the auaile of your Country.

Example for Nauigation: Sir Francis Drake, a man greatly honoured for his knowledge, is not the tallest man, and yet hath made as great a venture for the honour of his Prince and Countrey, as euer Englishman did.

Scholar. Sir, I thanke you for your good encouragement. My minde, though I be little is as desirous of knowledge, as any other: I haue pondred now a little of it, and thus I set forth the worke.

	Rod.	Men.
	200	300
Z	Rod	
	3400	

Saying, if 200 Rod require 300 men, what shall

shall 2400 rods require? I multiply 3400 by 300 and it yeeldeth 1020000, which I divide by 200, and my quotient is 5100 men.

Then must I say for my second worke, if in 8 houres 5100 men be able to discharge it, how many shall performe the same in three houres? Now if I would worke by the Golden rule of proportion forward, I should finde a lesse number of men: because three houres is lesse than 8 houres: but because reason teacheth me that the lesser the time is, wherein the trench must be made, the more labourers I ought to haue, thereupon I vse now the Backer rule, as in example. And I haue in my Quotient 5000. So many Pioners must I haue to entrench the Campe in three houres.

Maker. You haue answered the question very artificially: And truely I commend you for your diligence and apt vnderstanding: and now according to my promise, I will (in whole number) giue you a little taste of the rule of 3, compounded of 5 numbers.

The

The Rule of 3. compound of 5. Numbers.

THis Rule of 3 compo-sed, is distinct for most needfull questions, into two seuerall parts or workings: And there belongeth vnto it al-waies fiue numbers, whereof in this rule be-ing the first part, the second number & the fift are al waies of one nature and like denomina-tion, which rule is to be wrought thus: you must multiply the first number by the second, and that shall be your diuisor: Then againe, multiply the other 3 numbers, the one by the other, and their product shall be your diuidend.

And now according to my promise, we will first worke the question of waight and carri-age, which I deliuered you in ỹ double rule of 3, to be absolued by this rule, which was this.

If the carriage of 1 C weight 30 miles cost 12 d; what will the carriage of 5 C weight stand me in being carried 100 miles?

C. weight.	Miles.	Pence.	C. weight.	Miles.
1	30	12	5	100

Now marke well how these fiue numbers stand: Then multiply the first number by the second

second, as 30 by 1, which maketh but 30, and that number kepe for your Diuisor. Then multiply the other 3 numbers the one into the other: that is to wit, 12 by 5, which maketh 60: lastly 60 by 100, which as you see here in our Tables, ariseth to 6000, which 6000 you shall diuide by the product of the 2 first numbers, which here is 30. And as you see there is found 200 pence, which is the dutie that you ought to pay for the carriage of 5 C. waight 100 miles, after the rate of 12 d a hundred, and agreeth with the conclusion of the double Rule of three.

Scholar. Sir I thanke you, it is euen so.

Master. Yet note this for a generalitie in this rule, take what nature or denomination your middle number is of (which here are pence) and of the like denomination or nature is alwayes is your quotient.

Scholar. Well now and if it please you, by your patience, I will see how I can end the question next following of thirtie bushels of wheat sowed, which in one yeare yelded 360: how many then will 80 bushels yeld in 7 yeare sowing euery yeare of these 7 full. 80 bushels, and according to your reasons I set my numbers thus.

Bushels.	Yeare,	Bush.	Yeer.
20	1	360	80
			7
		80	
		28800	
		7	
		201600	

Where

Where I multiply 30 by 1, and it maketh 30 my Diuisor: then multiplying the other 3 numbers the one into the other, as here appeareth in my Tables, they make 201600 which I diuide by 20: and my quotient is 6720 bushels my desire, for so much also it came to at two workings by the Rule of 3.

Master. Yet one question more I will propound vnto you, and so leaue this rule till it please God hereafter, that I may make you worke it in broken numbers.

What comes the interest of 258 ℔ for fiue moneths to, after the rate of 8 ℔ taken in the 100 ℔ for 12 moneths?

Scholar. Sir, this is yet within the compasse of some reasonable diuance. Therefore to minister equitie in this case, I will see how I can worke.

The same which ℔ moneths. ℔ ℔ moneths
I set downe 100–10——8–248–5
thus, praying
you if I haue not done well, to shew me mine errour.

Master. Proceed, you haue done very well.

Scholar. Then I doubt not by the grace of God but to end it: I multiply 100 by 12, it yeeldeth 1200, and the 3 other numbers multiplied together produceth 10320: which I diuide by 1200: and my quotient is eight pounds. Then according as you haue taught me heretofore, I turne the 720 ℔ that I left,

into

into shillings: and diuiding it by the first number, my quotient is 12 s. So I answere that the loane of 258 l for 5 moneths, after the rate of 8 l in the 100 l for a yeere, comes to 8 l, 12 s.

Master. You say true, I commend your diligence: now behold the manner of the second part of this rule.

The 2 part of the rule of 3 compound.

¶ Master. In the second part of this rule of 3 compoſed, the third number is like vnto the first. And the rule is to be wrought as thus: you shall now, contrary to the last rule, multiply the third number and the fourth together, and that Product shall be your Diuisor. Then multiply the first by the second, and the Product thereof by the first: and that is the number that shall be diuided. For example, I propound this question, for a proofe of my last question of interest. A Merchant hath receiued 8 l, 12 s, for interest of certayne money for 5 months terme, which he receiued after the rate of 8 l in the 100, for a yeere. The question is now, how much money was deliuered to raise this interest: Behold therefore the manner,

The proof of the last question.

how the question is set	l	moneths.	l	moneths.	l	s
	100	12	8	5	8	12

forth.

Scholar. Sir I perceiue it very well: and according to the doctrine which you prescribed for the working thereof: if it please you now

it

it is set downe, I thinke I can follow the worke.

Master. May stay a while, and before you worke, marke well how I deliuer a reason for the perfect vnderstanding of this rule, which is thus: If 8 l. in 12 moneths doe yeld me 100 l. to take 8 l. 12 s. for 5 moneths, must needs yeld a great deale more.

So vpon the knowledge that I haue in this Art, the first part of this rule is answerable to the rule of the 3 forward: And this latter part accordeth to the rule of 3 backward.

Scholar. Sir, I yeld you most heartie thankes for these your last instructions, they haue giuen me great light into these two rules, whereby I may the better by deliberation conceiue how to vse them hereafter, when occasion shall require.

Master. You say well, goe to now if you will, and trie your cunning in the question: But this note take with you by the way, in as much as heere is mention made of shillings: turne all your money as you worke into s. for your more ease in worke.

Scholar. If it please you to behold me a little, I will quickly end it: for I haue but my first, my second, and my last number to be multiplied together for my diuidend: And my third into my fourth for my Diuisor.

100 l.

Moneths	l'	Moneths.	l',	s.
100 — 12	— 8	— 5	— 8	— 12
20	20		20	
2000	160		172	
12	5			
4000	800			
2000				
24000				
172				
48000				
168000				
24000	4128000	}		
4128000	8	60 {		

Which 4128000 I diuide by 800, and my quotient is 5160 s, which in pounds yeelds 258, my desire.

Master. I will heare for this time in whole numbers end this rule, and I will instruct you in the rules of fellowship. You may at your conuenient leasure for your exercise worke the same by the Rule of 3 at twice. And for your aide and incouragement therein, I set downe heere a proffer how to apply it.

Moneths	A	l'	s	Pound.	B	l'
5	Z	8	12	8	Z	100
12		413 $\frac{1}{3}$		412$\frac{1}{2}$		258 l'

The Rule of Fellowship.

But now will I shew you of the Rule of Fellowship or Company, which hath sundrie operations, according to the diuers number of the Company. This Rule is sometime without difference of time, and sometimes there is in it difference of time. First I will speake of that without difference of time, of which let this be an example.

Foure Merchants of one company made a banke of money diuersly: for the first laid in 30 l. the second 50 l. the third 60 l. and the fourth 100 l: which stocke they occupie so long, till it was increased to 3000 l. Now I demaund of you, what should each receiue at the parting of this money?

Scholar. I perceiue that this rule is like the other, but yet there is a difference, which I perceiue not.

Master. Then will I shew it to you. First by Addition you shall bring all the particular summes of the Merchants into one summe, which shall be the first summe in your working by the Golden Rule, & the whole summe of the gaines by that stocke shall be the second summe. Now for the third summe you shall

set the portion of each man one after another, and then worke by the Golden Rule, and the fourth summe will shew you each mans gaines: as in example.

The parcels of those foure Merchants make in one summe 240 pounds: set that in the first place, the gaines in the second, and the first mans portion of stocke in the 3 place thus.

240 Z 3000
30

Now multiply the second by the third, and it will be 90000, which you shall divide by 240, and there will appeare 375 l. thus.

And that is the gaines for the first man.

240 Z 3000
30 375

Now for the second man, set the 50 l. that he brought, in the third place, and worke as before: and his part will be 625 l: as this figure sheweth.

240 Z 3000
50 625

Likewise for the third man, set his money which was 60 l, and his part of gaines will be 750 l, as heere appeareth.

240 Z 3000
60 750

And so for the fourth man, if you set his sum which is 100 l, his gaines will be 1250 l, as the worke will declare.

240 Z 3000
100 1250

Scholar. This I perceiue: but is there

any

any way to examine whether I haue well done or no?

Master. For the triall hereof, adde together all their foure portions, and if their addition make the whole summe of their gaines, then is the worke well done.

Scholar. That will I trie by and by, the foure parcels are these,

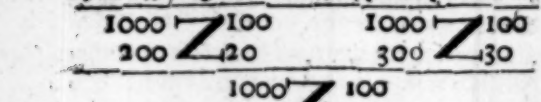
which added together,	375
make 3000, which is	625
the iust summe of money	750
that they gained where-	1250
by I know the worke is	
well done.	3000

Master. Well, now another example will I put to you, not of gaines, but of losse: for one reason serueth for both.

If three Merchants in one shippe, and of one fellowshyp, had bought Merchandise, so that the first had laid out 200 l. the second 300 pound, and the third 500 l. and it chanced by tempest that they did cast ouer board into the Sea merchandise of the value of 100 l. how much should each man beare in this losse?

Scholar. If I shall doe in this, as you did in the other question, then must I ioyne their three portions together, 200, 300, and 500, which maketh 1000. Then say I, if 1000 lose 100, then shall 200 lose 20, and 300 shall lose 30, and 500 shall lose 50, as by

these three figures it doth appeare plaine.



Master. Well, sith now you haue done these, I will propound a question of more importance, which shall make you not onely the abler to vnderstand this rule, but also it will greatly aide you in the next Rule of Fellowship with time,, if such neede be that your money be of diuers denominations.

Note. For this may not be forgotten in all such questions: if the number be of diuers kindes, you must by Reduction bring it into one kinde, that is to say, to the least value that is named in the question. And likewise shall you doe, if the time be of diuers kindes, as some yeeres, some moneths, weeks & dayes, you shall make all moneths, weeks or dayes, according as the last name of time in the question, is as for example.

Question of Sheepe. First in diuersitie of money. Three companions bought 2000 sheepe, and paid for them 241 l.—13—4 d. of which summe one paid 101 l.—10 s. The second 82 l. 17 s. 10 d. And the third paid 52 l. 5 s. 6 d: How many sheepe must each of them haue? Answere: The first shall haue 840. The second 686. And the third 474. And that must you work thus.

First

First, considering that your money is of diuers denominations, you shall (by Reduction) bring it all into the smallest denomination which is in it, that is to say, pence; and so will the totall summe be 58000 pence.

Now, if you turne each mans money into pence also, the first mans sum will be 24360 pence: The second mans summe 19894 pence: And the third mans money will be 13746 pence.

Now to know how many shæpe euery man shall haue, let the whole summe of money, that is, 58000 pence be set in the first place, and in the second place set the number of shæpe, and then orderly in the third place set each mans money, and then multiplying the third and the second summes together, and diuiding that that amounteth by the first, there will appeare the number of shæpe that each man ought to haue: as these three figures doe shew.

a

58000 — 2000
24360 — 840

b

58000 — 2000
19894 — 686

c

58000 — 2000
13746 — 474

Scholar. Why doe you set the money in the first place, seeing in the question you say

2000 sheepe cost 58000 pence, and not thus, 58000, cost 2000 sheepe.

Master. You remember I taught you at the beginning of this Golden Rule, that the first and third numbers must be of one name, and of like things: and euermore the number that the question is asked of, must be set in the third place. Now is the question plainly this: If foure men bought 2000 sheepe for 58 thousand pence, how many sheepe shall each man haue.

But seeing in this question, there ought more respect to be had to the summe of money than to the summe of the persons, (for in the summes of money is their proportion toward the sheepe, and not in the number of persons) therefore must we turne the question thus.

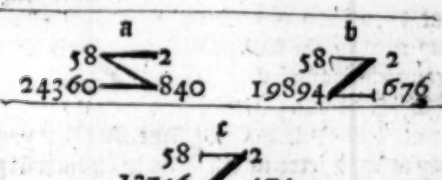
If 58000 pence bought 2000 sheepe, how many did 24360 buy? Againe, how many did 19894 pence buy? And how many bought 13746 pence?

Scholar. I perceiue it reasonably, and so shall I doe in all like questions.

Master. Even so. But for easinesse of the worke, marke this: Whensoeuer the first and second numbers haue ciphers in the first places, you may both in the multiplication and in the diuision leaue out those ciphers, so that you leaue out like many out of both summes, as in this question, the first number 58000 hath three ciphers, and so hath the second, that

is

is 3000 : therefore caſt away their ciphers, and ſo will the firſt number be 58, and the ſecond 2 : ſet them in their places, and worke according to the rule, and you ſhall perceiue that it will be all one, ſauing that this is the ſhorter and eaſier way, as theſe three figures doe ſhew.



And this you ſee is both eaſier, and alſo the more certayne way to know the anſwere to this queſtion.

Scholar. Truth it is as you ſay : but ſir me ſeemeth I might aſke a further queſtion here, not onely how many ſheepe each man ſhould haue, but alſo what euery ſheepe coſt.

Maſter. That queſtion doth not onely belong to this rule, but may alſo be diſcuſſed by Diuiſion, eſpecially if the queſtions number be one onely : as thus : Diuide the totall ſom 58000 pence by 2000 (or 58 by 2, omitting the ciphers) and the Quotient will bee 29 pence, that is 2 s, 5 d; howbeit, by this rule you may doe it, and beſt when the number of the queſtion doth exceede 1 : as if I ſhould aſke

this question, 2000 sheepe cost 58000 d, how much doe 20 cost? Then shall I set my figure thus.

200 ⊢ 58000
20 ⊥

And doing after the rule there will amount 580 pence, that is 2 l—8 s—4 d, the price of one score: But if you will vse that easie way that I did teach you, you may change the first and second number thus.

2 ⊢ 58
20 ⊥

Thus do you perceiue the vse of the rule without time. And that you may as well perceiue the same with diuersity of time, I propose this example.

The rule of fellowship with time.

Foure Merchants made a common stocke, which at the yeeres end was increased to 55145 l. Now to know what shall be each mans portion of gaines, you must know each mans stocke and time of continuance.

Question of a banke.

The first man of these foure laid in 669 l, which he did take from the stocke againe, at the end of 10 moneths. The second man laid in 81 l for eight moneths. The third laid in 900 l for seauen moneths. And the fourth laid in 1040 l for twelue moneths.

Note. A generall rule.

This question shall you examine as you did the other before, sauing that whereas in the third place of the figure you did set each mans summe alone, heere you shall set the same being multiplied by the number of their time, & likewise in the first place of the figure you shall

shall set the number which amounteth of their whole summes so multiplyed by their time and added into one whole summe, as thus.

The first mans summe is 669 ℔, which I multiply by 10 (that was the number of his time) and it maketh 6690. The second mans summe 810 ℔, multiplyed by 8, (which was his time) maketh 6480. The third mans summe 900 ℔ multipled by 7 (for that was his time) yeldeth 6300. The fourth mans summe was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

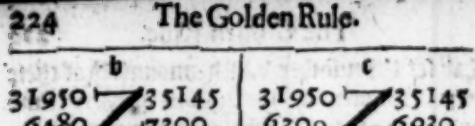
These foure sums thus multiplied by their time, must be set orderly in the third place of the figure, and in the first place must be set the whole summe of all foure, which is 31950, and the gaine must be in the second place, which is 35145. Now to end the question, I say first, If 31950 did get 35145, what did 6690 get? Answere.

7355 ℔, as by this figure appeareth.

a	
31950	35145
6690	7359

Likewise the second man had to his part 7128 pound, the third must haue 6930 pound. And the fourth man shall haue for his part 13728 pound, as these figures doe partly declare.

Scholar.



Scholar. This I like very well: but what profe is there of this worke?

Master. The summe that I taught you for the other, to adde all the portions together, and it may agree to the whole summe, then is your worke well done.

Scholar. Then will I proue in this example. The foure parcels are these, which if I adde together, there will amount 35145, and that was the whole summe, whereby I perceiue the worke is well done.

	7359
	7128
	6930
	13728
	35145

Master. If it fall out otherwise, be sure it is not well.

Scholar. Then doe I vnderstand this worke also very well: but what haue I now to learne?

Master. There are many other excellent parts behinde, of which I will not as now make mention, because that without the knowledge of fractions they cannot be duely taught, and much lesse vnderstood. Therefore

fore will I propose to you two or three questions more, (that thereby you may better perceiue the vse of this rule and all other the like) and so may make an end for this time.

¶ Three partners, by some ill aduenture sustayned the losse of 160 l, whereof the first laid into the common stocke 200 l for terme moneths. The second laid in 350 l, and the third 100 l, but for how long the two latter is vnknowne. But breaking off their partnership, the first found himselfe a loser, 80 l, the second 56 l, and the third 24 l. The question is, for how long time was the money of the two latter in company?

For the solution hereof, and of such other like, you must also multiply the first mans 200 l, that he put into the stocke by his time of continuance, which was 10 moneths, and it maketh 2000: wherefore now I affirme if his money that lost 80 l, multiplied by his time, make 2000: what shall his money make that lost 56 l, and his that lost 24 l, which 2 numbers I commit to the triall of the rule of 3, at two workings, thus:

80	2000
56	1400

If 80 giue 2000, what gineth 56? Anoagaine, if 80 giue 2000, what gineth 24.

80	2000
10	600

To conclude, if you now diuide 1400, the second mans portion by 350, which was

his stocke that he laid into company, you shall finde in your quotient 4 moneths, and for so long time did the second man put his money into the common stocke.

Lastly, if you diuide the third mans new laying in, which was 600 by 100, which was his stocke that he put into company: the quotient declareth his time of continuance, which was 6 moneths. And thus is the question resolued.

Scholar. Sir, I haue attentiuely beheld your working, and the more we trauell herein, the more me thinke I am in loue with this excellent Art.

Master. Then what say you to this question?

There is in a Cathedrall Church 20 Cannons, and 30 Vicars, those may spend by yeere 2600 l. but euery Canon must haue to his part 5 times so much as euery vicar hath: how much is euery mans portion, say you?

Scholar. I pray you make the answere your selfe also, so shall I perceiue best the meanes to answere to such other like.

Master. In this question, you must doe as in those before said, that haue diuersitie of time, for here is diuersitie of portions: Therefore shall you multiply the number of the persons by their difference of portion: (as you did in the other by time.) Then must you multiply the 20 (which is the number of Cannons) by

by 5 (for that is the number of their portion) so will it be 100: Then 30 (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put those two summes together, and they make 130: then say thus: If 130 spend 2600 l. what may 100 spend? The rule sheweth 2000 pound.

Againe for Vicars: If 130 spend 2600 l. what may 30 spend? Answere, 600 l. as these figures shew.

130 Z 2600	130 Z 2600
100 Z 2000	30 Z 600

But if every Canon should have so often times 4 l. as the Vicar should have 3 l. then should I multiply 20 by 4 (that were 80) and 30 by 3 (that were 90) and then both were 170. Then should the figures be set thus.

l	s	d	l	s	d
170 Z 2600			170 Z 2600		
80 Z 1223	10.	7	90 Z 1376	9	5

But this sort is too hard for you, by reason of the fractions, therefore I will let it rest to that place.

And by this rule you see what the 20 Canons may spend, which summe if you divide by 20, you shall see each Canons portion: and so of the Vicars, if you divide their summe by 30, the quotient will declare every Vicars portion.

The

The second Dialogue.

The accounting by Counters.

Master.

NOw that you haue learned the common kindes of Arithmetick with the pen, you shall see the same Art in Counters: which feat doth not onely serue for them that cannot write and read, but also for them that can doe both, but haue not at sometime their pen or tables ready with them.

This sort is in two formes commonly. The one by lines, and the other without lines. In that that hath lines, the lines doe stand for the order of places: and in that that hath no lines, there must be set in their stead so many counters as shall neede, for each line one; and they shall supply the stead of lines.

Scholar. By examples I should better perceiue your meaning.

Master. For example of the lines, loe here you see fiue lines, which stand for fiue places, so

100000
10000
1000
100
10
1

that

that the neathermost standeth for the first place, and the next about it for the second, and so vpward till you come to the highest, which is the first line, and standeth for the first place. Now what is the value of euery place or line you may perceiue by the figures which I haue set on them, which is according as you learned before in Numeration of figures by the pen: for the first place is the place of vnits or ones, and euery counter set in that line, betokeneth but one: and the second line is the place of 10, for euery counter there standeth for 10. The third line the place of hundreds, the fourth of thousands, and so forth.

Scholar. Sir, I doe perceiue that the same order is here of lines as was in the other figures by places, so that you shall not need longer to stand about Numeration, except there be any other difference.

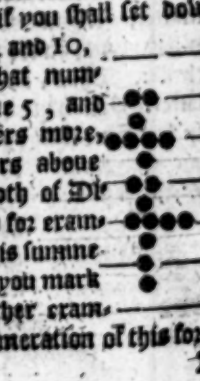
Master. If you doe vnderstand it, then how will you set 1543? 1——
Scholar. Thus, as I suppose. 5——
Master. You haue set the places truly, but your figures be not mete for this vse: for the meetest Figures in this behalfe, is the Figure of a Counter round, as you see here, where I haue expressed that same sum. 4——
3——

Scholar.

Scholar. So that you haue not one figure for 2, nor 3, nor 4, and so forth, but as many Digits as you haue, so many Counters you set in the lowest line, and for euery 10 you set one in the lowest line: and so of other. But I know not by what reason you set that one Counter for 500 betweene two lines.

Master. You shall remember this, that whensoeuer you neede to set downe 5, 50, or 500, or 5000, or so forth any number whose Numerator is 5, you shall set one counter for it in the next place aboue the line that it hath his denomination of: as in this example of that 500, because the numerator is 5, it must be set in a void space, and because the Denominator is a hundred, I know that his place is the void space next aboue hundreds, that is to say, aboue the third line.

And further you shall marke, that in all working by this sort, if you shall set downe any summe betweene 4 and 10, for the first part of that number you shall set downe 5, and then so many Counters more, as there rest numbers aboue 5. And this is true both of Digits and Articles. And for example I will set downe this summe 297955, which sum if you mark well, you need none other examples for to learne ỹ numeration of this forme.



But

But this shalt you marke, that as you did in other kindes of Arithmeticke, set a pricke in in the places of thousands, in this worke you shall set a Starre, as you see before.

Scholar. Then I perceiue Numeration: but I pray you how shall I doe in this Art to adde two summes or more together.

Addition.

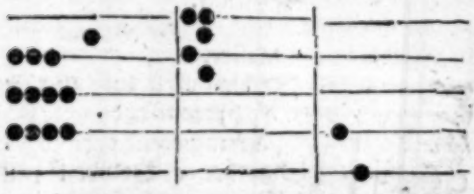
Master.

THe easiest way in this Art is to adde but two summes at once together: Howbeit you may adde more, as I will tell you anone. Therefore when you will adde two summes you shall first set downe one of them, it forceth not which, and then by it draw a line crosse the other lines. And afterward set downe the other summe, so that the line may be betweene them: as if you would adde 2659 to 8342, you must set your summes as you see here.

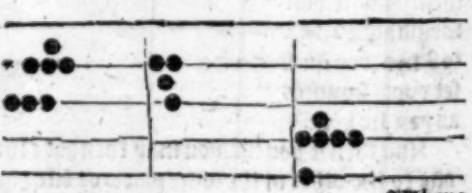
And then if you list, you may then adde the one to the other in the same place: or else you

may adde them both together in a new place: which way because it is most plaine, I will shew you first.

Therefore will I begin at the vnites which in the first summe is but 2, and in the second summe 9, that maketh 11. Those doe I take vp, and for them I set 11 in the new rome, thus.

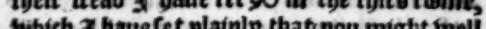


Then doe I take vp all the Articles vnder a hundred, which in the first summe are 40, and in the second summe 50, that maketh 90: or you may say better, that in the first summe there are foure Articles of 10, and in the second summe 5, which maketh 9, but then take hede that you set them in their right lines, sée héere.



Where

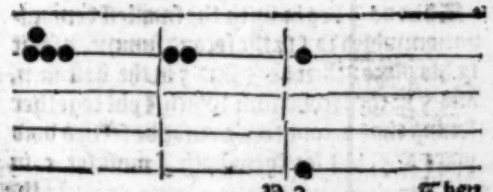
Where I haue taken away 40 from the firſt ſumme, and 50 from the ſecond, and in their ſtead I haue ſet 90 in the third rome, which I haue ſet plainly, that you might well perceiue it: howbeit, ſeeing that 90 with the 10 that was in the third rome already doth make 100, I might better for thoſe 6 Counters ſet 1 in the third line thus.



For it is all in one ſumme, as you may ſee, but it is beſt neuer to ſet fiue counters in any line, for that may be done with one Counter in a higher place.

Scholar. I iudge that good reaſon, for many are vnneedfull where one will ſerue.

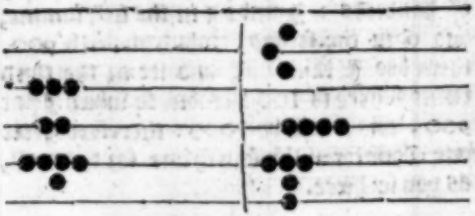
Maſter. Well, then will I adde forth of hundreds: I finde 3 in the firſt ſumme, and 6 in the ſecond, which maketh 900, them doe I take vp, and ſet in the third rome, where is 100 alreadie, to which I put 900, and it will be 1000: therefore I ſet one Counter in the fourth line for them all, as you ſee heere.



Then adde I the thousands together, which in the first summe are 8000, and in the second 2000, that maketh 10000: them doe I take vp for those two places, and for them I set one counter in the fift line, and then it appeareth as you see to bee 11001, for so many doth amount of the Addition of 8342 to 2659.

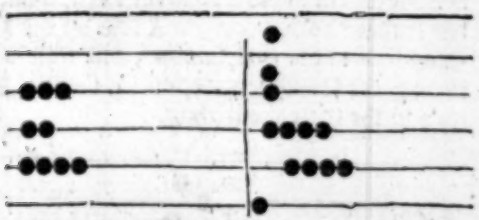
Scholar. Sir, this I do perceiue: but how shall I set one summe to another, not changing them to a third place?

Master. Marke well how I doe it: I will adde together 65436 and 3245, which first I set downe thus.



Then do I begin with the smallest denomination, which is 1 in the second summe, & set it in his place: then do I find 5 in the first sum. and 5 in the second sum which I put together sauing that 2 counters cannot be set in a void place of 5, but for them both I must set 1 in the

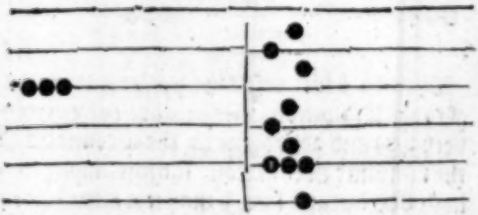
the ſecond line, which is the place of 10, therefore I take vp the fiue of the firſt ſumme, and the 5 of the ſecond, and for them I ſet 1 in the ſecond line, as you ſee heere.



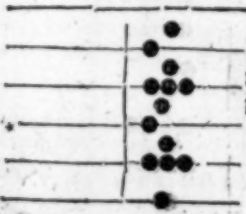
Then do I likewiſe take vp the 4 counters of the firſt ſumme and ſecond line (which maketh 40) and adde them to the 4 counters of the ſame line in the ſecond ſumme, and it maketh 80: but as I ſaid, I may not conuenientlyſet aboue 4 counters in one line, therefore to thoſe 4 that I toke vp in the firſt ſumme, I take one alſo of the ſecond ſumme, and then haue I taken vp 50: for which 5 Counters I ſet downe one in the ſpace ouer the ſecond line: as heere doth appeare.

And then is there 80, as well with thole 4 Counters, as if you had let downe the other 4 alſo.

Now do I take the 200 in the firſt ſumme, and adde them to the 400 in the ſecond ſum, and it maketh 600, therefore I take vp the 2 Counters in the firſt ſumme, and three of them in the ſecond ſumme, and for them 5, I ſet 1 in the ſpace aboue, thus.



Then take I the 3000 in the firſt ſumme, vnto which there are none in the ſecond ſum agreeing, therefore I doe onely remoue thoſe three Counters from the firſt ſumme into the ſecond, as here doth appeare.

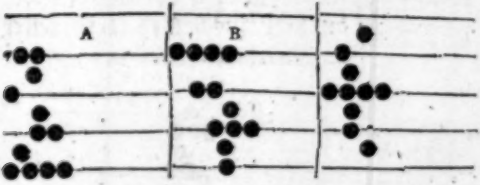


And ſo you ſee the whole ſumme that a-mounteth of that Addition of 65436 with 3245, to be 68681.

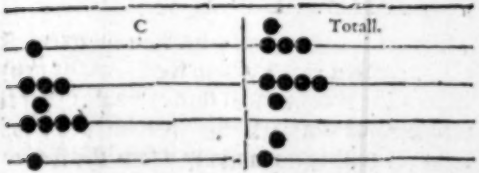
And if you haue marked

marked these two examples well, you nede no further instruction in Addition of 2 onely summes: but if you haue more than two summes to adde, you may adde them thus.

First adde two of them, and then adde the third and fourth, or more if there be so many: as if I would adde 2679 with 4286, and 1391. First I adde the two first sums, thus.



And then I adde the third thereto, thus.



And so of more, if you haue them.

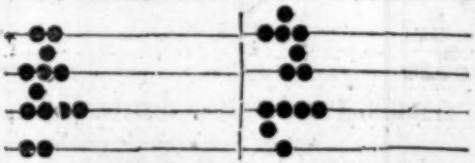
Scholar. Now I thinke it best that you passe forth to subtraction, except there be any way to examine this manner of Addition, then I thinke that were good to be knowne next.

Master. There is the same proose here that is in the other Addition by the pen, I meane Subtraction; for that onely is a sure way: but

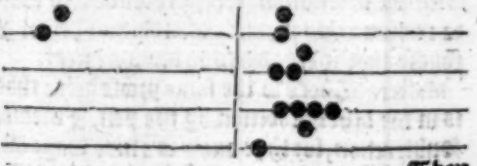
conſidering that Subtraction muſt bee firſt knowne, I will firſt teach you the Art of Subtraction, and that by this example.

Subtraction.

I Would Subtract 2892 out of 8746. Theſe ſummes muſt I ſet downe as I did in Addition: but here it is beſt to ſet the leſſer number firſt, thus.

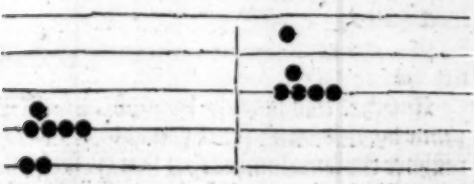


Then ſhall I begin to ſubtract the greateſt numbers firſt (contrary to the vſe of the pen) that is, the thouſands in this example: therefore I finde amongſt the thouſands 2, for which I withdraw ſo many from the ſecond ſumme (where are 8) and ſo remayneth there 6, as this example ſheweth.



Then

Then doe I likewise with the hundreds, of which in the first summe I finde 8, and in the second summe but 7, out of which I cannot take 8, therefore this must I doe: I must take how much my summe differeth from 10, which I finde here to be 2, then must I abate for my summe of 800, one thousand, and set downe the excesse of hundreds, that is to say, 2, for so much as 1000 is more than I should take vp: therefore from the first summe I take that 800, and from the second summe (where are 6000) I take vp one thousand, and leaue 5000, but then I set downe the 200 vnto the 700 that are there already, and make them 900, thus.

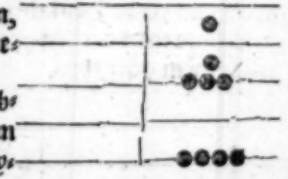


Then come I to the Articles of tennes, where in the first summe I finde 90, and in the second summe but onely 40. Now considering that 90 cannot be abated from 40, I take how much that 90 doth differ from the next summe aboue it, that is 100, or else (which is all to one effect) I take how much 9 doth differ from 10, and I finde it to be 1: then in the

the ſtead of that 90, I do take from the ſecond ſumme 100 : but conſidering that is 10 to much, I ſet doiyne 1 in the nert line beneath foꝛ it, as you ſee heere.

Sauing that here I haue ſet 1 counter in the ſpace in ſtead of 5 in the nert line.

And thus haue I ſubtract all ſaue 2, which I muſt bate from 6 in the ſecond ſum, and there will remayne 4, thus.



So that if I ſubtract 2892 from 8746, the remayner will be 5854.

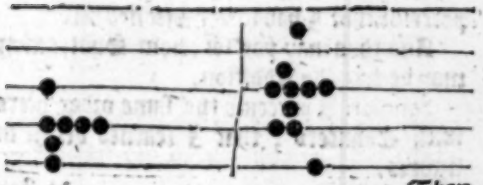
And that this is truly wꝛought, you may pꝛoue by Addition : foꝛ if you adde to this remayner the ſame ſumme that you did ſubtract, then will the foꝛmer ſumme 8746 amount againe.

Scholar. That will I pꝛoue, and firſt I ſet the ſumme that was ſubtracted, which was 2892, and then the remayner 5854, thus.

Then

Then doe I adde the first 2 to 4, which maketh 6: so take I vp 5 of those counters, and in their stead I set 1 in the space, and one in the lowest line, as here appeareth.

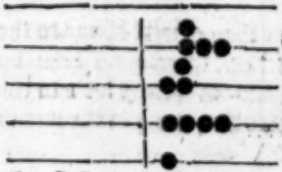
Then doe I adde the 90 next aboue to the 50, and it maketh 140, therefore I take vp those 6 counters, and for them I set 1 to the hundreds in the third line, and 4 in the second line, thus.



Then

Then doe I come to the hundreds, of which I find 8 in the first summe, and 8 in the second, that maketh 1600, therefore I take vp those 8 counters, and in their stead I set 1 in the fourth line, and 1 in the space next beneath, and 2 in the third line, as you see here.

Then is there left in the first summe but onely 2000, and in the second 5000, which is 7000, which I shall take vp from thence, and set in the same line in the second summe to the one that is there alreadye: and then will the whole summe appeare, as you may well see, to bee 8746, which was the first grosse summe, and therefore I doe perceiue that I had well subtracted before.



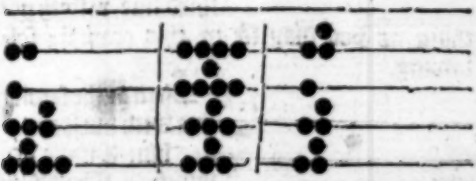
And thus may you see, how Subtraction may be tried by Addition.

Scholar. I perceiue the same order here with Counters, that I learned before in figures.

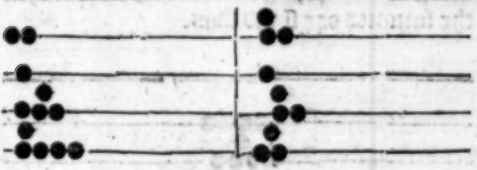
Master.

Master. Then let me see how you can trie Addition by Subtraction.

Scholar. First I will set forth this example of Addition, where I haue added 2189, to 4983. And the whole summe appeareth to be 7177.

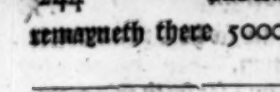


Now to trie whether that summe be well added or no, I will subtract one of the first two summes from the third, And if I haue well done, the remayner will be like that other summe, as for example, I will subtract the first summe from the third, which I set thus in their order.



Then doe I subtract 2000 of the first summe from the second summe, and then remayneth

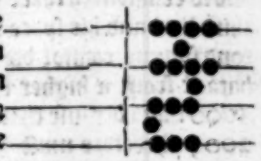
remayneth there 5000, thus.



Then in the third line I ſubtract the 100 of the firſt from the ſecond ſumme, where is onely 100 alſo: and then in the third line reſteth nothing, as you may ſee in this example following.

Then in the ſecond line with his ſpace ouer him I finde 80, which I ſhould ſubtract from the other ſumme: then ſeeing there are but onely 70 I muſt take it out of ſome higher ſumme, which is heere onely 5000: therefore I take vp 5000: and ſeeing that is too much by 4920, I ſet downe ſo many in the ſecond rowme, which with the 70 being there alreadie, do make 4990, and then the ſummes doe ſtand thus.

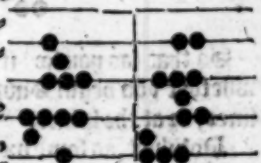
Yet remayneth therein the first summe 9, to be abated from the second summe, wherein that place of vnites doth appeare onely 7: then must I abate a higher sum, that is to say 10, but seeing that 10 is more then 9 (which I should abate) by 1 therefore shall I take vp one counter from the second, & set down the same in the first or lowest line, as you see heere.



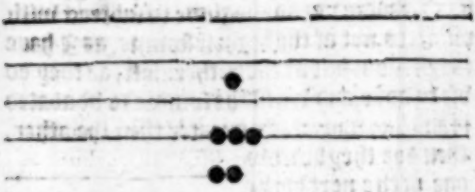
And so haue I ended this worke, and the summe appeareth to be the same which was the second summe of mine Addition, and therefore I perceiue I haue well done. Another way of Addition.

Master. To stand longer about this, it is but folly, except that this you may also vnderstand, that many doe beginne to subtract with counters, not at the highest summe, as I haue taught you, but at the nethermost, as they do vse to adde, and when the summe to be abated in any line appeareth greater then the other, then doe they borrow one of the nert higher rome, as for example.

If I should abate 1846 from 2378, they set the summes thus.



First they take 6, which is the lower line, and his space from 8 in the same romes in the second summe, and yet there remayneth 2 counters in the lowest line. Then in the second line must 4 be subtracted from 7, and so remayneth there 3. Then 800 in the third line and his space, from 300 of the second summe cannot be, therefore doe they bate it from a higher rome, that is, from 1000: and because that 1000 is too much by 200, therefore must I set downe 200 in the third line, after, I haue taken vp 1000 from the fourth line. Then is there yet 1000 in the fourth line of the first summe, which if I withdrawe from the second summe, then doe all Figures stand in order, thus. 532.



So that (as you see) it differeth not greatly whether you begin Subtraction at the higher lines, or at the lower.

Howbeit, as some men like that one way best, so some like the other: therefore you now knowing both, may vse which you list.

Multi-

Multiplication.

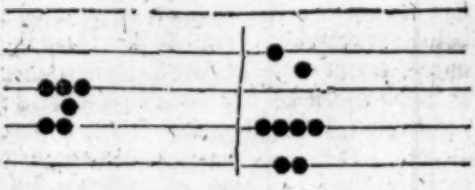
But now touching Multiplication: you shall set your numbers into two rowes (as you did in those other kindes) but so that the multiplier be set in the first rowe: then shall you begin with the highest numbers of the second rowe, and multiply them first after this sort.

Take the outermost-line in your first working, as it were the lowest line, setting on it some moueable marke (as you list) and take how many counters be in him, take them vp, and for them set downe the whole multiplier so many times as you take vp counters: reckoning (I say) that line for the vnites. And when you haue done with the highest number, then come to the nert line beneath, and doe so euen with it, and so with the nert, till you haue done all. And if there be any number in a space, then for it shall you take the Multiplier 5 times: and then must you reckon that line for the Vnites, which is nert beneath that space. Or else after a shorter way, ye shall take onely halfe the multiplier, but then shall you take the line nert aboue the space for the line of vnites. But in such working, if by chance your Multiplier be an odde number, so that

you cannot take the halfe of it iuſtly, then muſt you take the greater halfe and ſet downe that, as if that it were the iuſt halfe: and further you ſhall ſet one counter in the ſpace beneath that line, which you reckon for the line of vnites, or elſe onely remoue forward the the ſame that is to be multiplied.

Scholar. If you ſet forth an example hereof, I thinke I ſhall perceiue you.

Maſter. Take this example: I would multiply 1542 by 365, therefore I ſet my numbers thus.



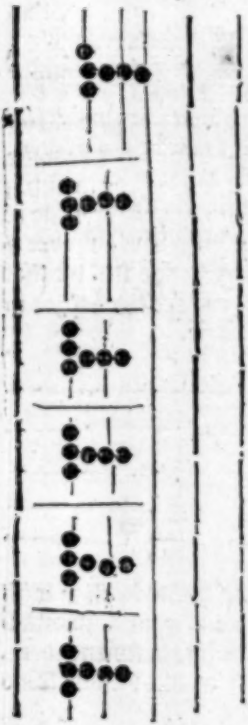
Then firſt I begin at the 1000 in the higheſt rowne as if it were the firſt place, and I take it vp ſetting downe for it ſo often (that is once) the multiplier, which is 365, thus as you ſée héere: where, for the one Counter taken vp from the fourth line, I haue ſet downe other fiue which make the ſumme of the multiplier, reckoning the fourth line, as if it were the firſt, which thing I haue marked by the ſtarre ſet at the beginning.

Scholar.

Scholar. I perceiue well, for indéede this ſumme that you haue ſet downe is 365000: for ſo much doth amount of 1000, multiplyed by 365.

Maſter. Well then to goe forth, in the next ſpace I finde one Counter, which I remoue forward, but take it not vp, but (as in ſuch a caſe I muſt) ſet downe the greater halfe of my multiplier (ſeeing it is an odde number) which is 182, and héere I doe ſtill let that fourth place ſtand as if it were the firſt, as in theſe examples you ſhall ſée.

Where I haue ſet the Multiplication with other; but for the eaſe of your vnderſtanding, I haue ſet a little line betwéene them. Now ſhould they both in one ſumme ſtand thus.



Howbeit another forme to multiply such Counters in spaces this: first to remoue the finger to the next line beneath the space, and then to take vp the Counter, and to set downe the multiplier 5 times: as heere you see.

Which summes if you do adde together into one summe, you shall perceiue that it will be the same that appeareth of the other working before, so that both sorts are to one intent: but as the other is shorter.

so

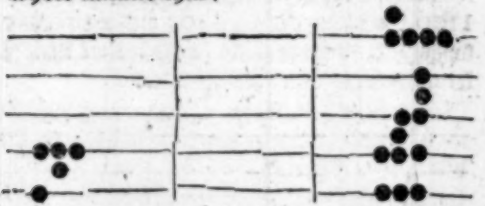
so this is playner to reason, for such as haue had small exercise in this Art.

Notwithstanding you may adde them in your minde before you set them downe: as in this example you might haue said, 5 times 300 is 1500, and 5 times 60 is 300, also 5 times 5 is 25, which all put together do make 1825, which you may at one time set downe if you list.

But now to goe forth. I must remoue the hand to the next counters which are in the second line, and there must I take vp those foure counters, setting downe for them my multiplier foure times seuerally, or else I may gather the whole summe in my minde first, and then set it downe: as to say, 4 times 300 is 1200: 4 times 60 are 240: and 4 times 5 make 20, that is in all 1460: that shall I set downe also, as heere you see.

Which if I ioyne in one summe with the former numbers, it will appeare thus.

Then to end this Multiplication, I remoue the finger to the lowest line, where are onely 2, them doe I take vp and in their stead doe I set downe twice 365, that is 730, for which I set one in the space aboue the third line for 500 and 2 more in the third line with that one that is there alreadie, and the rest in their order, and so haue I well ended the whole summe thus.



Whereby you see, that 1542 (which is the number of yeeres sith Christ his incarnation) being multiplied by 365 (which is the number of dayes in one yeere) doth amount to 562830, which declareth the number of dayes since Christs incarnation vnto the end of 1542 yeeres, beside 385 dayes and twelue houres for leape yeeres.

Scholar.

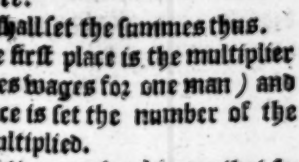
Scholar. Now will I proue by another example, as this, 40 labourers) after 6 d the day for each man) haue wrought 28 daies, I would know what their wages doth amount vnto.

In this case must I worke doubly: first I must multiply the number of the labourers by the wages of a man for one day, so will the charge of euery day amount.

Then secondarily shall I multiply the charge of 1 day by the whole number of dayes, & so will the whole sum appeare: first therefore I shall set the summes thus.

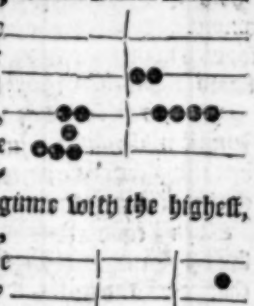
Where in the first place is the multiplier (that is one dayes wages for one man) and in the second space is set the number of the workemen to multiplied.

Then say: If 6 times 4 (reckoning that second line of the line of vnites) maketh 24, for which sum I should set 2 counters in the third line, and 4 in the second, therefore doe I set 2 in the third line, and let the 4 stand still in the second line thus.

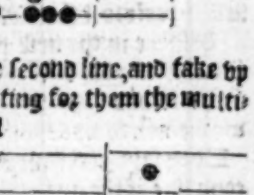


So appeareth the whole dayes wages to be 240 d, that is 20 s.

Then doe I multiply againe the same sum by the number of dayes, and first I set the numbers thus, then because there are Counters in diuers lines I shall begynne with the highest, and take them vp, setting for them the Multiplier so many times as I tooke vp Counters, that is twice, then will the Summe stand thus.



Then come I to the second line, and take vp those 4 counters, setting for them the multiplier 4 times, so will the whole summe appeare thus.



So is the whole wages of 40 workemen for 28 dates after 6d each day for a man) 6720 d, that is, 560 s, or 28 l.

Maister. Now if you would proue Multiplication, the surest way is by Diuision, therefore will I ouerpasse it, till I haue taught you the Art of Diuision, which you shall worke thus.

Diui-

Diuision.

FIrſt ſet downe the Diuiſor, for feare of forgetting, and then ſet that number that ſhall be diuided at the right ſide ſo far from the diuiſor that ẏ quotient may be ſet betwæn them as for example.

If 225 ſhæpe coſt 45 ℔, what did euery ſhæpe coſt? To know this I would diuide the whole ſumme, that is 45 ℔, by 225, but that cannot be, therefore muſt I firſt reduce that 45 ℔, into a leſſer denomination, as into ſhillings; then I multiply 45 by 20, and it is 900: that ſumme ſhall I diuide by the number of ſhæpe which is 225, theſe two numbers therefore I ſet thus.

Then begin I at the higheſt line of the diuidend therein, and that I may doe foure times: then ſay I foure times 2 are 8, which if I take from 9, there reſteth but 1 thus.

And

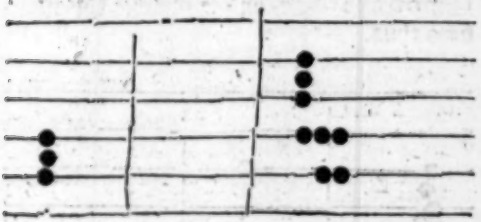
And because I found the Diuisor 4 times in the diuidend, I haue set, as you see, 4 in the middle rome, which is the place of the quotient: but now must I take the rest of the diuisor as often out of the remayner, therefore come I to the second line of the Diuisor, saying: two times 4 make 8, take 8 from 10, and there remayneth 2, thus.

Then come I to the lowest number which is 5, and multiply it 4 times, so is it 20, that take I from 20, & there remayneth nothing, so that I see my quotient to be 4, which are in value shillings, for so was the diuidend: and thereby I know that if 225 sheepe did cost 45 l. every sheepe cost 4 s.

Scholar. This can I do, as you shall perceiue by

by the example. If 160 souldiers doe spend euery monete 68 l, what spendeth each man?

First because I cannot diuide the 68 by 160, therefore I will turne the l, into pence by multiplication, so shal there be 16320 pence: now must I diuide the summe by the number of souldiers, therefore I set them in order thus. Example of wages.



Then begin I at the highest place of the diuidend, seeking my Diuisor there, which I finde once, therefore I set 1 in the neather line.

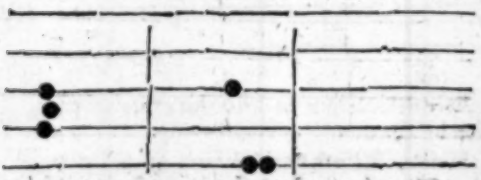
Master. Not in the neather line of the whole summe, but in the neather line of that worke, which is the third line.

Scholar. So standeth it with reason.

Master. Then thus doe they.

Then

Then ſeeke I againe the reſt: how often I may finde my diuiſor: and I ſee that in 300 I might find 100 three times: but then the 60 will not be ſo often found in 20, therefore I take 2 for my quotient: then take I 100 twice from 300, and there reſteth 100, out of which with the 20 that maketh 120, I may take 60 alſo twice, and then ſtand the numbers thus.



Where I haue ſet the quotient 2 in the loweſt line: So is euery ſouldiers portion 102 d, that is 8 ſ, 6 d.

Maſter. But yet becauſe you may iuſtly perceiue the reaſon of Diuiſion, it ſhall be good that you ſet your Diuiſor ſtill againſt thoſe numbers from which you doe take it, as by this example I will declare.

Example of purchaſe.

If the purchaſe of 200 acres of ground did coſt 290 pound, what did one acre coſt?

Firſt will I turne the pounds into pence, ſo will there be 69600 pence. Then in ſetting downe theſe numbers I ſhall doe thus.

Firſt ſet the diuidend on the right hand as

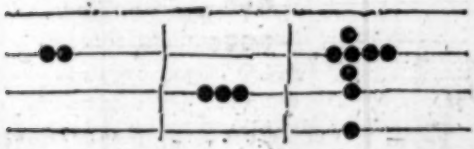
ought

ought, and then the diuisor on the left hand against those numbers from which I intend to take him first, as heere you see where I haue set the diuisor two lines higher then his owne place.

Scholar. This is like the order of Diuision by the pen.

Master. Truth you say, and now must I set the quotient of this worke in the third line, for that is the line of vnites in respect of the Diuisor in this worke.

Then I seeke how often the Diuisor may be found in the dividend, and that I find 3 times, then set I 3 in the third line for the quotient, and take away that 60000 from the dividend, and further I set the Diuisor one line lower, as you see heere.



And

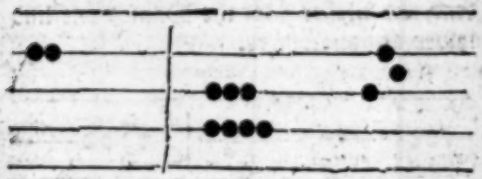
And then take I how often the Diuisor will be taken from the number against it, which will be foure times, and 1 remayning.

Scholar. But what if it chance that when the Diuisor is so remoued, it cannot be once taken out of the diuidend against it?

Master. Then must the Diuisor be set in another line lower.

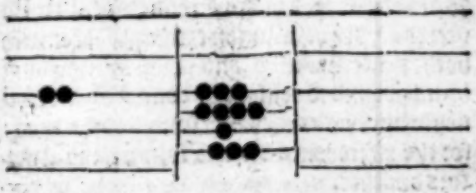
Scholar. So was it in Diuision by the pen, and therefore was there a cipher set in the Quotient: but how shall that be noted here?

Master. Here needeth no token, for the lines doe represent the places, onely take, that you set your Quotient in that place which standeth for vnites in respect of the Diuisor. But now to returne to the example: I finde the Diuisor foure times in the diuidend, and 1 remayning: for 4 times 2 makes 8, which I take from 9, and there resteth 1, as this figure following sheweth: and in the middle space for the Quotient, I set 4 in the second line, which is in this worke the place of vnites.



Then remoue I the Diuisor to the next lower

lower line, and seeke how often I may haue it in the Diuidend, which I may doe heere 8 times iust, and nothing remayne, as in this forme.



Where you may see that the whole Quotient is 348 li, that is 29 s, whereby I know that so much cost the purchase of one Acre.

Scholar. Now resteth the proues of Multiplication, and also Diuision.

Master. Their best proues are each one by the other, for Multiplication is proued by Diuision, and Diuision by Multiplication, as in the worke by the pen you learned.

Scholar. If that be all, you shall not neede to repeate againe that which was sufficiently taught alreadie: and except you will teach me any other feate, heere may you make an end of this Art, I suppose.

Master. So will I doe as touching whole number, and as for broken number, I will not trouble your wit with it, till you haue practised this so well that you be full perfect,

so

ſo that you need not to doubt in any point that I haue taught you, and then may I boldly inſtruct you in the Art of Fractions or broken numbers: wherein I will alſo ſhew you the reaſons of all that you haue now learned. But yet before I make an end, I will ſhew you the order of common caſting, wherein are both pence, ſhillings, and pounds, proceeding by no grounded reaſon, but onely by a receiued forme, and that diuerſly, of diuers men: for the Merchants vſe one forme, and Auditors another.

Merchants vſe.

But firſt for Merchants forme marke this example heere, in which I haue expreſſed this ſumme 198*l*, 19*s* 11*d*. So that you may ſee that the loweſt line ſerueth for pence, the next aboue for ſhillings, the third for pounds, and the fourth for ſcores of pounds.

And further you may ſee that the ſpace betweene pence and ſhillings may receiue but one counter (as all other ſpaces likewiſe doe) and that one ſtandeth in that place for 6 *d*.

Likewiſe betweene the ſhillings and the pounds, one counter ſtandeth for 10 ſhillings.

And

And betweene the pounds and 20 l, one Counter ſtandeth for 10 l.

But beſide thoſe, you may ſée at the left ſide of ſhillings that one number ſtandeth alone, and betokeneth 5 s.

So againſt the pounds, that one Counter ſtandeth for 5 l. And againſt the 20 pounds, the one Counter ſtandeth for 5 ſcore pounds, that is, 100 pounds, ſo that euery ſide Counter is fiue times ſo much as one of them againſt which he ſtandeth.

Auditors account.

NOw for the account of Auditors, take this example.

Where I haue expreſſed the ſame ſumme 198 l — 19 — 11 d:

But héere you ſée the pence ſtand towards the right hand, and the other increaſing orderly toward the left hand.

Againe you may ſée, that Auditors will make two lines (yea and more) for pence, ſhillings, and all other values, if their ſummes extend thereto. Alſo you ſee, that they ſet one Counter at the right end of each row, which ſo ſet there, ſtandeth for fiue of that rowe, and

on the left corner of the row it standeth for 10 of the same row.

But now, if you would adde, or subtract after any of both those sorts, if you marke the order of the other leafe which I taught you, you may easily doe the same heere without much teaching: for in Addition, you must first set downe one summe, and to the same set the other orderly, and in like manner if you have many: but in Subtraction, you must set downe first the greatest summe, and from it must you abate the other, euery denomination from his due place.

Scholar. I doe not doubt but with a little practise I shall attayne these both: but how shall I multiply and diuide after these forms?

Master. You cannot duely doe any of both by these sorts, therefore in such case you must resort to your other Arts.

Scholar. They that vse such accounts that it exceed 200 in the summe they set not 5 at the left hand of the scores of pounds, but they set all the hundreds in another farther row, and 500 at the left hand thereof, and the thousands they set in farther row yet, and at the left side thereof they set the 5000, and in the space ouer they set the 10000, and in a higher row 20000, which all I haue expressed in this example which is 9869 l, 12 s, 9 d, ob, q. Nineteene seauen thousand, eight hundred, three-score and nine pounds, twelue

twelue shillings and 9 pence halfe penie farthing, for I had not told you before where, neyther how you should set downe farthings, which (as you see here) must be set in a void place sliding beneath the pence, for farthing one Counter, for ob 2 Counters, for ob. farthing, 3 Counters, and more there cannot be: for 4 farthings make 1 d, which must be set in his due place.

And if you desire the same summe after Auditors manner, loe here it is.

But in this thing you shall take this for sufficient, and the rest you shall obserue as you may see by the working of each sort, for the diuers wits of men haue inuented diuers and sundry wayes, almost innumerable.

The second part of Arithmeticke, touching Fractions, briefely set forth.

Scholar.

ALbeit I perceiue your manifold busines doth so occupie or rather oppresse you, that you cannot as yet completely end the Treatise of Fractions Arithmeticall, which you haue prepared, wherein not onely sundrie workes of Geometrie, Musicke, and Astronomie be largely set forth, but also diuers conclusions and naturall workes, touching mixtures of mettals, and compositions of medicines, with other strange examples: Yet in the meane season, I cannot stay my earnest desire, but importunely craue of you some briefe preparation toward the vse of Fractions, whereby at the least I may bee able to vnderstand the common workes of them, and the vulgar vse of those rules, which without them cannot well be wrought.

Master. If my leasure were as great as my will is good, you should not neede to vse any importunate crauing, for the attayning of that thing whereby I may be perswaded that I shall any wayes profit the com-

mon-

common-wealth, or helpe the honest studies of any good members in the same: wherefore, while mine attendance will permit me to walke and talke, I am well willing to helpe you as I may.

Therefore first to begin with the explication of this name, Fractions, what take you it to bee? What a fraction is.

Scholar. Mary sir, I thinke a Fraction (as I haue heard it often named) to be a broken number, that is to say, to be no whole number but a part of a number.

Master. A Fraction indeed is a broken number, and so consequently the part of another number: but that must be vnderstood of such another number as cannot be diuided into any other parts then Fractions, for although I may take the third part of 60, or the fourth part of it, and so of other parts diuersly, yet these parts be not properly, nor ought not to be called Fractions, because they may be expressed by whole numbers; for the third part of it is 20: the fourth part is 15. the twelfth part is 5. and so forth of other parts, which all be whole numbers.

Wherefore properly a Fraction expresseth the parts or part onely of a vnite, that is to say, that the number which is the whole or entire summe of any Fraction, may not be greater then one: and therefore it followeth that no one Fraction alone can be so great, What a fraction is properly

that it shall make 1, as by examples I will declare as sone as I haue taught you to know the forme how a Fraction is expressed or representeb in writting.

Numeration.

But first to begin with expressing of a Fraction, which is the Numeration of it: you must vnderstand that a Fraction is represented by 2 numbers, set one ouer the other, and a line drawne betweene them, as thus, $\frac{1}{3}\ \frac{3}{4}\ \frac{2}{7}\ \frac{10}{17}$, which 4 Fractions you must pronounce thus: $\frac{1}{3}$ one third part, $\frac{3}{4}$ 3 quarters, $\frac{2}{7}$ two fift parts, $\frac{10}{17}$ ten 17 parts.

Scholar. I vnderstand the forme of their expression and pronunciation: but their meaning or valuation seemeth more obscure. Yet I thinke that by the two first fractions I vnderstand the valuation of the two latter fractions, and consequently of other.

Master. Value them then, that I may perceiue your taking of them.

Scholar. $\frac{2}{7}$ Betokeneth two fift parts, that is to say, if one be diuided into 5 parts, that fraction doth expresse two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be diuided into 17 parts, I must take tenne of them. And

this

this I gather of the two first examples: for $\frac{1}{3}$ that is one third part, both easily declare, that if any one thing be diuided into three parts, I must take out one of them: so $\frac{3}{4}$, that is three quarters, both declare that one being diuided into foure quarters, I must take (for this Fraction) three of those quarters.

If there be no more difficultie in their Numeration, then I pray you goe forward to their Addition and Subtraction, and so to the other kinds of workes. For I vnderstand that the same kindes of workes be in fractions, that be in whole numbers.

Master. There are the same kindes of workes in both, albeit the order of them is diuers, as I will anone declare: but yet more in Numeration before we leaue it. You must vnderstand, that those two numbers which expresse a Fraction, haue seuerall names. The ouermost, which is aboue the line, is called the Numerator, and the other beneath the line, is called the Denominator.

Scholar. And what is the reason of their diuers names? (For, in mine opinion, both be Numerators, seeing both they doe expresse the numeration of the Fraction.

Master. You are deceiued: for one onely (which is the ouermost) doth expresse the Numeration: and the Denominator doth declare the number of parts, into which the vnite is

diuided, as in this example, when I say: Diuide a pound weight of Gold betweene foure men, so that the first man shall haue $\frac{2}{15}$, the second $\frac{3}{15}$, the third $\frac{4}{15}$ and the fourth $\frac{6}{15}$.

Now doe you perceiue that by the Denominator (which is one in all foure fractions) it is intended that the pound weight should be diuided into so many parts, I meane 15, and by the foure seuerall Numerators is limited the diuers portion that each man should haue, that is, that when the whole is parted into 15, the first man shall haue two of those 15 parts: the second man 3 of them: the third man 4: and the fourth man 6. And so may you see the seuerall offices (as it were) of those two numbers, I meane of the Numerator and the Denominator.

And hereby you perceiue that a man can haue no more parts of any thing then it was diuided into, neyther yet aptly so many: so that it were vnaptly said: You shall haue $\frac{15}{15}$ that is 15 parts of any thing, seeing it were better said you shall haue the whole thing.

Scholar. So doth it appeare reasonably, for the labour is vaine to diuide any thing, and then to apply the Diuision to no vse. And much lesse reasonable were it to say $\frac{16}{15}$: for if the whole be diuided into 15 parts onely, it is not possibly to take 16 of them, that is to say, more then another.

Master. This is true touching the proper

and

and apt vse of the name of a Fraction: yet improperly, and after a vulgar acceptation (for easinesse in worke) both those formes be called Fractions, because they be written like Fractions, although they be none indeede: for $\frac{15}{15}$, and generally all such other, where the Numerator and Denominator be equall, are not Fractions, but the whole thing with all his parts. And so $\frac{15}{12}$ is not to be called a Fraction, but a mixt number, of a whole number and a Fraction, for it is as much as $1\frac{3}{12}$, that is one whole 1 & $\frac{3}{12}$ parts, as shall be declared in Reduction. Therefore they doe abuse the names that call them Fractions where the Numerator is eyther equall or greater then the Denominator.

Scholar. But is there any needfull cause, why they should so abuse the name?

Master. There is cause why they shall sometimes for easinesse in worke write some numbers after that sort like Fractions: but they needed not to call them Fractions, but (as they be) whole numbers or mixt numbers, (that is, whole numbers with Fractions) expressed like Fractions.

Now must you vnderstand, that as no Fraction properly can be greater then 1, so in smalnesse vnder one the nature of Fractions doth extend infinitely, as the nature of whole numbers is to increase aboue one infinitely, so that not onely one may be diuided into infinite

Fractions

fractions or parts, but also euery fraction may be diuided into infinite Fractions or parts, which commonly be called Fraction of Fractions, and they be expressed diuersly: as for example $\frac{1}{4}$ of $\frac{2}{3}$ that is, three quarters of two third parts of one halfe part. Whereby is signified, that if one be diuided into two halues, and the one halfe into three parts, and two of those three parts be diuided iointly into foure quarters, this fraction of fractions doth represent three of those quarters.

Scholar. I pray you let me proue by an example in common money, whether I doe rightly vnderstand you or no. One Crowne, which I take for an vnite, doth containe 60 d, therefore the halfe of it is 30 pence: $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{1}{4}$ is fifteene pence, so then 15 pence is $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a shilling.

Master. You perceiue this well enough: yet this note I giue you by the way, that the forme of expressing these fractions is voluntarie, and hath no other reason than the will of the Diuisor, which forme many follow: for some expresse them thus $\frac{121}{415}$ without any figure of distinction betweene them, which forme many follow. Some other doe make lines betweene euery fraction, and adde words of distinction, after this sort, $\frac{1}{4}$, of $\frac{2}{3}$, which forme is best.

Some

Some other expresse them thus in slope forme, to distinct them from seuerall fractions of one whole number, for if they were set in one right line thus, $\frac{121}{415}$: then ought it to be pronounced, three quarters, and two third parts and a halfe, which maketh almost two whole vnites, lacking but one 12 part. And so is it nothing agreeable with the other fraction of fractions: wherefore it is a great ouersight in certaine learned men, which doe expresse them so confusedly with such seuerall fractions, that a man cannot know the one from the other.

$\frac{3}{4}$
$\frac{2}{3}$
$\frac{1}{4}$

Therefore some men (as Stifelius) doe expresse without a line numbers of proportion, being applied to Addition or Subtraction, because they must be taken as two, where the line in fractions maketh them to be taken for one: For the Numerator and Denominator is made one number.

Scholar. Then I perceiue there be three seuerall varieties in fractions: first, when one onely fraction is set for one number, as $\frac{4}{5}$, that is, foure fift parts. The second is, when there be set two or more seuerall fractions of one number: as $\frac{4}{9}$, that is foure ninth parts, and two fift parts. The third sort is fractions of fractions, as $\frac{4}{9}$ of $\frac{2}{5}$, that is foure ninth parts of two fift parts.

Master. You haue said well, if you vnder-

stand

ſtand well your owne wordes.

Scholar. If it ſhall pleaſe you I will by an example in the parts of an old Engliſh Angell expreſſe my meaning.

Maſter. Let me here you.

Scholar. The old Engliſh Angell did contayne 7 ſhillings 6 d, that is 90 pence. Now $\frac{2}{3}$ of it is 72 d. And of the ſame 90 pence, if I take $\frac{4}{5}$ and $\frac{3}{7}$ that is foure nine parts, and 2 fift parts, $\frac{2}{5}$ is 40, and $\frac{4}{7}$ is 36, which both make 76: but if I take $\frac{4}{7}$ of $\frac{3}{7}$, that is 4 nine parts of two fift parts, ſaying $\frac{3}{7}$ is but 36, then $\frac{2}{5}$ of 36 will yeld but 16, for $\frac{2}{5}$ of 36, is but 4, and that taken 4 times maketh 16.

Maſter. This is plainly expreſſed, and truly, and hereby (I doubt not) but you doe perceiue, that as great a difference as is betweene 16 and 17, ſo much difference is betweene thoſe two Fractions $\frac{4}{5}$ and $\frac{3}{7}$: and $\frac{4}{5}$ of $\frac{3}{7}$.

And now that you vnderſtand theſe varieties, I will proceed to the reſt of the workes: firſt admoniſhing you, that there is another order to be followed in Fractions, then there was in whole numbers: for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diuiſion, & Reduction: but in fractions (to follow the ſame aptneſſe in proceeding from the eaſieſt workes to the harder) we muſt vſe this order of the workes, Numeration, Multiplication, Diuiſion, Reduction, Addition, and Subtraction.

Scholar.

Scholar. That Multiplication and Diuision shoulo goe together, and Subtraction to follow Addition, naturall order doth perswade, but why Multiplication should be first in order heere next to Numeration, and Reduction in the middle, I desire to vnderstand the reason.

Master. As in the Art of whole numbers, Order would reasonably begin with the easiest, & so goe forward by degrees to the hardest: euen so reason teacheth in Fractions the like order. And, considering that Addition or Subtraction of Fractions can very seldome be wrought without Multiplication and Reduction: and contrariwise, Multiplication and Reduction may bee wrought without this forme of Addition or Subtraction: therefore was it orderly required, that Multiplication and Reduction should goe before Addition and Subtraction, and the same reason serueth for the placing of Multiplication before Reduction.

Scholar. Then if Multiplication be the easiest, I pray you declare the forme of it first by Rule, and then by example.

Master. Your example is good.

Multi-

Herefore when any two fractions be propoued to be multiplied together, the Numerator of the one muſt be multiplied by the Numerator of the other: and the ſumme that amounteth thereof muſt be ſet for a new Numerator: likewiſe the Denominator of the one muſt be multiplied by the Denominator of the other, and that that firſt amounteth, ſhal be ſet for the common Denominator: and this new third fraction expreſſeth the product of the multiplication of the two firſt fractions propoued. whereof take this example, $\frac{3}{5}$ multiplied by $\frac{5}{12}$ doth make $\frac{15}{60}$.

Scholar. I perceiue then that 3, being the Numerator of the firſt fraction, is multiplied by 5, being the numerator of the ſecond fraction, whereof amounteth 15, the numerator of the third fraction. And ſo likewiſe, 5 being denominator of the firſt fraction, is multiplied by 12 the denominator of the ſecond fraction, whereof amounteth 60 the new denominator, ſo that I perceiue how the worke is done, but I doe not perceiue how $\frac{15}{60}$ is greater than $\frac{1}{5}$: for I ſhall not vſe my former manner of examination by the parts of ſome coyne. I ſee

that

that $\frac{1}{5}$ of a crowne is 360, and $\frac{5}{12}$ of a crowne is 250, whereof the one multiplied by the other, doth make 900 d, which is 15 crownes: but by your Multiplication there amounteth $\frac{15}{60}$, which is but 15 pence, and that is much lesse then any other of both the first fractions.

Master. That difference is betweene multiplication in whole numbers, and multiplication in broken numbers, that in whole numbers the summe that amounteth is greater then both the other whereof it came: but in fractions it is contrariwise: for the sum that amounteth is lesser then any of the other two fractions whereof it came.

Scholar. I desire much to understand the reason thereof.

Master. Although I purposed to reserue the reasons of workes Arithmeticall for the perfect booke of Arithmetick, yet I will shew you this, because of the strangenesse of the worke.

You see in whole numbers, that of two numbers being multiplied together, is made the third number, which third number, doth beare the same proportion to the number multiplied, that the multiplier doth beare to an vnite. And so in fractions the third number which amounteth of multiplication, beareth the same proportion to each of the two first fractions, that the other of those two fractions doth beare to an vnite.

Scholar. Sir, I vnderstand your words thus:

thus: when 40 is multiplied by 12, there doth amount 480, which 480 doth contayne 40 so many times in it, as twelue doth contayne vnites, that is to say, twelue times. And so it appeareth that 480 doth contayne twelue so many times also as 40 doth contayne vnites, that is, 40 times. But now I see not how the third number in this example of Fractions can contayne any of the two former (as it happened in whole numbers) seeing it is lesser then eyther of them.

Master. No marueil if you cannot see that thing which is not possible to be seene of any man, how the third number in multiplication of Fractions should be greater then any of the two former fractions: but yet this may you see (which I said) that the third number in Fractions so multiplied, doth beare the same proportion to any of the two former fractions, that the other of those two Fractions doth beare to an vnite, as in your example, $\frac{1}{2}$ being multiplied by $\frac{1}{30}$, doth make $\frac{1}{60}$. Now say I, that $\frac{1}{60}$ doth beare the same proportion to $\frac{1}{30}$ that $\frac{1}{30}$ doth beare to an vnite, as you may in your owne forme of examination by toyntrie it: for in an old Angell (which in times past was current for 7 s 6 d, are 180 halfe pence) which I set for the intire vnite whose parts (according to the fractions aforesaid) are these, for $\frac{1}{60}$ set 45 ob, for $\frac{1}{30}$ take 180 ob, and for $\frac{1}{2}$, put 75 ob. Now doth 45 beare

the

the same proportion to 108 that 75 doth beare to 180, for 45 is $\frac{5}{12}$ of 108, and so is 75 also $\frac{5}{12}$ of 180.

But these reasons may be better reserued till another time, when the knowledge of proportions in due order shall be taught. Yet in the meane season I will shew you how it commeth to passe, that in fractions the third summe must needes be lesser than any of the other two.

Consider this, that when a fraction is proponed, as in the former example $\frac{1}{5}$ if it be multiplied by more than 1, it will make more than one entire number. As if I multiply $\frac{1}{5}$ by 5: that is to say, if I take it 5 times, it will make three entire vnites: example in a crowne, $\frac{1}{5}$ of it maketh 3 s̄, which if I take fiue times, it will amount to 15 s̄, that is, three entire crownes: so if I take the same $\frac{1}{5}$ but twice, it will yeld 6 s̄, that is, one entire Crowne, and $\frac{1}{5}$. Now if I take it but once, it cannot be more than it was before, that is 5 s̄. And if I take it lesse than once, it cannot be so much as it was before. Then seeing that a fraction is lesse than one, if I multiply a fraction by another fraction; it followeth that I doe take the first fraction lesse than once, and therefore the summe that amounteth, must needes be lesse than the first fraction.

Scholar. Sir, I thanke you much for this reason. And I trust I doe perceiue the thing,

as by example of this same fraction $\frac{1}{7}$ I will expresse. If I take $\frac{1}{7}$ of a crowne once, that is to say, if I multiply $\frac{1}{7}$ by 1, it will be as it was before, but 3 s: so if I doe multiply it by $\frac{1}{12}$, that is, if I take but halfe one time, then will it be but halfe so much: likewise if I multiply it by $\frac{1}{3}$ that is, if I take but the third part of one, it will yeld but 12 pence, that is, the third part of the first fraction.

And so to make an end: If I take it but the twelfth part of once, that is, If I doe multiply it by $\frac{1}{12}$, it will yeld but the twelfth part of the first fraction, which is but three pence. And it followeth, that if $\frac{1}{7}$ make 3 pence, then $\frac{1}{12}$ must nedes make five times so much, that is, 15 pence, which was the summe that hath given the occasion of all this doubt.

Master. Then I perceive you have sufficient understanding in this sort of Multiplication for this time, wherefore I will omit that I might say more of Multiplication, till we come to Reduction, and will passe to the other workes, and first to Division, whose place followeth Multiplication, both by naturall order, and also in easinesse of worke.

Division.

Diuision.

Whensoeuer two Fractions be propoſed, that one ſhould be diuided by the other, I muſt ſet downe firſt the Fraction that ſhall be diuided, (which is called the Diuidend) and then after it the other, which is the Diuiſor. Then ſhall I multiply the Numerator of the Diuidend by the Denominator of the Diuiſor, and that which amounteth I muſt put for a new Numerator. Againe, I ſhall multiply the Denominator of the Diuidend by the Numerator of the Diuiſor, and the number that amounteth thereof, I muſt put for the new Denominator. And this third fraction is the Quotient of the ſaid Diuiſion.

Scholar. This ſeemeth eaſie in forme, as be example thus: If I would diuide $\frac{5}{8}$ by $\frac{6}{2}$, firſt I muſt multiply 5 (being the Numerator of the Diuidend) by 6, which is the Denominator of the Diuiſor, and thereof riſeth 30: then I multiply 8 being the Denominator of the Diuidend) by 2, being the Numerator in the Diuiſor: and ſo riſeth 16, the which I muſt make in a third Fraction, thus $\frac{30}{16}$.

Maſter. We ſeemeth you are quicker in vnderſtanding now, then you were when I taught you the Art of whole number: but

that is no maruell: for the more knowledge that any man getteth, the readier shall he finde his wit, and quicker in vnderstanding: but yet of two things I will admonish you, which you might haue obserued heere for the ease of worke, and lightnesse of vnderstanding, the nature of the Quotient.

Whensoeuer you diuide one fraction by another, eyther they be both equall together, eyther else the one is greater then the other: if they be equall, their Quotient shall be such, that the Numerator and the Denominator of it shall be equall also. And if the two first fractions be vnequall, their quotient shall declare the same by the vnequality of the Numerator and Denominator, as in these examples following shall appeare.

First of equall fractions: $\frac{4}{9}$ and $\frac{12}{27}$ be equall together, and if the one be diuided by the other, the quotient will be $\frac{108}{108}$, as you may perceiue by that rule aforesaid.

Now in the vnequall fractions, as $\frac{4}{9}$ and $\frac{1}{10}$, the quotient will be $\frac{40}{39}$: where the Numerator is greater then the Denominator.

Scholar. I see it is so: but I see not the reason why it should be so.

Note how to know the proportion betweene two numbers.

Maister. The reason is this: When any Fraction is diuided by another, the quotient declareth what proportion the diuidend beareth to the Diuisor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$, maketh 2, which I must be founded, not 2, but twice,

twice, declaring that $\frac{1}{4}$ is contained twice in $\frac{2}{3}$.

And note this, that the Numerator in the quotient representeth the diuidend, and the Denominator representeth the Diuisor. And this is alwaies true, whether the greater fraction be diuided by the lesser, or the lesser by the greater. But this proportion will not be exactly knowne till you haue learned the Art of proportions: notwithstanding somewhat of it I will declare in the next rule of Reduction. But now for the easie remembrance of the quotient in Diuision: as sone as you haue set downe your two fractions the one against the other, then make a straight line for the quotient: and as sone as you haue multiplied the Numerator of the diuidend, by the Denominator of the Diuisor, set the number that amounteth ouer the said line, and then multiply the other two numbers, and set their totall vnder the same line.

Scholar. I perceiue you would not haue me trust to memory till I were better expert, lest oftentimes I happen by misse remembrance to be abused. This example I take for that declaration.

If I would diuide $\frac{2}{3}$ by $\frac{1}{4}$, I must set the numbers one against the other, (as heere doth appeare) and then make another line for the quotient in some good distance, where I may set the numbers of the quotient, as sone as any of

2 3
3 4

them is multiplied. So then as soone as I haue multiplied 2 by 4, which maketh 8, I shall set that 8 ouer that line, thus: 8 And then multiply 3 by 3, which yeeldeth 9: and that 9 must be set vnder the same line, and then will the whole quotient appeare thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$, as 3 is to 9: but how may I perceiue that?

Maister. Although you shall better perceiue it by the Rule of Reduction, yet this example may be declared in common coynes, as in a common shilling of 12 d, of which $\frac{2}{3}$ maketh 8 d, and $\frac{3}{4}$ doth make 9 d, and so you may easily see that their proportions doe agree. And if you had taken this example before when you twise the example of $\frac{2}{3}$ and $\frac{3}{4}$, your Quotient would appeare (as this doth) more easie to vnderstand; whereas that quotient being $\frac{12}{16}$ is not an easie proportion for you to perceiue, being yet little acquainted with proportions: whereof to giue you some taste, I will enter to the rule of Reduction: In which also I will declare other workes, both of Multiplication, and also of Diuision, which now I must for a time omit, as things that doe neede the helpe of Reduction.

Reduction.

THerefore will I now declare the diuersities of Reduction of Fractions, which commonly haue fiue varieties.

Fiue varieties of Reduction.

1 When there be sundrie Fractions of one intire vnite, they must be reduced to one denomination, and also into one fraction.

2 Secondarily, when there be proponed fractions of fractions, they must be reduced likewise into one fraction: for otherwise they cannot be brought into one denomination.

3 Thirdly, when an improper fraction is proponed, that is to say a fraction in forme, which indeed is greater then an vnite: it must be reduced into apt forme, expressing the vnite or vnites of it, & the proper fraction distinctly. And sometimes also it shall be needfull to conuert such a mixt number of vnites with fractions, into the forme of a fraction, that is, into an improper fraction: which two formes I esteeme but as one, because they worke on one kinde of number.

4 Fourthly, there happeneth sometimes fractions to be written in great numbers, which might be written in lesser numbers: therefore is there a meane to reduce such

great numbers into their smallest termes.

5 Fiftly, when any fraction betokeneth the parts of a whole thing, which hath by common partition certaine parts, but none of like denomination with that Fraction, then may you reduce the said Fractions into another, whose denomination shall expresse the common parts of that whole thing.

Scholar. This distinction in doctrine delighteth me much, but more with hope then present fruit: for as yet I do not understand scarcely the varieties, and much lesse the practise and vse of their workes.

Master. Reduction is an orderly alteration of numbers out of one forme into another which is neuer done orderly but for some needfull vse, as in euery of the said fiue seuerall varieties I will distinctly declare.

First therefore, when two or more seuerall fractions of any vnite be proponed, as for example $\frac{3}{16}$ and $\frac{4}{6}$, because it is hard to tell what proportion of the entire number those two fractions doe expresse, therefore was Reduction diuised, to be a meane whereby these seuerall fractions might be brought into one denomination and fraction.

And in these factions this is the Art for bringing them to one denomination.

Multiply first the denominators together, and the totall thereof you shall set twice down vnder two seuerall lines or two new Denominators,

minators, or rather for one common Denominator. Then multiply the Numerator of the first fraction, by the Denominator of the second, and set the totall thereof for the Numerator over the first line. Likewise multiply the Numerator of the second fraction by the Denominator of the first, and set that totall over the second line for the Numerator of that fraction, and so are those two first fractions of severall denominations, brought to one denomination.

Scholar. If I understand you, as I thinke I doe, my example shall declare the same. The fractions which you proponed were these, $\frac{3}{16}$, and $\frac{4}{6}$, whose denominators (being 16 and 6) I multiply together, and there amounteth 96 which I set under two lines thus:

Then I multiply the Numerator of the first fraction by the denominator of the second saying, 3 into 6 maketh 18, that set I over the first line for a new Numerator, and it will be thus: $\frac{18}{96}$.

Likewise I multiply the Numerator of the second fraction by the denominator of the first, saying, 4 times 16 maketh 64, that I set for the second Numerator, and the fraction will appeare thus.

So that both fractions brought to one denomination, must stand thus: $\frac{18}{96}$ and $\frac{64}{96}$.

Master. You have done well.

Scholar. I beseech you let me examine it after

after my accustomed forme, by common parts of coyne or other measure.

Master. Goe to.

Scholar. I haue a peece of Gold which is accounted worth 8 shillings, and contayneth 96 pence, whereof $\frac{1}{16}$, that is, the 16 part, is 6 pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{16}$. Againe $\frac{1}{6}$ of the same peece of Gold is 16 d, so that $\frac{4}{6}$ maketh 64 d, that is, $\frac{64}{96}$. And so I finde the summes to agree with the other before.

Master. So haue you now the Art to bring two such fractions into one denomination. And if there be more then two, then must you multiply all the denominators together, and set the totall thereof so many times downe as there be fractions, and then to get for each one a new Numerator, multiply the Numerator of the first, by the Denominator of the second, and the totall thereof multiply by the Denominator of the third, and so forth, if there be more. Likewise multiply the Numerator of the second, by the Denominator of the first, and the totall thereof by the Denominator of the third. And in the same sort multiply the Numerator of the third into the Denominator of the first, and the totall thereof into the Denominator of the second, and so forth, if there were moe. So these three fractions $\frac{232}{141}$ doe make by reduction these other 3 fractions of one denomination $\frac{244540}{606060}$. All which you may bring into one fraction by adding the numerators

merators together, and putting that totall for the common Numerator, reserving still that same common denominator. And those 3 fractions make one improper fraction thus, $\frac{107}{60}$.

Scholar. All this I perceiue, and also that this last fraction is more then an vnite, and therefore you did call it an improper fraction.

Master. There be certayne other formes of working in this Reduction, which I will briefely touch also, to giue you an occasion to exercise your wit therein.

The first varietie is this: When you haue made and written downe your common Denominator (as I haue taught before) then to get a Numerator for the first: do thus: Deuide the common denominator by the denominator of the first fraction, and the quotient multiplied by the Numerator of the same yeeldeth a new Numerator for the first new fraction. So likewise do with the second and the third, and with all the residue, if there be more. The first varietie of this Reduction.

Scholar. That will I proue in your last example of these 3 fractions, $\frac{2}{3}, \frac{1}{4}, \frac{2}{5}$. When the denominators be multiplied, they make 60: for 5 into 4 maketh 20, and 20 by 3 yeeldeth 60, that I set downe 3 times, thus: 606060: then to haue a numerator for the first I must deuide 60 by 5, (the denominator of the first) & the quotient is 12, which I must multiply by 2 (the numerator of the first) & that maketh 24, and so haue I for the first fraction $\frac{24}{60}$.

Likewise

Likewise for the second fraction: I diuide 60 by 4, and there commeth 15: which I multiply by 3, and so haue I 45, and the second fraction $\frac{45}{60}$. Then for the third in like sort will come $\frac{40}{60}$.

The second varietie.

Master. Another way is this: If it happen so, that the lesser Denominator can by any multiplication make the greater, then note the multiplier, and by it multiply the Numerator ouer that lesser denominator, and for the lesser denominator put the greater, as thus in these two fractions, $\frac{1}{12}$ and $\frac{3}{4}$, there being the lesser denominator multiplied by 4, will make 12, which is the greater denominator: therefore by the same 4 I doe multiply 2, which is the Numerator ouer 3, and that maketh 8: vnder which I doe put 2 being the greater denominator, which is also made by multiplication of 4 into 3, and so haue I these two fractions, $\frac{3}{12}$, $\frac{7}{12}$, thus shortly reduced, without altering the one fraction,

Scholar. This I vnderstand.

The third varietie.

Master. Then marke this third way: if the denominators do not happen so, that one by multiplication may make the other, then loke whether they both may be parts of any other one number, as in $\frac{5}{12}$, and $\frac{8}{12}$, although the lesser taken but twice, be too great to make 18, yet they both may be parts vnto 36: therefore loke how many times twelue is in 36, and that quotient being multiplied by the Numerator

tator ouer 12, the totall shall be put in stead of the Numerator ouer 12, and for 5 put 15, thus, $\frac{15}{16}$. So likewise looke how often is 18 in 36, and because it is twice, therefore by 2 multiply 7, which is ouer 18, and it will be 14, set that for the Numerator, and in stead of 18 put 36, and then shall your fractions reduced stand thus: $\frac{15}{16}\frac{14}{16}$ in stead of $\frac{4}{12}$ and $\frac{7}{18}$.

And if you will proue whether you haue wrought well or no, that may be proued by Reduction of them againe to their former denominations, which Art shall be taught in the fourth kinde of Reduction, where greater termes of fractions be reduced into smaller in number, but no smaller in proportion. And if in such Reduction of the same termes or numbers come againe that were before, then is the worke good, else not.

Scholar. Sir, I heare your words, but I doe not vnderstand many of them: which if it may please you declare.

Master. With a good will, when conuenient place serueth, but that must be in the said fourth kinde of Reduction. In the meane season I will declare the second forme of Reduction, which teacheth how to reduce fractions of fractions into one fraction, and so to one denomination.

When fractions of fractions bee proponed, you shall multiply the Numerators of each into other, and set the totall for the new Numerator,

Reduction of fractions into one denomination.

merator, and then multiply all the denominators likewise, and take their totall for the new denominator, and so are they speedily reduced.

Scholar. If that be all, then I understand it already, as by this example I will declare. These be the fractions $\frac{1}{2}$, of $\frac{1}{3}$, of $\frac{4}{7}$ of $\frac{7}{12}$, which I would reduce to one denomination.

Therefore beginne I with the numerators, and multiply them together, saying: 3 into 2 maketh 6, and 6 by 6 maketh 36, which multiplied by 7, yeldeth 252, that I set over a line for the numerator thus.

$\frac{252}{}$

Then I multiply the denominator, 4 by 3 maketh 12, and that by 7 bringeth 84. which multiplied by 9, yeldeth 756, the new denominator. And so the whole reduced fraction is this, which is to hard a fraction for me to understand yet.

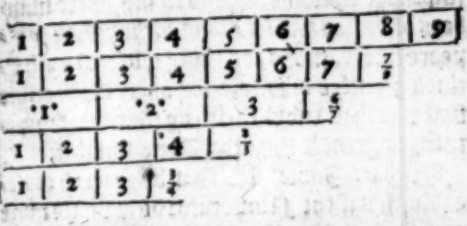
$\frac{252}{756}$

Master. You thinke so, and no marueil, but anone you shall learne to iudge it easily: for this fraction is no more indeed than $\frac{1}{3}$, although it be in greater termes, and therefore more straunger, and more obscure.

And this sufficeth for this Reduction, saue that I will shew you by a figure of measure, the iust rate and reason of this kinde of fractions, and also the due vnderstanding of the Reduction.

The

The entire measure parted into 9.



Heere you see the longest measure, (which standeth for the whole and entire quantitie) first parted into diuisions 9, whereof 7 are severed by the second measure: and thereof againe are parted out 6, and that 6 being distinct into three parts, two of them are parted by the fourth measure, of which fourth measure being diuided into foure parts, the lowest measure doth containe $\frac{1}{4}$, so that the same $\frac{1}{4}$ must be named, not $\frac{1}{4}$ of the whole measure, but indeed is $\frac{1}{4}$ of $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{7}{9}$.

Scholar. This example is so sensible that I cannot chuse but see it. And furthermore I see also that the same fraction is equall to $\frac{1}{9}$ of the entire measure, as the lines which runne vp and downe doe expressly set forth. Also I see heere that $\frac{1}{4}$ of $\frac{6}{7}$ is equall to $\frac{3}{7}$. And further yet that $\frac{3}{4}$ of $\frac{7}{9}$ is equall to $\frac{6}{9}$ or $\frac{2}{3}$.

Master. I am glad that you see it so well, not doubting but you will gather greater light of knowledge hereby.

But

The third forme of Reduction of improper fractions.

But now it is time that we come to the third forme of Reduction, which teacheth of improper fractions, that is to say, mixt numbers of vnites and fractions, although they appeare like fractions, as this $\frac{5}{7}$, which doth include 5 vnites wholly, and $\frac{1}{7}$ ouer. Wherefore first you shall know them, by that the Numerator is greater than the Denominator.

Scholar. Indeed sir, that appeareth reasonable, that if the Numerator do expresse more parts to be taken of any vnite, than the Denominator doth signifie that vnite to be diuided into, it must needs follow that such a fraction importeth more than the whole, that is to say the whole with certayne parts ouer. But what Reduction is there in it?

Master. There be two seuerall kindes of Reduction, concerning such fractions. Sometimes it shall be needfull to conuert these fractions into vnites, and the proper fractions that will remayne. And sometimes contrariwise, it shall be meet to reduce mixt numbers, that is, vnites, written with fractions, into the forme of one simple fraction, and so be there two wayes.

Scholar. What is the meane of the first way to turne improper fractions into vnites with their fractions?

Master. That is thus. Your Numerator being greater than the denominator, must be diuided by the same Denominator, and the

quotient

quotient thereof expresseth the vnites: the remayner shall be put for the Numerator of the fraction that resteth, and the Denominator must be the same that was before.

Reduction of improper fractions into vnites, with their proper fractions.

Scholar. For example, I take $\frac{17}{5}$. And dividing 17 by 5, the quotient will be 3, and there will remayne 2.

Master. That must you write thus, $3\frac{2}{5}$ where (you see) I haue written 3 without any line, as entire numbers ought to be written, and the 2 that remayned I haue set ouer the former Denominator with a line, as a proper fraction. And this number doth signifie new 3 vnites, and $\frac{2}{5}$ of one.

Scholar. Then if I would by vnites here vnderstand crownes, so it were 3 Crownes, and $\frac{2}{5}$, that is 2 s.

Master. Euen so, and therefore $\frac{17}{5}$ did signifie the same. But this happeneth sometime that when the reduction is so wrought, there remaineth nothing. And then it is not a mixt number but a simple entire number, represented like a fraction.

Scholar. As $\frac{15}{5}$ will make 3 iust, and $\frac{18}{3}$ will make euen 6. This I will remember. But now, what is the second forme of Reduction, that you speake of, for these sorts of fractions?

Reduction of whole numbers, eyther alone, or ioyned with fractions into fractions.

Master. Whensoeuer you haue any of these two sorts of numbers, that is to say, whole numbers without fractions, or whole numbers with fractions, and you would turne

them into the forme of a fraction, you must multiply the whole number by that denominator which you will haue to remaine still, & to the totall thereof adde the Numerator, which you haue alreadie, and all that shall you set for the new Numerator, keeping still the former Denominator: as if you haue $6\frac{1}{4}$ which you would conuert into an improper fraction, you must multiply 6 by 4, whereof commeth 24, and thereto adde the Numerator which is 3, and so haue you 27 for the Numerator, and 4 still for the Denominator.

Scholar. Then is $\frac{27}{4}$ equall to $6\frac{1}{4}$.

Master. Euen iust, and so backward (as appeareth by the former reduction) $6\frac{1}{4}$ maketh $\frac{27}{4}$. And thus one of these Reductions may be the proofe of the others worke.

Scholar. This I perceiue, but now if you would turne whole numbers without fractions into any fraction, I see not how that may be done, because there is no Denominator to make the multiplication by.

Master. That was well marked: but this you know, that no man intendeth to turne any whole number into a fraction, but he hath in his minde that denominator by which the multiplication must be made: for the proofe whereof I set downe 7, which is a whole number. And if you will haue this number conuerted into any certayne fraction, will me to doe it.

Scholar.

Scholar. I pray you reduce 7 into a fraction.

Master. Then you care not what the fraction be, so it be some fraction.

Scholar. No, I passe not for the sort of the fraction.

Master. Then how can you thinke that you require mee to doe any thing certayne, when you leaue me to doe as I list? And seeing you stand at that stay, whether thinke you that I must first intend in minde what fraction I will make of it, before I can doe it indeed?

Scholar. Else you should doe ignorantly.

Master. Then will I limit my selfe (seeing you will not) to turne it into quarters. And therefore I multiply 7 by 4 (which is the Denomination of quarters) and there amounteth 28 to be set for the Numerator, and the 4 must be set for the Denominator, and the fraction will be thus $\frac{28}{4}$.

Scholar. Indeed I perceiue this to be reasonable, for without much triall I vnderstand that $\frac{28}{4}$ of any thing doth make 7. And so then if I would turne 8 into 5 parts, it will make $\frac{40}{5}$, which is all one with 8: for 8 crownes turned into 5 parts (that is, into shillings) will make 40 shillings, that is $\frac{40}{5}$ of a crowne.

Master. Seeing you vnderstand now these three kindes of Reduction, I will declare vnto you the fourth kinde, that is, when fractions

be written in greater termes than they néede, how they may be brought to lesser termes.

Scholar. To write any thing in greater termes than néedeth, séemeth to bée a fault, and so this rule séemeth to amend that fault.

Master. It were a fault to doe any thing without néede, which after must be redressed: but in this case it is not so; neyther did I say absolutely (as you doe) that it néedeth not to expresse those fractions in so great termes; but that the fractions doe not néede, I meane for their value to be vnderstood: but yet it may be néedfull for the ease of these workes whereto they be applied, as for example: In the first kinde of Reduction this was your owne example: $\frac{1}{7}$, and $\frac{4}{6}$, which when you would reduce you were faine to turne them first into one denomination, and so appeared they thus, $\frac{18}{96}$, and $\frac{64}{96}$, where the fractions (for their owne vnderstanding) néeded not to be turned out of smaller termes into greater, but yet the easinesse of working néeded it.

Scholar. Sir, I vnderstand now, not onely the difference of this néed (for the fractions might better be vnderstood as fractions seuerall, each in his value, when they were in lesser termes, although they could not so well be reduced) but also I vnderstand what you meane by greater termes and lesser termes, whereof before I was in doubt: for I see you

call

call the Numerator and Denominator, the termes of the fraction.

Master. I am glad you understand it so well. Now then when you would value any fractions (because that they may best be done when the termes are smallest) you shall reduce them to the smallest that you can, which thing you may doe thus: Diuide the greatest of any such two termes by the lesser, and if any thing remayne, by that remayner diuide the last diuisor: and if any thing remayne now, by that diuide the first diuisor (which was before the remayner of the last diuision) and so continue still, till nothing doe remayne in the diuision: and then marke your last diuisor, for it is the number that will easily reduce your fraction, if you diuide both the numerator and the denominator by the same number, and put for the numerator the quotient of his diuision, and for the denominator also his quotient, that riseth by his diuision.

Scholar. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I diuide it by 18, and the quotient is 5, and there resteth 6: what shall I doe with this quotient?

Master. Nothing in this worke, but now seing there remayneth somewhat, by that remayner must you diuide the last diuisor.

Scholar. If I shall diuide 18, (which was the last diuisor) by 6, that was the remayner, so is the quotient 3, and nothing resteth.

Master. As for the Quotient I omit him yet: but because there doth remaine nothing, therefore is 6 (which was your last Diuisor) that number by which you may reduce the fraction proponed.

Scholar. Then as you taught me, I must diuide the Numerator 18, by 6, and the quotient is 3 which I must put for the Numerator ouer a line, thus: $\frac{3}{\ }$ And then by the said 6 must I diuide also the Denominator 96, and the quotient will be 16, which I must take for the Denominator, and so is the fraction $\frac{3}{16}$. And so me thinketh this Rule doth proue the worke of the first reduction.

Master. That is true if the first Reduction were made of fractions in their least termes, and else not, without some help, as the second number in that place will declare.

Scholar. The second number was $\frac{4}{6}$ which was turned into $\frac{64}{96}$ by that Rule. Now if I shall by this Rule reduce it againe into the least termes, I must diuide 96 by 64, and there remayneth 32, wherefore I must take that 32 for the Diuisor, to reduce the said fraction. Then doe I diuide 64 by 32, and the quotient is 2, which I set for my Numerator. Againe, I diuide 96, by 32, and the quotient will be 3, and so I haue but $\frac{2}{3}$.

Master. Muse not at the matter, for you haue done well enough: but you thinke you haue

haue not the fraction that you loked for, that is, $\frac{4}{6}$, yet haue you one equall to it, as by the the parts of a shilling you may proue.

Scholar. Truth it is, for each of them will bring forth 8 pence, so that $\frac{8}{12}$, and $\frac{4}{6}$ and $\frac{2}{3}$, be all three equall. And now I perceiue that because $\frac{2}{3}$ was not written in the least termes that it might bee, therefore this Reduction brought forth not it, but that other which is written in the least termes. Now vnderstand I this Rule well. But is there any other way to worke this Reduction?

Master. Yes, but first note this, that if you finde no such Diuisor to reduce the fraction till you come to 1, because one doth make no diuision: therefore that fraction is already in his least termes, as by $\frac{710}{760}$ you may proue, and so of $\frac{81}{98}$ and many other like.

But now for you better ayde to finde the due proportion in least termes, with more ease for a young learner, you shall meditate or take the halfe of the numerator, and also the Denominator as long as you may vpon a line, allwayes parting them with a right downe dash of your pen as you worke, which may easily be done, if the numbers be euen: as 2,4,6,8,or 10, but if they be odde (though it be but one of them) then must you abbreuiate them by 3,5,7,or 9, &c.

And because examples doe most instruct, I haue here set downe the manner of two or

thæe, whose last number at the end of the line sheweth the least terme or valuation of that fraction.

As for example, I would reduce $\frac{288}{576}$ into his least terme or value, whereupon I set forth $\frac{288}{576}$ with a long line drawne from it thus, $\frac{288}{576}$.—And because both the Numerator, and the denominator end in euen numbers, I see this may be abbreuiated by 2, or 4, or 6, &c. therefore on the other side of the right downe daſh toward the right hand, I first take the halfe of the numerator: saying, the halfe of 2 is 1, the halfe of 8 is 4: and againe, the halfe of 8 is 4: which 144 is now a new Numerator: and therefore I part it with a right downe daſh as before.

Then doe I also take the halfe of 576, in saying, the halfe of 5 is 2, and the halfe of 17 is 8, and the halfe of 16 is 8, and so haue I 288 for a new Denominator.

Then beginning againe: saying, the halfe of 144 is 72, and the halfe of 288 is 144: thus continuing the mediation or diuision by 2, vntill you come to the last worke, as appeareth heere in the example, where the same is reduced to $\frac{1}{2}$, which is equall to $\frac{288}{576}$.

288	144	27	36	18	9	3	1
576	288	144	72	36	18	6	2

Abbre-

Abbreuiated firſt by 5, then by 293.

1465	293	1
4395	879	3

Scholar. Sir I thanke you much, this is very eaſie and good for a young learner.

Maſter. So it is, but yet notwithſtanding if you can without that diuiſion by memory eſpie the greateſt number that may diuide exactly both termes of your fraction proponed, then neede you not to vſe that diuiſion, as in this fraction $\frac{60}{360}$, I ſee that 12 is the greateſt number that I can diuide them both: and therefore without any worke, by memory onely, I turne that into $\frac{5}{30}$, but this ability in knowledge is got by exerciſe.

Yet one other way of eaſie Reduction in this kinde there is, when your fraction hath any ciphers in the firſt places of both termes, then may you by caſting away the ciphers, make a briefe Reduction, as thus $\frac{100}{400}$: heere take away the ciphers, and it will be $\frac{1}{4}$, which is the ſame in value with $\frac{100}{400}$.

Scholar. And ſo if I haue $\frac{100}{600}$, it will be $\frac{1}{6}$.

Maſter. You are deceiued, for you take away more ciphers from the Numerator than you doe take from the Denominator, which you may not doe.

Scholar. I confeſſe my fault, which came of too much haſt, I was more gladder of the rule

rule than wise in doing it: but now I understand it I trust.

The fift kinde of Reduction

Master. Then may I goe in hand with the fift or last kinde of Reduction, which teacheth how to turne any fraction proponed into any other Denomination that you list, or into any parts of common coynes, weights, or measures, or such like.

For declaration whereof, first you shall marke whether your fraction be a simple fraction, eyther else a fraction of sundry parts, I meane of more termes than two. And if your fraction be a fraction of fractions, or otherwise compound, you must reduce it to one simple fraction. And then marke well the Denomination of that other fraction, into which you would turne this: for by that Denominator you must multiply the Numerator of your first fraction, and the totall product therof shall you divide by the Denominator of your first fraction, and that quotient shall be the Numerator of the Denominator proponed: as for example, I have this fraction $\frac{1}{2}$, which I would turne into ten parts: therefore I multiply this 10 by 3, that is the Numerator of my fraction, and there ariseth 30, which I divide by 5, and the quotient is 6, which must be the Numerator to 10, and so $\frac{1}{2}$ will be $6\frac{6}{10}$.

Scholar. This is easie enough to doe.

Master. Then shall you see another example of the same fraction that is not so easie:

as

as if I would turne ⅓ into 8 parts, proue you that worke.

Scholar. I must multiply 8 by 3, and there amounteth 24, which I diuide by 5, and the quotient is 4, then is the new fraction ⅘.

Master. And see you nothing doubtfull in this worke?

Scholar. I see that when 24 was diuided by 5, there remayned 4, which I did not passe of, because ye spake nothing of any remayner, but onely of the quotient.

Master. By likelyhood you remember what I said to you in Diuision of whole numbers, that you should not passe of the remayner there, but onely note it as a summe that could not be diuided without knowledge of fractions. Wherefore now marke this, that in all diuisions of whole numbers, when there is any remayner, you shall set it ouer a line as a Numerator, and set the Diuisor for the Denominator, and that fraction doth make the Diuision complete, and is part of the quotient: as if I would diuide 48 by 5, the quotient will be 9 ⅖: so in your former worke when 24 was diuided by 5, the quotient should be 4⅘, and so the new fraction should be thus: ⅘ and ⅘, of 8, that is, ⅘ of the entyre number, and ⅘ of ⅛ part of any thing, which you may proue by example of some coyne.

Scholar. Then I take a crowne, whose ⅛ is 3 s. Now if I would proue whether the 3 s be

⅘ and

$\frac{4}{5}$ & $\frac{4}{7}$ of $\frac{1}{7}$, I shall haue a cumbrous worke to doe.

Master. Indeed for whole pence, your example is a little troublesome: yet turning the crowne into halfe pence, it is easie enough.

Scholar. That will I trie.

¶ First I see that $\frac{4}{5}$ of a crowne is 3 s, which is 36 d, or 72 halfe pence. Now if I can find that this fraction $\frac{4}{5}$ and $\frac{4}{7}$ of $\frac{1}{7}$ be equall vnto 3 s, then am I fully answered.

Because I cannot take $\frac{4}{5}$ of a crowne, I turne the crowne into halfe pence, as you willed me, which makes 120, which I diuide by 8, my quotient is 15, which taken foure times make 60 ob. Now resteth me to haue $\frac{4}{7}$ of the $\frac{1}{7}$ part of a crowne, whereof $\frac{1}{7}$ part is 15 ob: the 15 being parted in 5 parts, the quotient is 3, which taken 4 times maketh 12 ob. which with my 60 before, amounteth to 72, which are then equall to $\frac{4}{5}$ my desire.

Master. I commend you for your diligence, you might haue wrought it thus eyther: $\frac{4}{5}$ being abbreuied as before I taught, is $\frac{1}{7}$. Now, halfe of a crowne is 2 s, 6 d. Then $\frac{4}{7}$ of $\frac{1}{7}$ is a fraction of fractions, which if you doe reduce into one entire fraction, as before you haue learned, in saying, 5 times 8 is 40, for a new Denominator; and once 4 is 4, for a new Numerator: it maketh $\frac{4}{40}$, and abbreuiated also make $\frac{1}{10}$. Now the tenth part of a crowne is 6 s, which put to 2 s 6 d, make also 3 s your desire.

But now one example more for this rule, and then we shall end it. If I haue $\frac{7}{17}$ of a Soueraigne (accounting the Soueraigne 20 shillings) how many shillings is that $\frac{7}{17}$?

Scholar. I must multiply 7 by 20, and that maketh 140, which I shall diuide by 15, & the quotient will be $9\frac{5}{17}$, or else in lesser termes $\frac{1}{3}$.

Master. That is 9 s, and one third part of a shilling, that is, 4 d, as by the same rule you may proue. And this for this time shall suffice for Reduction, saue that I must now repeat a little touching Multiplication and Diuision, and so goe forward.

Multiplication.

IN Multiplication it happeneth sometime, that there bee whole numbers to be multiplyed with Fractions: and may be in two sorts: for eyther the whole number is seuerall from the Fraction, and is the Multiplyer, or else the whole number is ioyned with one, or both of the Fractions, and so maketh a mixt number thereof. If it be in the first sort, then needeth there no Reduction, but onely multiply the Numerator of the Fraction by that whole number, and the totall thereof set for the new Numerator.

Reduction of whole numbers into fractions.

Scholar.

Scholar. I vnderstand you thus. If I haue $\frac{6}{23}$ to be multiplied by 16, then must I multiply that 16 with 6, which is the Numerator whereof commeth 96, and that must I set for the new Numerator, keeping still 23 for the Denominator, and so the fraction will be $\frac{96}{23}$, that is $4\frac{4}{23}$.

Master. And in this sort of worke you may abridge the labour, thus. If it happen the Denominator to be such a number, as may euenly be diuided by the said whole number proposed, then diuide it thereby, and set the quotient of that diuision for the former Denominator: but reserue still the Numerator, and so is the Multiplication ended.

Scholar. Then I faine this example $\frac{7}{20}$ to be multiplied by 5. And because 5 will iustly diuide 20, therefore I take the quotient of that diuision which is 4, and set in stead of 20, and so the fraction will be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Master. Which is all one with $\frac{35}{20}$ that would haue followed of the other sort of worke.

Scholar. I perceiue it very well.

How to multiply mixt numbers.

Master. Now then for the other sort, where the number is mixt take this way: first to reduce the said whole number, and fraction into one fraction improper (as I shewed you in Reduction) and then multiply them together as if they were proper fractions.

Scholar. $13\frac{1}{2}$ being set to be multiplied by $\frac{1}{2}$ first I must reduce the mixt number, as ap-

appeareth in the margent, by multiplying 13 by 5, and that maketh 65, whereto I muſt adde the Numerator 3, and ſo the fraction will be $\frac{68}{5}$, which two fractions now I ſhall multiply after the accuſtomed forme, and it will be $\frac{884}{5}$. $13\frac{3}{5}$ $\frac{68}{5}$.

Maſter. You haue done well: and ſo may you ſee, that although moſt part of the formes of Multiplication may be wrought without Reduction, yet ſome cannot, as namely mixed numbers.

And yet one note more will I tell you of Multiplication, before we leaue it: That is, whenſoeuer you would multiply any fraction by 2, which commonly is called Duplation, you may do it not onely by doubling the Numerator, but alſo by parting the Denominator into halfe, if it be euen.

Scholar. Then if I would double $\frac{5}{12}$, I may chuſe whether I will make it, $\frac{10}{12}$, or elſe $\frac{5}{6}$. And indeed I ſee that is all one, but that the diuiding of the Denominator ſeemeth the better way to make ſmaller termes of the fraction, and ſo they ſhall néede the leſſe reduction.

Maſter. It is ſo: and now I ſhall not néed to tell you that Multiplication is proued by Diuiſion, and Diuiſion likewiſe by Multiplication: but the like workes that I ſhewed you in Multiplication, will I ſhew you in Diuiſion alſo.

Diuiſion.

Diuision.

Diuision to diuide a whole number by a fraction.

When any whole number shall be diuided by a fraction, you must multiply the said whole number with the Denominator of the fraction, and set the totall thereof for the new Numerator, and for the Denominator set the Numerator of the fraction.

Scholar. Then 20 diuided by $\frac{1}{4}$ $\frac{20}{1}$ by $\frac{1}{4}$ will make $\frac{80}{1}$, as here appeareth $\frac{80}{1}$.

To diuide the fraction by the whole number.

Master. Euen so: but if you would diuide the fraction by the whole number, then multiply the Denominator by the same whole number, and set the totall for the Denominator, without changing the Numerator.

Scholar. Then to diuide $\frac{10}{33}$ by 4, it will be $\frac{10}{132}$, as here appeareth $\frac{10}{33}$ by $\frac{4}{1}$ in this example. $\frac{40}{132}$.

Another briefe way.

Master. You say well. And by the same example you giue me occasion to remember another briefe way to do the same: for if you had diuided the said Numerator by 4, & set the quotient for the Numerator, keeping still the old Denominator, it would haue bene not onely as wel done, but also in a fraction of lesser termes.

Scholar. I guesse it to be euen so, by a like worke that you taught me in multiplication: And for proofe thereof $\frac{10}{33}$ being the diuident, and 4 the diuisor, I diuide the Numerator 10

by

by 4, and the quotient is 5, which I set for 20 ouer 23, thus $\frac{5}{23}$: And I see that it is all one with $\frac{20}{92}$ as by dyuiding or abbreuiating both theſe termes by 4, and ſo reducing them to their leaſt denomination, I may eaſily proue: as appeareth by this example. $\frac{20}{92} \mid \frac{5}{23}$.

Maſter. You conceiue it well. And if there be mixt numbers (either one or both) you muſt firſt reduce that mixt number into an improper fraction, and then worke as you haue learned.

Scholar. That was ſufficiently taught in Multiplication. Therefore I pray you go forward to ſome other thing.

Maſter. Then take this note yet for Diuiſion, if the Denominators be like, then diuide the Numerators as if it were in whole numbers, and the quotient whether it be fraction, whole number, or mixt, is a good quotient for that diuiſion. And generally if one of the Numerators may iuſtly diuide the other, by that quotient multiply the Denominator of the leſſer Numerator, and ſet it that doth amount in the rome of the ſame Denominator, and then for a Numerator to it, ſet the Denominator of the other fraction.

Scholar. Then if I would diuide $\frac{1}{4}$ by $\frac{12}{27}$, I ſee that 3 will diuide 12, and the quotient will be 4, by which I muſt multiply the other 4, that is the denominator vnder 3, and then it is 16, which I ſet for the denominator 4, and

ouer it instead of 3, I must set 17 the other Denominator, and so is it thus, $\frac{17}{?}$.

Master. And so is $\frac{19}{?}$ instead of $\frac{57}{48}$, which would haue risen by the common $\frac{1}{4}$ by $\frac{12}{17}$ worke as heere appeareth:

$\frac{57}{48}$

And now for mediation (which is to diuide by 2) marke this, if the Numerator be euen, set the halfe of it in his place without the Diuisor, and so haue you done: and if the Numerator be not euen, then double the Denominator.

Scholar. That is, if I would mediate $\frac{6}{11}$ I may make the quotient $\frac{3}{11}$, and if I would mediate $\frac{7}{11}$ I must make it $\frac{7}{22}$.

Master. Now trust I that you haue sufficient knowledge in Reduction, Multiplication and Diuision: and therefore will I go in hand with Addition and Subtraction, which now will appeare easie enough.

Addition.

WHensoeuer you haue any Fractions to be added, you must consider whether they be of one denomination or not: and if they be of one denomination, then adde the Numerators together, and set that that aromiteth for the Numerator, ouer the common Denominator, & so haue you done: The reason is because that such differ little in Addition or Subtraction from the worke of vul-

gar

gar denominations, where the denominators be of odde numbers: as 3 d, and 5 d make 8 d, where the Denomination is not altred. But if the fractions be not of one denomination, or any of them be mixt of whole numbers & fractions, then must you first reduce them to one denomination, & after adde them. And if they be many then adde first 2 of them, and so the sum that doth amount of the Addition, & the third, and then the fourth, &c. if you haue so many.

Scholar. This seemeth easie enough, now that I haue already learned to multiply and to reduce, without which two I could neuer haue wrought this. And therefore now I see good reason, why you did place Multiplication, and Reduction before Addition.

Master. It is well considered, but yet refuse not to expresse your vnderstanding of it by an example.

Scholar. Then would I adde first $\frac{7}{18}$ with $\frac{5}{18}$, and because the denominators are like (and so needeth no reduction) I adde 7 to 5, which maketh 12, and then is my summe $\frac{12}{18}$, that is in smaller numbers, being abbreuiated, $\frac{2}{3}$.

And if I haue many numbers to be added as heere $\frac{349}{8510}$) first I must reduce them (because they haue diuers Denominators) into one denomination, and then they will be thus $\frac{150220360}{400400400}$ or in lesse termes $\frac{153236}{404040}$, which by Addition doe make $\frac{33}{40}$ that is $2\frac{3}{30}$.

Master. Now may we goe to Subtraction.

Subtraction.

Subtraction hath the same precepts that Addition had: for if the Denominators be like then must you subtract the one Numerator from the other, and the rest is to be set ouer the common Denominator, and so your Subtraction is ended: but and if you haue many fractions to be subtracted out of many, then must you reduce them to one Denomination, and into two seuerall fractions, that is, all must be subtracted into one fraction, and the residue into another fraction, and then worke as I said before.

Scholar. For the first example I take $\frac{15}{12}$ to be subtracted out of $\frac{17}{12}$ & the rest will be $\frac{2}{12}$ or $\frac{1}{6}$.

For another example, I take $\frac{3}{4}$ to be subtracted out of $\frac{7}{8}$, which I must reduce, and it will be thus $\frac{14}{32}$ and $\frac{28}{32}$.

Then doe I subtract 24 out of 28, and there resteth 4, which I set ouer the common Denominator for a Remayner, thus, $\frac{4}{32}$. that is $\frac{1}{8}$.

Now for the third example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to be subtracted from $\frac{7}{8}$ and $\frac{9}{10}$. And because their Denominators be diuers, I doe reduce them into one denomination thus: $\frac{144014001680172 8}{1020102019201020}$.

Then

Then doe I adde the two firſt, and they make $\frac{1040}{1920}$. Alſo I adde the two laſt, and they yelde $\frac{1408}{1920}$. Then doe I ſubtract 3040 out of 3408, and there reſteth 368, ſo is the remayner $\frac{368}{1920}$, that is in ſmaller termes, $\frac{120}{213}$. And thus haue I done with Subtraction, except you haue any more to teach me.

Maſter. Proue one example more of two fractions of diuers denominations.

Scholar. I take theſe two fractions, $\frac{7}{8}$ to be ſubtracted from $\frac{9}{24}$, which being reduced, will ſtand thus $\frac{168}{192}$ and $\frac{72}{192}$. Now wold I ſubtract 168 out of 72, but I cannot.

Maſter. Then may you perceiue that you miſtooke the fractions: for you can neuer ſubtract the greater out of the leſſer, although you may adde, multiply, or diuide the greater with the leſſer. And albeit that $\frac{7}{8}$ hath both his termes leſſer then $\frac{9}{24}$, yet is $\frac{9}{24}$ the leſſer fraction: for generally if you multiply the Numerators and the Denominators of two fractions croſſewaies, that fraction is the greateſt of whoſe Numerator commeth the greateſt ſumme, as in this example: 7 multiplied by 24, maketh 168: and 9 being multiplied by 8, yeldeth but 72: therefore is the firſt fraction 7 the greateſt of theſe two, ſo can you not ſubtract it out of a leſſer fraction.

But if you ſhould ſubtract a fraction out of a whole number what would you doe?

Scholar. Marry I would reduce the whole

number into a fraction of the same Denomination that my fraction is, and then worke by Subtraction.

Master. So may you do, but it is easier much, if your fraction be a proper fraction, that is to say, lesse then an vnite, to take an vnite from the whole number, and then turne it into an improper fraction, and so worke your Subtraction. As if I would subtract $\frac{1}{7}$ from 4, I may take 1 from 4, and turne it into $\frac{7}{7}$, from which if I abate $\frac{1}{7}$ there will remayne $3\frac{6}{7}$. And if the first fraction be an improper fraction, then may I take so many vnites from the whole number, that they may make an improper fraction greater then that first, and then worke by Subtraction: As if there be proponed $\frac{10}{7}$ to be subtracted from 6, because $\frac{10}{7}$ is more then 3, and not so much as 4, I must take 4 from 6, and turne them into thirds thus, $\frac{12}{7}$ then abate $\frac{10}{7}$, and there resteth $\frac{2}{7}$; so the whole remayner is $2\frac{2}{7}$. Or else you may at your pleasure take $3\frac{1}{7}$ which is $\frac{10}{7}$, from 6 whole: then set 1 vnder 6, as thus $\frac{6}{1}$: And then to reduce those two fractions into one Denomination, as heere appeareth, $\frac{1}{7}$, from $\frac{6}{1}$.

Then $\frac{10}{7}$ from $\frac{42}{7}$ resteth $\frac{32}{7}$ which maketh $2\frac{2}{7}$ your $\frac{10}{7}$ desire. $\frac{42}{7}$ $\frac{32}{7}$

And thus will I make an end of the worke of common fractions for this time, not doubting, but you can apply them both vnto the rule of Progression, and also vnto the Gol-

den

den rule, without any other teaching then you haue learned before, which might seeme tedious to repeat, saue that in some speciall diuersities, which be peculiar to fractions, I cannot ouerpasse, but instruct you somewhat by the way.

The Golden Rule.

Wherfore as touching the Golden rule, for the placing of the 3 numbers proponed in the question whereby to finde the third, and for the forme of their worke, with other like notes, I referre you to that which you haue alreadie learned.

But this easie forme of working by fractions shall you note, that if your 3 numbers be fractions, for an apt worke and certayne, multiply the Numerator of the first number in the question, by the Denominator of the second. And all that againe multiply by the Denominator of the third number, and the totall thereof shall you keepe for to be the Diuisor. Then multiply the Denominator of the first number by the Numerator of the second, and the whole thereof by the Numerator of the third, and the totall thereof shall be your dividend.

Now diuide this dividend by the Diuisor which you found out before, and that number

shall be the fourth number of the question, which you seeke for: as in this example. If $\frac{1}{4}$ of a yard of velvet cost $\frac{3}{7}$ of a Soueraigne estemed at 20 s, what shall $\frac{1}{6}$ cost?

Scholar. If it please you to let me make the answere, I would first place these three numbers, as I learned in whole numbers, thus. $\frac{1}{4}$ N $\frac{3}{7}$ $\frac{1}{6}$

And then according to your new rule, I must multiply 3, being numerator in the first number, by 3 the Denominator of the second, and thereof commeth 9, which I multiply againe by 6, the denominator of the third number, and so haue I 54, which I keepe for the Diuisor, then multiply I 4, the denominator of the first, by 2 the Numerator of the second, and there riseth 8, which againe I multiply by 5, the Numerator of the third, and it maketh 40: then must I diuide 40 by 45, and it will be 40 that is, $\frac{40}{54}$ in lesser termes, and then the figure will stand thus. $\frac{1}{4}$ N $\frac{2}{7}$ $\frac{5}{6}$ $\frac{40}{54}$

But what that is in money I cannot tell, except I shall worke it by Reduction, as you taught me.

Master. It forceth not now, you may reduce it when you list, but it were disorderly done heere to mingle diuers workes together where we do not seeke the value of the thing in common money, but in an apt number, which ye haue well done. And therefore will I yet shew you

you another like way of exercise in worke, how you may change your three fractions into three whole numbers, by which you shall worke as if the question were proponed in whole numbers. The first number you shall finde as I taught you: now to finde the Diuisor of the second number, take the Numerator for the second fraction: and for the third number take that that riseth of Multiplication of the Denominator of the first, by the Numerator of the third, and then worke your question.

Scholar. For example hereof, I put this question. If $\frac{11}{12}$ of 1 l. waight of siluer be worth $\frac{12}{4}$ of a Sousraigne, what is $\frac{1}{2}$ of 1 l. waight worth? For the answere, first I place the fractions in order thus. $\frac{11}{12}$ Z $\frac{12}{4}$ $\frac{1}{2}$

Then to turne these fractions into whole numbers, I multiply 11, which is the Numerator of the first by 4 (the Denominator of the second) and there commeth 44: which I multiply by 2 the Denominator of the third, and so amounteth 88, which I set for the Diuisor in the first place. Then in the second place I set 12, which is the Numerator in the second fraction, and in the third place I set the summe that amounteth of 12, being the Denominator in the first number, multiplied by one, being Numerator in the third number, and so the figure will stand as here you see. 88 Z 12 12

Then

Then to worke it forth, I multiply 12 by 12, and there amounteth 154, which I divide by 88, and the quotient will be $\frac{16}{88}$, or in lesser termes, $1\frac{7}{17}$, and then the figure will stand thus:

$\frac{11}{12}$ Z $\frac{11}{4}$
$\frac{1}{2}$ $1\frac{7}{17}$

The proof of the Golden Rule.

Master. These two formes now you understand well enough, and as for any other at this time I will not repeat, onely this shal you marke for the proofe of this rule whether your worke be well wrought or no: Multiply the first number by the fourth, and note what amounteth: then multiply the second by the third, and marke what amounteth also. Now if those two numbers so amounting be equal, then is your worke well done, else you have erred. And this shall suffice for the former rule: but in the backer rule, this shall you note for ease of worke, that you multiply the Numerator of the first, by the Numerator of the second, and the whole thereof by the Denominator of the third, and that amounteth thereof, shall be the Dividend. Then multiply the Denominator of the first by the Denominator of the second, and that whole by the Numerator of the third, and that that riseth thereof shall be the Divisor. Example of this:

The backer Rule.

A question of loane.

I did lend my friend $\frac{1}{2}$ of a Portegnise seaven moneths, upon promise that he should doe as much for me againe, and when I should borrow of him, he could lend me but $\frac{1}{12}$ of a Portegnise: now I demand how long time must

I

I kepe his money in iust recompence of my loane, accounting 13 moneths in the yere.

Scholar. The first number must be the first money borowed, that is $\frac{1}{4}$ of the pozteguise: the second number the 7 moneths, that is $\frac{7}{13}$ of a yere: and the third number the money that was lent in recompence, that is, $\frac{2}{7}$ of a pozteguise: then I $\frac{1}{4}$ $\frac{7}{13}$ $\frac{2}{7}$ set the numbers thus

Then (as you taught me) I multiply 3 (being Numerator in the first number) by 7, the Numerator of the second number, and it maketh 21, which I multiply by 12, the Denominator of the third, and so haue I 252 for the diuidend: then I multiply 4 the Denominator of the first, by 13 the Denominator of the second, and it yeldeth 52, which I multiply againe by 5, the Numerator of the third, and it wil make 260, that is the diuisor. Then must I diuide 252 by 260, so it will be in the small fraction $\frac{63}{65}$ of yere.

Statute of Assise of Bread and Ale.

Master. And thus doe you see some ease in worhing better then to multiply and diuide tediously so many fractions. Another question yet will I propose, to the intent you may see thereby the reason of the statute of Assise of Bread and Ale, which in all statute bookes, in French, Latine, and English is much corrupted for want of knowledge in this Art, for the right vnderstanding whereof I propone this question.

When

When the price of a quarter of wheat is 2 s, the farthing white loafe shall waigh 68 s, then I demand, what shall such a loafe waigh when a quarter of wheat is sold for 3 s?

Scholar. The question must be wrought as it is proponed in whole numbers, and not in Fractions.

Master. You seeme to say reasonably, howbeit, in the Statute of Assise, the rate is made by the proportion of parts in a pound waight Troy, else could it not be a Statute of any long continuance, seeing the shillings doe change often, as all other monies do: but this Statute being well understood, is a continuall rule for ever, as I will anone declare by a new table of Assise, converting the shillings into ounces and parts of ounces.

Therefore heere by a shilling you must understand $\frac{1}{20}$ of a pound waight, and so by ten $\frac{1}{240}$ of an ounce: wherefore although you might worke this question proponed by whole number well enough, for that time when the Statute was made, yet to apply it to our time, and to make it serue for all times generally, it is best to worke it by fractions, setting for 2 shillings $\frac{2}{20}$, and for 68 shillings $\frac{68}{20}$: and so for three shillings $\frac{3}{20}$, and then will the figure of the question stand thus: $\frac{2}{20}\ \frac{68}{20}\ \frac{3}{20}\ Z$

In which question because all the Denominators be like, you shall worke onely with the Numerators.

Scholar.

Scholar. Then I shall multiply 68 by 2, whereof commeth 136, which if I diuide by 3, the quotient will be $45\frac{1}{3}$: but how shall I make a Fraction of that to stand with the other?

Master. Haue you so soone forgotten what was taught you so lately? This is his forme.

Scholar. I remember it now, and then it signifieth 45 twentie parts, and the $45\frac{1}{3}$ third deale of one twentie part.

Master. So is it that maketh in shillings 45 s, 4 d, whereby you may note one great errour in the statute Bookes, which haue constantly 48 s in that Assise. And by this rule if you examine the Statute, you shall finde many summes false. Wherefore for the true vnderstanding of that Statute, and such like as I haue made mention of it, and somewhat recognised it, so doe I wish that all gentlemen and other students of the Lawes would not neglect this Art of Arithmetick, as vnneedfull to their studies. Wherefore, to encourage them thereto, and to gratifie both them and all other in generall, I will exhibite a Table of that part of the statute in two columes, and in a third colume I will adde the correction of those errours which haue crept into it.

Heere followeth the Table.

The

The price of a quarter of wheate.		The waight of a farthing white loafe by the ſtatute bookes.			The correction by iuſt Aſſiſe.		
ſ	d	l	ſ	d	l	ſ	d
1	0	6	16	0	6	16	0
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 1/7	2	14	4 4/7
3	0	2	11	0	2	5	4
3	6	2	2	0	1	18	10 3/7
4	0	1	16	0	1	14	0
4	6	1	10	0	1	10	2 3/7
5	0	1	8	2 1/2	1	7	2 2/7
5	6	1	4	8 1/2	1	4	8 6/7
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 1/13
7	0	0	19	1	0	19	5 1/7
7	6	0	18	1 1/2	0	18	1 1/7
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 1/4	0	15	1 1/7
9	6	0	14	1/4	0	14	3 1/19
10	0	0	13	7 1/2	0	13	7 1/7
10	6	0	12	1/4	0	12	11 1/7
11	0	0	12	4 1/2	0	12	4 4/11
11	6	0	11	10	0	11	9 2/11
12	0	0	11	4	0	11	4

In

In the common bokes there is no further rate of Assise made, then into 12 s the quarter of wheat, but in an ancient copy of 200 yeers old (which I haue) there is added the rate of assise vnto 20 s the quarter, but yet was that assise also eyther wrong cast at the first penning, or else corrupt sines that time, for lacke of true knowledge in the Rule of Proportion, which I will adde here also to gratifie such as be desirous to vnderstand truth exactly.

The price of a quarter of wheate.		The waight of a farthing white loafe by the Statute bookes.			The correction by iust Assise.		
s	d	l	s	d	l	s	d
12	6	0	11	0	0	10	10 $\frac{14}{21}$
13	0	0	11	0 $\frac{1}{2}$	0	10	5 $\frac{7}{13}$
13	6	6	10	1 $\frac{1}{2}$	0	10	$\frac{8}{3}$
14	0	0	9	7	0	9	8 $\frac{4}{7}$
14	6	0	9	2 $\frac{1}{3}$	0	9	4 $\frac{16}{29}$
15	0	0	9	1 $\frac{1}{2}$	0	9	0 $\frac{4}{5}$
15	6	0	9	1 $\frac{1}{4}$	0	8	9 $\frac{9}{31}$
16	0	0	9	0	0	8	6
16	6	0	8	6	0	8	2 $\frac{10}{11}$
17	0	0	8	2	0	8	0
17	6	0	7	10	0	7	10 $\frac{2}{37}$
18	0	0	7	6	0	7	6 $\frac{3}{7}$
18	6	0	7	3	0	7	4 $\frac{8}{37}$
19	0	0	7	2	0	7	1 $\frac{17}{19}$
19	6	0	5	10	0	6	11 $\frac{9}{11}$
20	0	0	5	6	0	6	9 $\frac{1}{5}$

These two tables I haue set seuerall, because no man should thinke that I would eyther adde or take away from any law those parts which might of right seeme eyther superfluous, eyther diminute; but yet I may not be so curious as to neglect manifest errours, which is not onely my part, but euery good subiects dutie with sobrietie to correct. And for auoyding of offence I haue rather done it in this priuate booke, then in any booke of the statutes it selfe, trusting that all men will take it in good part.

Scholar. I would wish so, but I dare not hope so, sith neuer good man that would reforme errour, could escape the venemous tongues of enuious detractors which because they eyther cannot, or list not to doe any good themselues, doe delight to barke at the doings of other, but I beseech you to stay nothing for their peruerse behauiour.

Master. I consider many things that some may obiect, whereunto I am not vnprouided of iust answeres, but I will not seeme so hastie to make the answers before I heare their obiections; but as I trust that most are of a better nature, and more gratefull now than some haue beene in times past, as I haue done in the Statute of Assise for bread in rate of s, so will I set forth the like table in pounds and ounces, and the parts thereof, that it may be easily applied to all times: but I meane not

by

by this to alter any word of the Statute (being so good an ordinance, and of so great continuance) but onely to make it as a kinde of exposition and declaration of the said Statute, trusting that thereby the Statute may be better understood, and consequently better put in execution. And heere you shall note that I haue accounted the shillings after the rate of 60 s, to the pound waight, because I esteeme it the most apt for our time. Wherefore in the first columne you finde the price of Wheate directly against it, in the second columne, you may finde the waight of a farthing white Loafe, in this our time: and if you double the number (as I haue done in the third columne) then haue you the waight of the halfe pennie white Loafe, and so in the fourth columne is set the waight of a pennie white Loafe. It needeth not to tell that, the sight doth testifie, how that euery columne is parted into three smaller pillars, where the first columne hath these three title words, ounces, and pennie waights. And as in the first columne 12 pence make a shilling, and 10 shillings make a pound, so in the other three columnes 20 pence waight maketh an ounce, and 12 ounces doe make a pound.

Scholar. Sir, I doe thanke you moſt heartily for this, not onely in mine owne name, and in the name of all Students, but alſo in the name of the whole Commons, to whom the reſtitution of this Aſſiſe (I truſt) ſhall bring reſtitution of the waight in bread, which long time hath beene abuſed. And if you know any like things more, wherein you would vouchſafe to declare the errours, and ſet forth the truth, you cannot but obtayne great thankes of all good hearted men that loue the Common-wealth.

Maſter. I haue ſundry things to declare, but I haue reſerued them for a priuate booke by it ſelfe, yet notwithſtanding becauſe the ſtatute of the rate of meaſuring of ground is ſo common that it toucheth all men, and yet no more common than needfull, but ſo much corrupt, that it is too farre out of all good rate, not onely in the Engliſh bookes of Statutes, commonly Printed, but alſo in the Latine bookes, and in the French alſo (for I haue read of each ſort, and conferred them diligently) I will giue you a Table for the reſtitution of thoſe errours, as may ſuffice for this preſent time. And firſt I will propoſe one queſtion to you touching the vſe of that Statute, whereby you may perceiue the order how to examine the whole ſtatute, and euery parcell thereof, and the queſtion is this.

When the Acre of ground doth containe foure

foure Perches in breadth, then muſt it containe 40 Perches in length: then doe I demand of you, how much ſhall the length of an Acre be, when there is in the breadth of it 13 perches: but before you ſhal anſwer to this queſtion, I will declare vnto you another Statute, which is the ground of the former Statute. And that Statute is this. It is ordayned that three Barly cornes, dry, & round, ſhall make vp the meaſure of an inch: 12 inches ſhall make a foote, and 3 foote ſhall make a yard (the common Engliſh bookes haue an Eine) fiue yards and a halfe ſhall make a perch, and 40 perches in length, and 4 in breadth, ſhall make an Acre. This is that Statute: whereby you may perceiue, that the intent of the Statute is, that one Acre ſhould containe 160 ſquare perches. Now let me heare your anſwere to the queſtion.

Scholar. As I perceiue by the words of the Statute, a perch to be the $\frac{1}{160}$ part of an Acre, ſo could I make thoſe numbers all in fractions, and ſo worke the queſtion, but ſeeing I may do it alſo in whole numbers I take that forme for the moſt eaſie, therefore thus I ſet the queſtion in forme. Then doe I multiply 40 by 4, and it maketh 160, which I diuide by 13 — 4 — 40 — 13, and the quotient is $12\frac{4}{13}$.

Maſter. Now turne that $\frac{4}{13}$ into the common parts of a perch, as they be named in

the former statute: howbeit it shall be best to take one of the least parts in Denomination for auoyding of much labour, as fote, whereof the Perch containeth $16\frac{1}{2}$.

Scholar. Then to returne $\frac{1}{11}$ into fote I multiply $16\frac{1}{2}$ by 4, and it maketh 66, which I must diuide by 13, and the quotient is $5\frac{1}{11}$.

Master. So I finde that if the Acre hold in breadth 13 Perches, it shall containe in length 12 Perches, 5 fote, and $\frac{1}{11}$ of a fote, which is not fully an inch, for the inch is $\frac{1}{12}$ of a fote. But here all the Statute bookes in Latine and English (that I haue seene) doe note it to be 13 Perches, 5 fote and one inch: which maketh aboue 13 Perches, too many in the Acre: so that I would haue thought the errour to haue crept into the printed bookes by the great negligence that Printers in our time doe vse, saue that in written copies of great antiquitie, I doe finde the same. Yet haue I one French copie, which hath 12 Perches $\frac{1}{2}$ and one fote, and that misseth very little of the truth.

Scholar. Then I see it is true that I haue often heard say, that the truest copies of the statutes be the French copies.

Master. That is often true, but not generally, as I haue conference tried diuersly: but in this statute the French booke is most corrupt in all other places lightly.

But now to performe my promise, I will set

set forth the Table for measuring of an Acre of ground onely by such parts as the statute doth mention, because at this time I doe of purpose write it for the better understanding of that statute, and hereafter with other things intend to set forth this same more at large.

In this Table following, I haue not done as in the other statute before compared by restitution with the faults crept into the statute, but onely haue written that true measure, which the equitie of the statute doth pretend. For it were vile to iudge of so noble Princes and worthy Counsellors, as haue authorised and set forth this statute, that they would make one Acre in any forme greater than another, but euery one to be iust and equall with each other, which is the ground also of my worke: and hereby may all men perceiue how needefull Arithmeticke is to the students of Law. But now I thinke best to make an end of these matters for this present time, sith the Table hath it in none obscuritie, that I should needs to declare.

The breadth	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of an inch.
10	16	0	0	0
11	14	9	0	0
12	13	5	6	0
13	12	5	0	33/73
14	11	7	0	6/7
15	10	11	0	0
16	10	0	0	0
17	9	6	9	9/17
18	8	14	8	0
19	8	6	11	7/19
20	8	0	0	0
21	7	10	2	4/7
22	7	4	6	0
23	6	15	9	9/23
24	6	11	0	0
25	6	6	7	1/2
26	6	2	7	6/13
27	5	15	3	1/2

The

The breadth	The length of the Acre.			
Perches.	Perches.	Feete.	Inches.	Parts of an inch.
28	5	11	9	3/7
29	5	8	6	12/29
30	5	5	5	0
31	5	2	7	29/31
32	5	0	0	0
33	4	14	0	0
34	4	11	7	23/34
35	4	9	5	1/7
36	4	7	4	0
37	4	5	4	8/37
38	4	3	5	13/19
39	4	1	8	12/39
40	4	0	0	0
41	3	14	10	28/41
42	3	13	4	2/7
43	3	11	10	32/43
44	3	10	6	0
45	3	9	2	0

Scholar. Indeede Sir, I vnderstand the Table (as I thinke) by those other which you set forth before. For in the first columne is set the Perches of the breadth of any Acre, and then in the two columnes following appeareth how many Perches, and how many foote that same Acre must haue for his length.

Master. You take it well: howbeit to speake exactly of breadth and length, and the first columne doth sometime betoken the breadth, and sometime the length: for properly the longest side of any square doth limit the length, and the shorter side doth betoken the breadth; yet it is no great abuse in such Tables, where a man cannot well change the title, to let the name remayne, although the proportions of the numbers doe change: for still by the first columne is expressed the measure of the one side, and by the two other pillars in one columne, is set forth the measure of the other side. And this shall be sufficient now for the vse of the Golden Rule.

The Rule of Fellowship.

NOw somewhat will I touch certaine other rules, which for their seuerall names may seeme diuers rules and distinct from this, but indeede they are but branches of it: yet because they haue

haue seueral workings in appearance, but also pleasant in vse, I will giue you a taste of each of them. As for the rule of Fellowship, both single and double, with time and without time, I shall neede to say little more than I haue alreadie said in teaching the workes of the whole numbers: yet an example or two will we haue to refresh the remembrance of the same, and to declare certayne proper vses and applications of it, as this for one.

Foure men got a bootie or prise in time of warre, the prize is in value of money. 8190 pound, and because the men be not of like degree, therefore their shares may not be equall, but the chiefest person will haue of the bootie the third part, and the tenth part ouer: the second will haue a quarter, and the tenth part ouer, the third will haue the fift part: and so there is left for the fourth man a very small portion, but such is his lot, whether he be pleased or wroth) he must be content with one 20 part of the pray: Now I demand of you, what shall euery man haue to his share?

Scholar. You must be faine to answere to your owne question, else it is not like to be answered at this time.

Master. The forme to vnderstand the solution of this question, and all such like is thus: Reduce all the denominators into one number by Multiplication, except that any of

them

them be parts of some other of them, for all such parts you may overpasse, and take for them all those numbers, whose parts they be: as in this example the shares be these. $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{10}$ $\frac{1}{6}$ $\frac{1}{20}$ if I multiply all the denominators together, beginning with 3, and so goe on unto 20, it will make 144000: but considering that 3 is a part of 6, I shall omit that 3, and likewise 10, which is a part of 20, I may overpasse also, and then is there but three denominators to multiply, that is, 4, 6, and 20 which make 480: which summe I take for my worke, because all the denominators will be found in it. Then I take such parts of it as the question importeth, that is, for the first man $\frac{11}{120}$, the $\frac{1}{3}$ 160, the $\frac{3}{10}$ is 48: which I put in one summe for the first mans share, and it maketh 208. Then for the second mans share, I take $\frac{1}{4}$, which is 120, and $\frac{1}{10}$ which is 48, and that maketh in the whole 168. Now for the third man which must haue $\frac{1}{6}$, I take 80. And for the fourth man there remaineth but 24, which is $\frac{1}{20}$ of the whole sum: so that if the whole pray had borne but 480 l, then were the question answered: but because the summe was of greater value, by this meanes now shall I know the partition of it. I must set my numbers by the order of the Golden Rule, putting in the first place the number that I found by multiplying the Denominators, and in the second place the summe of

the

the bottie. And loke what proportion is betweene the first number and the second, the same proportion shall be betweene the parts of that first number, and the parts of the second, comparing each to his like. Therefore I must put in the 3 place, one of the parts or shares, and then worke by the former Rule of proportion or Golden Rule. And because I haue foure seuerall parts of the first number, by which I would finde out foure like parts of the second number, therefore must I make foure seuerall figures. The reason of this rule.

Scholar. Now I trust I can answere to your question, as by your fauour I will proue.

A
480 Z 8190
208

B
480 Z 8190

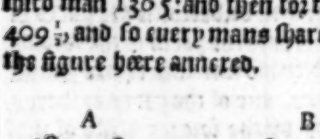
C
480 Z 8190
80

D
480 Z 8190
24

And to trie it, I set the foure figures thus, marked with A, B, C, D, to shew their order. And then in each of them I multiply the second number by the third, and diuide their totall by the first, and so amounteth the fourth summe which I seeke for: for if I doe multiply 8190 by 208, it maketh 1703520, which being diuided by 480, maketh in the quotient 3549 for the first mayne portion.

And

And so working with the other three figures, I finde for the second man $2866\frac{1}{3}$, and for the third man 1365: and then for the fourth man $409\frac{1}{2}$, and so every mans share is set forth in the figure here annexed.



The proof by addition.

And thus I thinke I haue done well.

Master. If you mis doubt your working, and list to proue it, adde all the shares together, and if they make the totall, then seemeth it well done.

Scholar. I may set them thus: and then by Addition the iust summe doth amount, that is, 8190, and therefore (as you say) it seemeth to be well wrought.

	3549
	$2866\frac{1}{3}$
	1365
	$409\frac{1}{2}$
	8190

The iust proofe

But I beseech you, is there any doubt in this triall, that you vse that word, Seemeth?

Master. You may easily coniecture, that if you did assigne the first mans share to the last, and so change all the rest, and one had anothers share, yet would the Addition appeare

all

all one, and therefore is not the proofe exact.

But if you will make a iust proofe for the first mans part, take $\frac{1}{3}$ and $\frac{1}{10}$ of the whole sum, and if it agree with the number in the figure, then it is well done. And so doe for the second, third, and fourth summes, and this proofe fayleth not. Now will I propound certayne other questions which haue beene set forth by certayne learned men, albeit not without some ouersight, which questions I protest heartily, I doe not repeate to deprayue those good men, whose labours and studies I much prayse, and greatly delight in. But onely according to my profession, to seeke out truth in all things, and to remoue all occasions of errour as much as in me lieth: and for that cause I will onely name the questions without hurting the Authors name. The first question is this.

Foure men did build an house, which cost them 3000 crownes, their shares were such that one man should pay $\frac{1}{2}$ of the summe, and 6 crownes ouer: the second should pay $\frac{1}{3}$ and 12 crowns ouer: the third man must lay out $\frac{1}{4}$ abating 8 crownes, & the fourth man should pay $\frac{1}{4}$ and 20 crownes more: can you answere to this question?

Scholar. No in good sooth sir, and that you know best of any man, for I know no more than you haue taught me.

Master. Then I dare say you cannot doe

it

it neither yet be beſt learned man that euer did propoſe it, for the queſtion is impoſſible. For declaration whereof, I will be bold to vſe firſt the repreſentation of the numbers in their aptest forme, (although I haue not yet taught that manner of worke) becauſe it may appeare plainely that the queſtion in not poſſible. For hære I haue ſet the parts, and added them, and they make the whole ſum, and ¼ and 30 more. Now, how is it poſſible to diuide truly either gains, either charges, ſo that the particulars ſhall be more than the totall?

An impoſſible queſtion.

⅓		6
1	·	12
½	·	7
¼	·	20
1¼	·	30

Scholar. It is againſt the forme of proofe by addition of parts.

Maſter. You ſay truth. And (becauſe you ſhall perceiue it the better) I will trie it after the vulgar forme, as in this figure you ſee, where the ⅓ with 6 ouer is 1506: for the totall as you heard before, 2000 the ⅓ and the 12 more is 1020: the ⅓ would be 2000 but then abating 8, it is but 1992, and then laſt of all, the ¼ is 750, and the 20 more maketh 770: which all being added in one ſumme, doe make 5280, where the totall ſumme ſhould be but 3000, which ſumme of 3000 if you diuide by ¼, ſo ſhall you

	1506
	1022
	1992
	770
	5110

you haue ½ of it, that is 2250, and therto adde 30 more, then will those 3 sums make 5280: whereby you may see how this forme (as well as ỳ other) doth declare that the particulars in that question would make more the whole sum by ½ and 30 more: and therefore can that question not be accepted as a possible thing: but yet do certaine learned men propound such questions, and answere to them. Therefore somewhat to say to their excuse (rather of their good meaning, then for their doing) I will anon declare what may be said for their defence: but, in the meane season, I will propound the question as it may be wrought by good possibilitie. As if foure men build a house together, and it cost them 3000 crownes, and then for the partition they agree thus: that as often as the first man doth pay 6 crownes, so often the second man shall pay 4, the third man 8, and the fourth man 3. Or else, thus: that the first man shall pay double so much as the fourth, and the second man shall pay ½ of the first mans charge: the third man shall pay double so much as the second: (and these two waies are to one end:) but further for their agreement it is appointed also, that the first shall giue 6 crownes ouerplus; and the second 12, and the fourth shall giue 20: but the third man shall giue no ouer-

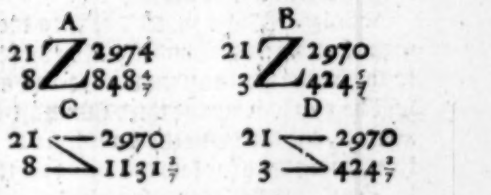
plus

	2000
	2250
	30
	5280

plus, but shall haue 8 crownes abated of his charge. Now is the question possible to be assoyled, and this is the way to do it. Marke the proportion of the seuerall charges, and set out small numbers in that rate, by which you may reduce the worke to the Golden Rule, as heere in the first forme, the numbers are already named, 6, 4, 8, 3 : and in the second forme, (although they be but plainely named) yet they may be the same numbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6 : and againe 8 is double to 4. Now adde these together, and they make 21, which 21 must be set in the number in the Golden Rule: for if it, with the ouerplus of each mans charge would make the totall summe of the charges : then were those seuerall sums the charges of each man besides his ouerplus: but now it is not so.

The rule. But yet, this is true, (so excellent are conclusions Arithmeticall,) that loke what proportion each of their seuerall summes doth beare to 21, the same proportion doth the iust charges of euery man (besides his ouerplus) beare to the totall of the charges, the ouerplus being deducted : wherefore this may you note, that, before you doe apply the totall of his charges to the Golden rule, you must deduct the ouerplus which is 6, 12, & 20 that is in the whole 38: but then 8 must be reserued for the abatement of the 3 man, and then remaineth to be deducted 30: Take 30 therefore

forth out of 3000, and there will reſt 2970, which I muſt ſet in the Golden Rule for the ſecond ſumme: and for the third ſumme I muſt put each of the ſmall numbers before mentioned, which although they be not ſeuerall charges, yet they repreſent them in proportion. And ſo making for euery mans charge a ſeuerall queſtion, the figures will be 4, which I marke with foure letters, A, B, C, D, thus.



Where I haue ſet for briefeneſſe the ſummes of euery mans charge in the fourth place, preſuppoſing that you can tell how to trie out that fourth ſumme by ſo many examples as ye haue had.

Scholar. As I truſt that I vnderſtand this forme, ſo I deſire much to know what may be ſaid for them that miſtake this queſtion.

Maſter. You ſeeme ſo deſirous to know this errour, that you haue forgotten to examine, whether this worke be without fault.

Scholar. It ſeemeth this worke to be well done, becauſe the Addition of the 4 ſeuerall

numbers doth make the totall summe of 2970, which was to be diuided into such foure parts.

Master. But then haue you forgotten that the first man must pay 6 Crownes more besides his share, and the second man twelue crownes more, the third man 8 crownes lesse, and the fourth man 10 crownes more, for without these your first totall of 3000 crownes will not be made.

Scholar. Then must I adde to the first mans summe 6 more, and it will be $854\frac{4}{7}$: and to the second summe I must adde 12, and it will be $577\frac{1}{7}$: from the third summe I must abate 8, and then will the summe be $1123\frac{1}{7}$: then adding vnto the fourth summe 20, it will be $444\frac{20}{70}$, and these foure summes will make 3000, which is the whole charge, as in this example it may appeare, where first I gather the $\frac{14}{7}$, that maketh 2, and so proceede I in the Addition to the end.

	$854\frac{4}{7}$
	$577\frac{1}{7}$
	$1123\frac{1}{7}$
	$444\frac{20}{70}$
	3000

Master. Now haue you well done, and this worke in the same summes is brought of other learned men for the true solution of the question, as it was first proponed, which (as I said) was impossible: and now examine it by these seuerall summes, and see whether it doth agree with the summes in the question proponed.

The

The first man must pay $\frac{1}{2}$ and 6 ouer of the totall summe: how thinke you, is $854\frac{2}{7}$ the halfe and 6 more of 3000?

Scholar. No, that it is not, for it should be 1506: and for the second man 1012: and for the third man 1992. and for the fourth man 770, whereof not one summe agreeth to this worke. But I maruell, that so wise men could be so much ouerseene.

Master. It is commonly seene that when men will receiue things from elder writers, and will not examine the thing, they seeme rather willing to erre with their ancients for company, than to be bold to examine their workes or writings. Which scrupulositty hath ingendred infinit errors in all kindes of knowledge, and in all ciuill administration, and in euery kinde of Art. But these learned men did not meane any other thing by this question than to find such numbers as should beare the same proportion together, as those numbers in the question proponed did beare one to another: which thing you shal perceiue more plainly by another question of theirs, that is this.

A man lying vpon his death-bed, bequeathed his goods, (which were worth 3600 crownes) in this sort. Because his wife was great with childe, and he yet vncertaine whether the childe were male or female, he made his bequest conditionally, that if the wife bare a daughter, then should the wife haue half his

goods, and the daughter $\frac{1}{3}$, but, if she were deliuered of a sonne, then that sonne should haue $\frac{1}{2}$ of the goods, and his wife but $\frac{1}{3}$. Now it chanced her to bring forth both a sonne and a daughter, the question is: How shall they part the goods agreeable to the testator his will?

Scholar. If some cunning Lawyers had this matter in scanning, they would determine this Testament to be quite void, and so the man to die intestate, because the Testament was made insufficient, sith this condition was not expressed in it: and also, it might haue chanced, that she should haue brought forth neyther sonne nor daughter, as often hath bene seene: so is the will insufficient in that point also.

Master. Such scanners should seeme to cunning, and yet not so cunning as cruell: for the minde of the Testator is to be taken fauorably, for the ayde of the Legatories, when there ariseth such doubt. But let vs trie this worke, not by force of Law, but by proportion Geometricall, seeing the Testator did minde to prouide for each sort of them.

Scholar. If the sonne shall haue $\frac{1}{2}$ by force of the Testament, so must the mother haue $\frac{1}{3}$. Againe, because she hath a daughter also, therefore ought she to haue $\frac{1}{3}$, and the daughter $\frac{1}{3}$, that is, both waies $\frac{1}{2}$ $\frac{1}{3}$, and $\frac{1}{3}$ $\frac{1}{2}$, which commeth to the whole goods, and $\frac{2}{3}$ more.

Wherefore it seemeth also impossible.

Master.

Master. In this matter the minde of the Testator is so to be vnderstood, that such proportion should be betweene the portion of the wife & the sonne, as is betweene ½ for ⅔ to, that is the sonne must haue ½ for ⅔ to his mother, so shall he haue 3 to 2, that is, as much as his mother, and halfe as much more, & the mother must haue the like rate in comparison to her daughter. Then must I finde out 3 numbers in such proportion that the first may be as much as the second and halfe as much more (that is) in proportion sesquialtera, and the second to the third in that same proportion: such numbers be 6, 9, 4.

Scholar. I pray you Sir, how shall I finde out those numbers.

Master. That will I gladly tell you.

Whatsoeuer the proportion be of any three numbers, multiply the Termes of that proportion together, and the number that amounteth, shall be the middle number of the 3: then multiply that middle number by the lesser terme, and diuide that totall by the greater, and the least number of the 3 will amount. So if you multiply that middle number by the greater extreame, and diuide that totall by the lesser extreame, then will the greatest number of that progression amount.

Scholar. Then in this example, to finde the proportion of ½ to ⅓, I must diuide (as you taught me in diuision) ½ by ⅓, and the quotient

will be $\frac{1}{2}$, that is $1\frac{1}{2}$, whereby I perceiue that the proportion in this question is as 3 to 2: Therefore (as you taught me euen now) I multiply 3 by 2, and the summe is 6, which must be the middle number: then, I multiply the middle number 6 by 2, which is the least terme, and the summe is 12: that I doe diuide by 3 being of the greater terme, and the quotient is 4: so is 4 the least number of the 3. Then I multiply 6 by 3, whereof commeth 18, and that I diuide by 2, and so haue I nine, which is the greatest number of the 3.

Master. Another way yet may you finde the third number in any Progression, if you haue two of them: for if the middle number be one of them which you haue, then multiply it by it selfe (as in this example 6 by 6 maketh 39) and that totall diuide by the other number which you haue, and the third number will be the quotient.

Scholar. Then if I diuide 36 (which commeth of 6 multiplied by it selfe) by 4, the quotient will be 9: and, if I diuide 36 by 9 the quotient will be 4. But what if I know the first number and the third, and would haue the middle number?

Master. Multiply the two numbers together, & in their totall you must seeke the roote of that number, and it shal be the middle number: but, because as yet you haue not learned how to extract rootes, therefore vse the first terme which

which I haue taught you, till I teach you to extract rootes. And now goe forward with the answere to the same question.

Scholar. I perceiue then that the sonne must not haue $\frac{1}{2}$ of the goods, neyther the mother $\frac{1}{3}$, nor yet the daughter $\frac{1}{3}$, but yet must the goods be diuided into such proportion, that the sonne shal haue 9 crowns for 6 to his mother, and the mother shall haue 6 crownes for euery 4 to her daughter. Then I apply it to the golden Rule in 3 examples, thus:

where the first number is the Addition of those three numbers, 9, 6, 4, and the third is 1 of them seuerally: the second is the total of the goods in that testament: & then by the worke of the Golden rule I finde out the fourth number in euery worke, that is for the sonne,	19 9	Z 3600
	19 6	Z 3620
	19 4	Z 3600
	1705 $\frac{5}{19}$	
for the mother, 1136 $\frac{16}{19}$, and for the daughter, 757 $\frac{17}{19}$: the which sums added together doe make the sum of the whole goods, as may be seene by this example.	1705 $\frac{5}{19}$	$\frac{29}{16}$
	1136 $\frac{16}{19}$	
	757 $\frac{17}{19}$	
		3600

And this (me thinketh) I doe perceiue, that because in this case there is a necessary remedie deuised against an vrgent inconuenience, therefore those learned men thought they might vse ỹ like liberty in that other question.

Scholar. Your ghesse is good, but they had so

good reason for them in the one, as they haue in the other: as in another example of theirs it may better appeare, as in this.

A man left vnto his three sonnes 7851 crowns to be parted in such sort, that the first sonne should haue $\frac{1}{3}$, the second sonne $\frac{1}{5}$, and the third sonne $\frac{1}{7}$, which is not possible, for $\frac{1}{3}$ and $\frac{1}{5}$ and $\frac{1}{7}$ doth make $\frac{26}{14}$: or $\frac{23}{15}$ that is $1\frac{8}{15}$, so is it more than the whole: but reduce these fractions into one denomination, the left that they will come to, and they will be $\frac{5}{15}, \frac{3}{15}, \frac{1}{15}$, and so may you part the goods into such proportion as these three numerators beare together, that is, the first to haue 5 for euery 4 to the second: and the second to haue 4 as often as the third hath 3: and so their portions will be for the first, $3623\frac{7}{11}$ for the second $2415\frac{9}{11}$: & for the third $1811\frac{10}{11}$, and these three shares added together, will make the totall summe of the whole goods, as you may easily see in this example.

	$3623\frac{7}{11}$
	$2415\frac{9}{11}$
	$1811\frac{10}{11}$
	7851

Another question is there proponed thus.

There are 450 crownes to be diuided betweene three men, so that the first man must haue $\frac{1}{3}$ and $\frac{1}{5}$, the second man $\frac{1}{3}$ and $\frac{1}{7}$, the third man shall haue $\frac{1}{5}$ and $\frac{1}{7}$.

Scholar. I maruell that any man should be so ouerseene to propound that question as a thing possible, sith $\frac{21}{35}, \frac{11}{14}, \frac{21}{45}$, doe make $1\frac{31}{14}$ that is almost double the whole summe.

But

But I perceiue it might be thus propoſned, that as often as the firſt man did receiue 50 crowneſ, ſo often the ſecond man ſhould receiue 35, and the third man 27, for $\frac{1}{3}$ and $\frac{1}{4}$, is equall to $\frac{12}{60}$, and is $\frac{1}{35}$, and equall to $\frac{12}{60}$, and $\frac{1}{2}$ and $\frac{1}{7}$ is $\frac{27}{60}$, ſo working the queſtion, the three figures will appeare in this ſorme: whereby the firſt mans portion is found to be $200\frac{10}{16}$: the ſecond mans part is 140 $\frac{35}{16}$, the third mans ſhare $110\frac{24}{16}$: which in the whole doth make 450 crowneſ to be diuided betweene them.

112 — 450
50 — $200\frac{10}{16}$

112 — 450
35 — $140\frac{35}{16}$

112 — 450
27 — $108\frac{27}{16}$

Maſter. And thus you are (I thinke) ſufficiently inſtructed in the rule of fellowſhip.

The Rule of Alligation.

NOw will I goe in hand with the Rule of Alligation: which hath his name, for that by it there are diuers parcels of ſundrie prices, and ſundry quantities alligate, bound or mixed together: whereby alſo it might be well called the Rule of Mixture, and it hath great vſe in compariſon of medicines, and alſo in mixtures of metals, and

ſome

The reason of this rule.

some vse it hath in mixtures of wines: but I with it were lesse vsed therein than it is now adaies. The order of the rule is this: When any summes are proponed to be mixed, set them in order one ouer another, and the common number whereunto you will reduct them set on the left hand: then marke what summes be lesser then that common number, and which be greater, and with a draught of your penne euermore linke two numbers together, so that one be lesser than the common number, & the other greater then he (for two greater or two smaller cannot well be linked together:) and the reason is this, that one greater and one smaller may be so mixed that they will make the meane or common number very well: but two lesse can neuer make so many as the common number, being taken orderly: no more can two summes greater than the meane, euer make the meane in due order, as it shall appeare better to you hereafter. And, as it is of necessitie to linke euery smaller (once at the least) with one greater, and euery greater with one smaller: so it is at liberty to linke them oftner than once: and so may there be to one question many solutions. When you haue so linked them, then marke how much each of the lesser numbers is smaller than the meane or common number, & that difference set against the greater numbers, which be linked with those smaller, each with his match

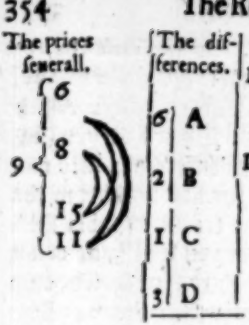
still

ſtill on the right hand, and likewiſe the exceſſe of the greater numbers aboue the meane, you ſhall ſet before the leſſer numbers which be combined with them. Then ſhall you (by Addition) bring all theſe differences into one ſumme, which ſhall be the firſt number in the Golden Rule: and the ſecond number ſhall be the whole maſſe that you will haue of all thoſe particulars: the third ſum ſhall be each difference by it ſelfe, and then by them ſhall be found the fourth number, declaring the iuſt portion of euery particular in that mixture: As now by theſe examples I will make it playne.

There are foure ſorts of Wine, of ſeuerall prices, one of 6 d a gallon, another of 8 d, the third of a 11 d, and the fourth of 15 d the gallon. Of all theſe Wines would I haue a mixture made to the ſumme of fiftie gallons, and ſo that the price of each gallon may be 9 d. Now demand I how much muſt be taken of euery ſort of wine? A queſtion of mixing of wine.

Scholar. If it ſhall pleaſe you to worke the firſt example, that I may make the applying it of the rule, then I truſt I ſhall be able not onely to doe the like, but alſo to ſee the reaſon in the order of the worke.

Maſter. Marke then this forme and the placing euery kinde of number in it.



Heere (you ſee) I haue ſet downe the ſeuerall prices which be 6, 8, 11, 15, and haue linked together 6 with 15, & 8 with 11. The common price, 9, I haue ſet on the left ſide, and the difference betweene it, and euery particular price, I haue ſet on the right hand, not againſt the ſumme (whoſe difference it is) but againſt the ſumme that it is linked withall: ſo the difference of 15 aboue 9, is 6, which I haue ſet not againſt 15 but againſt 6, that is linked with 15, and the difference betweene 6 and 9 (that is 3) I haue ſet againſt 15. So likewiſe, the difference betweene 8 and 9 is but 1: that I haue ſet againſt 11, and the difference of 1 aboue 9 (which is two) I haue ſet againſt 8. Then add I all thoſe foure differences, and they make 12, which I ſet for the firſt number in the Golden Rule: the ſecond number I make 50, which is the ſumme of gallons that I would haue, and the third ſumme

ſumme is euery particular difference. Now, if you worke by the Golden rule, you ſhall find the number of Gallons that ſhall be taken of each ſort of wine: For the better diſtinction, whereof, I haue ſet theſe letters A, B, C, D, both againſt the numbers for which the workes doe ſerue, and ouer the workes alſo, which ſeuerally ſerue for each of them. And now (if you liſt to examine the truth of theſe workes,) adde thoſe foure ſummes together, and they will make fiftie, that is this totall which I would haue, as by this example you may eaſily perceiue. And (for to proue how the prices doe agree) doe this: multiply the totall ſumme 50, by the common price 9 and it will make 450: then keepe that ſum by it ſelfe, and afterward multiply euery ſeuerall ſumme of Gallons by the price belonging to the ſame Gallons: and if that ſumme do agree with this which you haue kept firſt, then is your worke well done. As here 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you ſhall ſet downe, then multiply $8\frac{1}{6}$ by 8 which is the price for the number of Gallons, and it will make $66\frac{4}{6}$: for againe $4\frac{1}{6}$ multiplied by 11: doth make $45\frac{5}{6}$. And laſt of all $12\frac{5}{6}$ multiplied by 15, maketh $187\frac{1}{6}$ theſe added together doe

The proof of this rule.

	35
	$8\frac{1}{3}$
	$4\frac{1}{6}$
	12
	50
	450
	150
	$66\frac{4}{6}$
	$45\frac{5}{6}$
	$187\frac{1}{6}$
	450

doe make 450, as in the example annexed you may see: wherefore seeing it doth agree with the former summe of 50 multiplied by 9, I may iustly affirme this worke to be good, and well done.

And now to proue how you can doe the like, I propound the same question, onely willing you to vse some other forme of combining or linking the summes.

Scholar. That shall I proue with your fauour, and therefore I combine 8 with 15, and 6 with 11, and then the forme will be as followeth.

Whereby amounteth the same summe in totall of the differences, as did before: and yet now the differences be altered, as the combination is changed, whereof I vnderstand the reason by your former worke. And therefore heere appeareth no strange thing, but that now I must haue 8 1/6 gallons of fire pence, and 25 gallons of 8 d, and 12 gallons and 1/6 of 11 d, and so consequently 4 gallons and 1/6 of 15 d, so

50	
200	
137 1/2	
62 1/2	
450	

that

that multiplying 8 ½ by 6, it maketh 50, & then 25 multiplied by 8, maketh 200: likewise 12 ½ multiplied by 11, yeld 137 ½, and 4 ½ multiplied by 15, maketh 72 ½, which 4 sums added into one, will yeld in the totall 450, which agreeth with the multiplication of 50 (being the totall summe of Gallons) by 9 the common or meane price.

Master. Seeing you conceiue this worke so well, I will propound another example vnto you of more varietie in the Alligations or combinings, as thus.

A Merchant being minded to make a bargaine for spices in a mixt masse, that is to say, of Cloues, Nutmegges, Saffron, Pepper, Ginger, and Almonds, the Cloues being at 6 s a pound, the Nutmegges at 8 s, Saffron at 10 s, Pepper at 3 s, Ginger at 2 s, and Almonds at 1 s. A question of spices.

Now would he haue of each sort some, to the value of 300 pound in the whole, and each pound one with another to beare in price 5 s: how much shall he haue of each sort?

Scholar. That will I trie thus.

First, I set downe those fiue seuerall prices, and at the left hand I set the common price 5 s. Then, I linke them thus, 1 with 10, 2 with 6, and 3 with 8, as in the example following.

Master.

5	a	a		b	
1	b	18	Z 300	18	Z 300
2	c	5	Z 83 1/3	3	Z 50
3	d	b		e	
2	e	18	Z 300	18	Z 300
4	f	1	Z 16 2/3	2	Z 33 1/3
18		c		f	
		18	Z 300	18	Z 300
		3	Z 50	2	Z 69 2/3

Maſter. I had minded to haue combined them in more varitie: but I am content to ſee your owne workes firſt, and then more varieties in combination may follow anon.

Scholar. Then to continue as I began, I ſeeke the difference betweene 1 and 5 (which is 4) and that I ſet againſt 10: then againſt 1 I ſet 5, which is the exceſſe of 10 aboue 5, ſo I gather the difference betweene 2 and 5, which is 3, and that I ſet againſt 6, becauſe it is combined with 2: and likewiſe the difference of 6, aboue (which is 1) I ſet againſt 2. Then take I the difference of three from fiue, which is two, and that I ſet againſt 8, and before that 3 I ſet the difference of 8 aboue 5, which is three, Then gather I all theſe differences by Addition, & they make 18, which I ſet for my firſt number in the Golden Rule, and ſo appeareth by thoſe workes, that of Almonds I muſt take 83 1/3, of Ginger 16 1/3, of Pepper 50 pounds, of Cloues, 50 pounds of

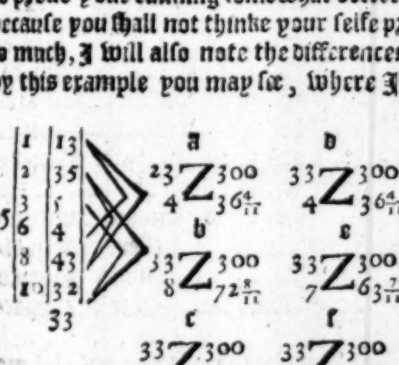
Nutmegs

Nutmegs 33 pounds $\frac{1}{3}$, and of Saffron 66 pounds $\frac{2}{3}$. Then for triall hereof, I multiply euery parcell by his seuerall price, as $83\frac{1}{3}$, which is the summe of Almonds, I multiply by one, which is their price.

	$8\frac{1}{3}$
	$33\frac{1}{3}$
	150
	300
	$266\frac{2}{3}$
	$666\frac{2}{3}$
	1500

Also $16\frac{2}{3}$ the summe of Ginger I multiply by two, which is the price of it. And so each other in his kinde, as this table annexed doth represent, and then adding them altogether, I finde the totall to be 1500, which also will amount by the multiplication of the grosse masse of 300 by the common price 5: wherefore it appeareth will wr ought.

Master. Now will I make the alligation to proue your cunning somewhat better: but because you shall not thinke your selfe pressed so much, I will also note the differences, as by this example you may see, where I haue



alligate

alligate 1 with 6 and 8, and therefore haue I set against 1 both their differences, that is, 1 and 3. Likewise because 2 is combined with 8 and 10, I set before him their differences, 3 and 5. Against 3 I haue set onely 5, which is the difference of 10, with whom 3 is combined onely. Likewise, 6 is onely alligate to 1, and therefore is the difference of 1 from 5: which is 4 onely set against it: 8 is linked with 1 and 2, and therefore hath set 4 against him, both their differences, 4, and 3, and 10 is ioyned with 2 and 5, therefore hath he their differences, 3 and 2. And because of ease for you, in another columne I haue set the differences reduced into one number for euery seuerall sort, and haue also added them together, whereby appeareth that they make 33, and so consequently you see the workes of the Golden Rule set forth. For the fire drugges, I haue added letters, a, b, c, &c. as before. But I would not wish you to cleaue still to these elementary aydes, but accustome memorie to trust her selfe, so shall occasion of negligence be best auoyded. And as for the proofe, trie it at more leysure, because the time now is short, and you sufficiently instructed in that proofe. And there resteth diuers things behinde yet, of which I would gladly giue you some taste before our departure.

Scholar. But if it may please you to let me

for

ſee all the variations of this queſtion, before you goe from it, for me thinketh I could vary it two or three wayes more yet.

Maſter. I am content to ſee you make two or three variations, but I would be loth to ſtay to ſee all the variations, for it may be varied aboue 300 wayes, although many of them would not well ſerue to this purpoſe.

Scholar. I thought it impoſſible to make ſo many variations.

Maſter. Marueil not thereat: for ſome queſtions of this rule may be varied aboue 1000 waies: but I would haue you forget ſuch fantaſies till a time of more leiſure. And now goe forward with ſome variation of this queſtion.

Scholar. For the firſt variation, I linke the firſt number 1 with 8 and 10, and 2 I combine with 6 and 10, then ioyne I 3 with 6, 8, and 10, as in this forme.

A

1	35	8	43	Z 300
2	15	6	8	5 3/43
3	135	9	B	
6	32	5	43	Z 300
8	42	6	6	41 17/43
10	432	9	C	
	43	43	Z 300	
			62 34/43	

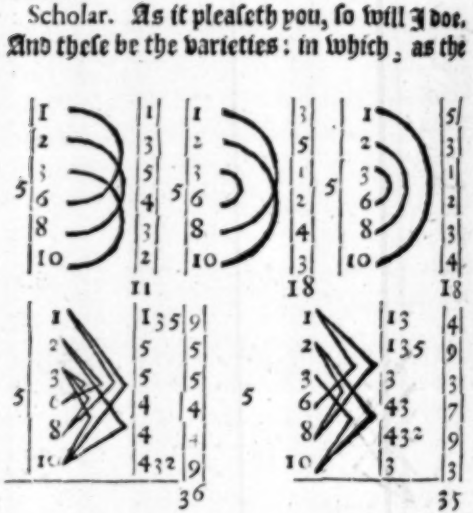
D

43	Z 300
5	34 18/43
E	
43	Z 300
6	41 17/43
F	
43	Z 300
9	62 34/43

And ſo doth there appeare the proportion of waight for euery kinde of drugge in this mixture. Now for the triall.

Master. Nay, stay there: you shall not neede to make triall in one example so often, or if you list to doe it by your selfe, I am content. But now set forth (for declaration that you conceive the rule) two or three examples of severall combinations, and then will we passe to some other example, and so end this Rule.

Scholar. As it pleaseth you, so will I doe. And these be the varieties: in which, as the

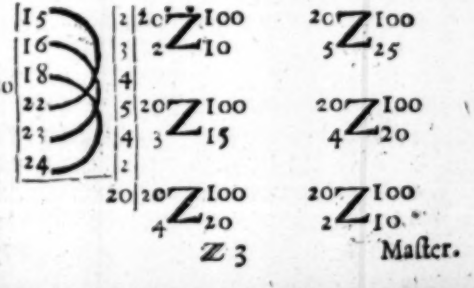


combinations are severall, so doth it plainely appeare, that the differences by which the proportion of each severall kinde is taken, are also severall. And yet I see in the three first of these five

fiue varieties, and in one other before, the totall summe of the differences to be one, that is to say, 18, whereby I perceiue that the varietie of their mixture doth depend on the varietie of their differences seuerall, and not of the varietie of their totall summe.

Master. So is it. And seeing you conceiue it so well, I will make an end of this Rule, onely exhibiting vnto you one question or two of the mixture of metals, that by it you may deuise other like, and exercise your selfe therein also, because the vse of it serueth often in busines of charge, not so much for Goldsmiths, as of coynage in Mints. First I demand of you this question: If a Mint-master haue gold of 22 karects, and some of 23 karects, some of 24: Againe, some of 15, some 16 and some of 18 karects, and would mixe them so that he might haue 100 ounces of 20 karects, how much shall he take of euery sort?

Scholar. To know that I answere in order thus.



Z 3 Master.

Master. You haue wrought the question well: but how chanced you made no doubt of that new name Barrel?

Scholar. Because I thought it out of time to demand such questions now, seeing you make so much haste to end: and againe in this case the proportion of the number is sufficient for my purpose in this worke: trusting that another time you will instruct me as well of this, as of sundry other things, which as I haue heard you talke of, so I haue a great desire to them.

Master. Your answere is reasonable, and your request and trust (with Gods helpe) I intend to satisfie: and to goe forward with this matter, let me see your examination of this last worke.

Scholar. First for the one part I adde together all the particular sums as they appeare in the worke, and they make 100, as here by their addition doth appeare.

And so it seemeth that the summes are well gathered: but for the further triall of them: I multiply first

10	150
15	240
20	360
25	550
20	460
10	240
100	2000

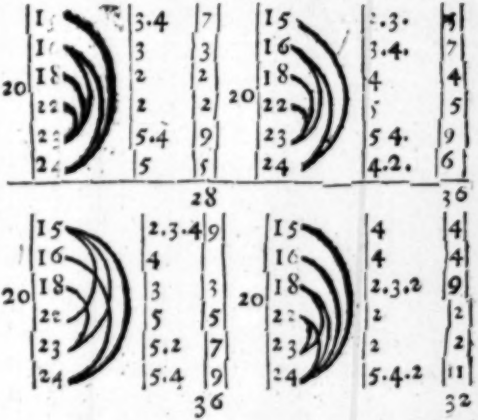
20, which is the common or meane summe of the barrels by 100, which is the summe of the whole masse, which I would haue, and it maketh 2000. Then I multiply euerie particular summe by the barrels that it doth containe, as 10

by 15, and that maketh 150.

Likewise I multiply 15 by 16, and it yéeldeth 240: so 20 by 18 maketh 360. And 25 by 22 yéeldeth 550: likewise 20 by 23 bringeth forth 460, and last of all 10 multiplied by 24 yéeldeth 240: which summes all ioyned together make 2000, that doth agrée with the like summe before, wherefore I may well say, that the worke is good. And now if (it please you) I would set forth some varieties of this question to proue my wit.

Master. Goe to, let me sée.

Scholar. Héere be foure varieties.



Master. You haue wrought the question well: but how chanced you made no doubt of that new name Barrel?

Scholar. Because I thought it out of time to demand such questions now, seeing you make so much haste to end: and againe in this case the proportion of the number is sufficient for my purpose in this worke: trusting that another time you will instruct me as well of this, as of sundry other things, which as I haue heard you talke of, so I haue a great desire to them.

Master. Your answere is reasonable, and your request and trust (with Gods helpe) I intend to satisfie: and to goe forward with this matter, let me see your examination of this last worke.

Scholar. First for the one part I adde together all the particular sums as they appeare in the worke, and they make 100, as here by their addition doth appeare.

And so it seemeth that the summes are well gathered: but for the further triall of them: I multiply first

150	20, which is the common or meane
240	summe of the barrels by 100, which
360	is the summe of the whole masse,
550	which I would haue, and it ma-
460	keth 2000. Then I multiply eue-
240	rie particular summe by the ba-
2000	rrels that it doth containe, as 10

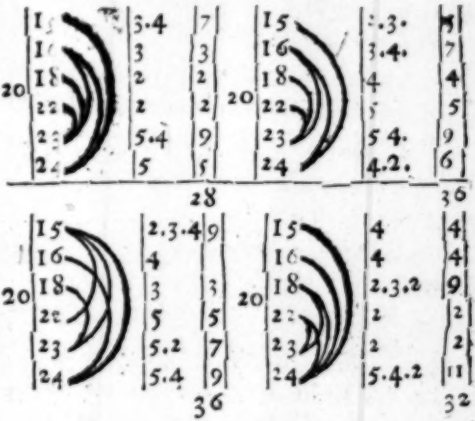
10	
15	
20	
25	
20	
10	
100	

by 15, and that maketh 150.

Likewise I multiply 15 by 16, and it yéeldeth 240: so 20 by 18 maketh 360. And 25 by 22 yéeldeth 550: likewise 20 by 23 bringeth forth 460, and last of all 10 multiplied by 24 yéeldeth 240: which summes all ioyned together make 2000, that doth agrée with the like summe before, wherefore I may well say, that the worke is good. And now if (it please you) I would set forth some varieties of this question to proue my wit.

Master. Goe to, let me sée.

Scholar. Héere be foure varieties.



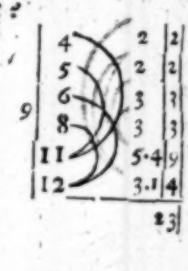
And more yet could I make, but not like to the number that you speake of in the variation of the other question.

Master. That will I teach you at more leisure, seeing it is a thing rather of pleasure than of any necessitie.

But now for your exercise in this rule, one other question I will propose. A Mint-master hath 6 ingots of silver of sundry fineneſſe, some of 4 ounces fine, and some of 5 ounces, some of 6, and other of 8, some of 11, and other of 12, and his desire is to mixe 500 pounds waight, so that in the whole masse every pound waight should beare 9 ounces of of fine silver: how much shall he take (say you) of every sort of silver?

A question of mixing of silver.

Scholar. To finde out that, I set the numbers thus in order.

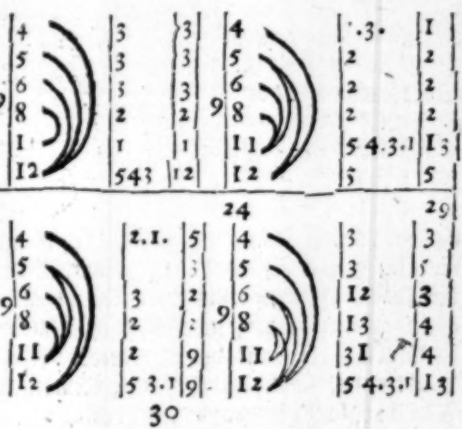


And gathering the differences, it will appeare, that, of the first sort there must be $43\frac{11}{37}$: of the second like much: of the third sort $65\frac{5}{37}$: and of the fourth sort as much: of the fift sort $195\frac{15}{37}$: and of the first sort $86\frac{22}{37}$, which in the whole will make 500 ℔ waight, and in ounces after 9 ounces fine 4500, that is of the first sort $173\frac{21}{37}$: and of the second sort

$127\frac{9}{37}$: of the third sort $391\frac{27}{37}$: of the fourth sort $521\frac{1}{37}$, of the fift sort $2152\frac{4}{37}$, and of the first sort $1043\frac{11}{37}$, which altogether doe make 4500 ounces, agreeable to the multiplication of 9 by 500.

Master. This is well done of you, therefore now make three or foure varieties, and so an end of this rule.

Schol. These 4 varieties I set for example.



Master. And by these it appeareth, that you can finde out more, with which I will not now meddle, saue onely (for to shew you an easie helpe in drawing the lines of Combination) I will set forth two varieties here.

And

And this shall suffice now for the rule of Alligation or mixture: for by these examples may you easily coniecture such other as doe appertaine to it, as well for the due working, as for varietie of drawing the lines of combination.

Scholar. Sir, albeit it pleased you erewhile, to put me from my musing at the manifold varieties that may fall in these combinations, and termed them fantasies; yet my fantasie giueth me, that the consideration of this should in many other examples and cases of importance be very needfull, and the knowledge of it most profitable. Therefore ye may well thinke, that at another time conuenient I will request you to ayde me herein.

Master. Truth it is, that this consideration may fall in practise as well Politike as Philosophicall, and sundry waies in them be applied: therefore when time shall fall fit for the discussing of this consideration, you shall not want my helping hand.

The

The Rule of Falshood.

NOw will I briefely alse teach you somewhat of the Rule of Falshood, which beareth his name, not for that it teacheth any fraud or Falshood, but for that by false numbers taken at all aduentures, it teacheth how to finde those true numbers that you seeke for. The occasion of the name.

Scholar. So might any other rule be called the rule of Falshood, for they work by wrong numbers, and by them find out the right numbers: so doth the Rule of Alligation, the Rule of Fellowship and the Golden Rule partly.

Master. In the Golden rule, the rule of Fellowship, & the rule of Alligation, although the numbers that you worke by, be not the true numbers that you seeke for, yet are they numbers in iust proportion, & are found by orderly worke: whereas in this rule the numbers are not taken in any proportion, nor found by orderly worke, but taken at all aduentures.

And therefore I sometimes being merry with my friends, and talking of such questions, haue caused them that proponed such questions, to call vnto them such children or Idiots as happened to be in the place, and to take

take their answere, declaring that I would make them solue those questions, that seemed so doubtfull.

And indeed, I did answere to the question, and worke the triall thereof also by those answeres which they happened at all aduentures to make: which numbers seeing they be taken as manifest false, therefore is this rule called the Rule of false Positions, & for briefenesse, the Rule of Falshod: which Rule, for readinesse of remembrance, I haue compriſed in these few verses following, in forme of an obscure Riddle.

Gheſſe at this worke as hap doth leed,
By chance to truth you may proceed.
And first worke by the question,
Although no truth therein be don.
Such falshood is so good a ground,
That truth by it will soone be found.
From many bate too many, mo,
From too few take too few also;
With too much ioyne too few againe:
To too few adde too many plaine.
In crosse-wise multiply contrary kinde,
All truth by falshood for to finde.

The sense of these verses, and the summe of this Rule is this:

When any question is proponed appertayning to this Rule, first imagine any number that you list, which you shall name the first

first position, and put it in stead of the true number, and then worke with it as the question imporeth: and if you haue missed, then is the last number of that worke eyther too great or too little: that shall you note as heere after shall be taught you, and you shall call it the first errour.

Then beginne againe, and take another number, which shall be called the second Position, and worke by the question: if you haue missed againe, note the excesse or default as it is, and call that the second errour. Then multiply crosse-wise the first Position by the second errour, and againe the second Position by the first error, & note their totals severally by the names of totals. Then marke whether the two errours were both alike, that is to say, both too much, or both too little: or whether they be vnlike, that is, the one too much, and the other too little: for, if they be like, then shall you subtract the one totall from the other (I meane the lesser from the greater) and the remayner shall be your Dividend: so must you abate the lesser errour out of the greater, and the residue shall be the Diuisor. Now diuide the Dividend by that Diuisor, and the quotient will shew you the true number that you seeke for. But, and if the errours be vnlike, then must you adde both those totals (which you noted) together, and take that whole number for the Dividend, so

shall

shall you adde both errours together, and that whole number shall be the Diuisor, and the Quotient of that Diuision shall giue you the true number that the question seeketh for, and this is the whole rule.

Scholar. This rule seemeth so vnlike any other, that without some example I shall not easily vnderstand it.

A question of Masonrie.

Master. With a good will: propose halfe a score sundry questions and examples of varietie, for the better vnderstanding of the worke hereof: and for the first take this example: A Mason was bound to build a wall in 40 daies, and it was couenanted so with him, that euery day that he wrought, he should haue for his wages 2 s, 1 d, and euery day that he wrought not, he should be amerced 2 s 6 d: so that when the wall was made, and the reckoning taken of the daies that he wrought, and of the other that he wrought not, the Mason had cleerely but 5 s 5 d for the worke. Now do I demand how many daies did he worke of those 40, and how many did he not worke?

The first example.

Scholar. I pray you expresse the order of the worke, that I may partly by imitation, and partly by comparing it with the rule, bee able againe to doe the like.

Master. This order shall you keepe in the worke of this rule: first take some number (as you list) at aduenture, as for example, I say he played 12 daies, & wrought 28 daies. Now cast

cast you the wages of euery day, and see whether it will agree with the summe of 5 ℔ 5 d.

Scholar. The 28 daies that he wrought after 25 pence the day, yeeldeth 700 pence. Then 12 daies that he wrought not, at 30 pence each day, doth amount to 360 pence which if I abate out of 700 pence, there resteth 340: but you say he had not so much.

Master. He had but 65 pence, and by this supposition he should haue had 340: therefore is this summe too much by 275, which summe I must set downe after this sort, as you see here, where first I haue made a crosse (commonly called S. Andrewes crosse) and at the ouer corner, on the left hand I haue set the first position 12: and at the other corner vnder it I haue set 275, which is the first errour, with this figure ✝, which betokeneth too much, as this line—plaine without a crosse line betokeneth too little. On the right hand of the crosse I haue left two like romes for the second position and his errour. Therefore to prosecute the worke, I suppose he played 16 dayes, and wrought 24.

12
275 ✝

Scholar. I was a while in doubt why you named the daies of his working, seeing they be not set in the figure: and I doubted how you knew them, or else whether that you did suppose them at all aduentures, as you did the dayes that he played: but now I gather

ther, that seeing 40 dayes is the whole time limited, then the dayes that he played being supposed, the rest of 40 must needs be ỹ dates that he wrought, and therefore 28 followed 12 of necessitie, and 24 followeth 16 also of necessity: but yet I scarce perceiue why you set not in the figures as well 28 as 12.

Master. It forceth not which of them I take, so that in the second position I take the numbers of the same nature that is here both of working dayes, or both of idle: but now examine you this second position.

Scholar. If he played 16 dayes, then abating 16 times 30 d, the summe will be 480 d. And for 24 dayes that he wrought, euery day yelding 25 d, the totall is 600 pence: so that abating 480 out of 600, there resteth 120, and as you say, it should be but 65: therefore it is too much by 55: that must be set on the right hand of the figure at the neather part, and ouer it on the same side 16, which is the second position, thus:

12 16

275 + 55 +

And as I gather by your words, it were all one if I did set 28 in stead of 12, and 24 in stead of 16.

Master. So were it. But this shall you marke, that, of what nature soeuer the two positions be, of the same nature is the quotient. Therefore when the positions in this

question

question are 12 and 16, which both beeing numbers of the playing daies, the Quotient shal declare the true number of playing daies: whereas if the positions had beene 28 and 24, which are supposed to be the working dayes, then would the Quotient declare the true number of the working daies, & not of playing dayes, as it will doe now. And therfore to continue the worke of this question, and to finde the true number of playing daies, I must multiply crossewise the first position 12 by 55, that is the second errour, and the total will be 660. then I multiply 275 and 16, and it yeldeth 4400. Now because the errours are like, that is to say both too much, I must substract 660 out of 4400, and so remaineth, 3740, which is the Diuident. Againe, I must subtract the lesser errour 55 out of 275 that is the greater errour, and there will remaine 220. which shall be the Diuisor: then dividing 3740 by 220, the Quotient will be 17. Wherfore I say now constantly, that 17. is the true number of dayes that the Mason played: and then it followeth that he wrought 23 daies, and so is the question answered.

Now for the order of triall of this worke, there needeth none other triall but onely this, to worke with this number according to the question, and if it agree, then appeareth the number to bee it that you would haue. As heere now seeing hee wrought 23. dayes, and

The proof of this rule

must haue for euery day 25 pence, the whole summe commeth to 575. Then againe seeing he plaied 17 daies, and must abate 30 pence for euery day, the whole summe of the abatement will bee 510: therefore I subtract 510 out of 575, and there will remaine 65, which maketh 5 s 5 d, the cleere wages of the Mason for his worke, according to the question.

Scholar. Now I trust I vnderstand the worke and the rule so well (and the better by this proofe) that I can be able to doe the like. And for a proofe, I take the same question all saue the last number, where I will suppose that hee had 10 s for his wages cleere. And now to gesse at the number of the daies that hee wrought, I suppose first that he wrought 20 daies: then say I, If he wrought 20 daies, his wages must be 500 d, then did hee play other 20 daies, for which must be abated 600 d, and then he loseth 100 d. And so am I at a stay for it is not like vnto your former worke.

Master. You should haue required of me some question, and not haue taken a question of your owne phantasying, vntill you were more expert in this Art: for so might you as wel happen on an impossible question, as on a possible: but now to go forward, consider that this nũber is too little by 220, seeing he should gaine by your supposition 120 d, and in this position he loseth 100, those both make 220, which you shall set downe for the first errour with

with this signe ——, betokening too little, as here in this forme following doth appeare.

And now for the rest goe forward your selfe once againe.

Scholer. As my errour hath bttered my folly, so it hath procured me better understanding

Now therefore considering this position not to solue the question, I take another, supposing that he wrought 30 daies: then for his wages he must be allowed 750 d. and for the 10 daies which hee wrought not he must abate 300 d, and so remaineth cleere 450 d: but it shall be onely 120 d, therefore is it too much by 330, which I set downe in the figure with the former position and his errour, and the figure appeareth thus:

Now must I multiply in crosse waies 20 by 330, & it will be 6600.

Then againe I multiply 30 by 200, and it will be also 6600. Wherefore if I shal subtract the one out of the other, there will remaine nothing to bee the Diuident.

Master. In this you forget your selfe againe: for in as much as the signes in the errors be vnlike, therefore must you worke by Addition, adding together those two totals to make the Diuident, and also adding the two

errors to make the Diuisor. And because you shall no more forget this part of the rule, take this briefe remembrance:

Vnlike require Addition,
And like desire Subtraction.

Scholar. You meane, that if the errours haue like signes, then must the Diuident and the Diuisor be made by Subtraction, as is taught before. And if those signes be vnlike, (as in this last example they bee) then must I by Addition gather the Diuident and the Diuisor. Therefore must I adde 6600 to 6600, and it will be 13200, which shall be the Diuident. Then againe I adde 220 to 330, and it will be 550, which must be the Diuisor: wherefore diuiding 13200. by 550, the Quotient will be 24 whereby I know that the Mason wrought 24 daies, and then it followeth that he plaied 16. dayes.

Master. Examine your worke, whether it be agreeable to the question or no.

Scholar. For 24 dayes worke the wages must be 600 d, and for 16 daies which the Mason wrought not, there must be abated 480, and then remaineth cléer to the Mason 120 d, as the question importeth: wherefore it is euident that 24 is the true number of dayes that he wrought.

Master. Although you seeme now to vnderstand this worke, yet to acquaint your minde

minde the better with the newe trade of this rule, I think it good to propone to you fiue or sixe examples more, before I make an end of it.

Scholar. Sir I thanke you that you doe so consider my commodity and profit in knowledge, for vndoubtedly it is practice and exercise that maketh men prompt and expert in euery kinde of knowledge.

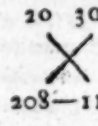
Master. You say well, so that they follow som certain precepts to gouern and rule their practice by, else may practice procure custom of errour, and a repugnance to exactnesse of knowledge; namely, as long as the errour is not plainly knowen to the vulgar sort. But to returne to our worke.

There is a seruant that hath bought of velvet and damaske for his master 40. yards, the velvet at 20 s a yard, and the damask at 12 s. and when hee commeth home, his Master demandeth of him how much he hath bought of each sort: I cannot tell (saith he) exactly: but this I knowe, that I paid for damaske 48. s more then I paid for velvet: now must you ghesse how many yards there is of each sort.

Scholar. Although the ghesse seemeth difficult, yet I will proue what I can doe: for I remember your saying, that it forceth not how fond or false the ghesse be, so it be some what to the question, and not an answere of a contrary matter.

Therefore first I imagine that hee bought

20 yards of Damaske, for which hee should pay after the former price 240 shillings: then must he needs haue of Velvet other 20 yards (to make vp the 40 yards) & that would cost 400 s. So that the totall of the price of the Damaske is lesse then the sum paid for Velvet 160 s, and should be more by 48: therfore the first errour is 208 too little. Then begin I againe, and suppose he bought of Damaske 30 yards, that cost 360 s: then had hee but 10 yards of Velvet, which cost 200 s: and now the price of the Damaske is greater then the price of the Velvet by 160 shillings, & should be but 48, therfore is the second error 112 too much, which I set in forme of figure as here both appeare: Then doe I multiply in crosse waies 208 by 30, and the summe will be 6240. Also I multiply 112 by 20, and there will amount 2240. And in as much as the signes of the errours be vnlike, I know I must worke by Addition, therfore ad I those two totals together, and they make 8480, which is the Dividend: then adde I also the two errors together, 208 and 112, and they make 320, which is the Divisor. Wherfore dividing 8480 by 320, the quotient will be 26 1/2, which is the true summe of yards of Damaske that he bought, and in Velvet 13 yards 1/2, and that appeared by examination,



thus

thus: 26 $\frac{1}{2}$ yards of Damaske at 12 s the yard, maketh 318 s: then in Veluet he had but 13 yards and $\frac{1}{2}$, that coſt 276 s, at 20 the yard. Now ſubtract 270 out of 318, and there will remaine 48, which is the number of ſhillings that the Damaske did coſt more then the Veluet.

Maſter. Now ſhall you haue a queſtion of another kinde.

There are three men that doe owe money to mee, and I haue forgotten what the totall ſumme is, and what the particulars be

Scholar. Why, then it is impoſſible to know the debt.

Maſter. Peace: ye are too haſtie: there is more helpe in it then yet you ſee. I haue three ſeuerall notes, whereby it appeareth that I did conferre their debts together, and found the debt of the firſt and the ſecond to amount to 47 l', the debt of the firſt man and the third man did make 71 l', and the ſecond man his debt with the third, did riſe to 88 l'. Now can you tell what euery man did owe, and what was the whole totall?

Scholar. Nay in good faith: but as I perceiue that it muſt be found by coniecture, ſo will I gheſſe at it, ſuppoſing that the firſt man did owe 20 l, the ſecond man 30, & the third.

Maſter. Nay ſtay there, for you are too far gone already: you may not ſuppoſe a ſeuerall ſumme for euery man, for it is enough to

ſuppoſe one ſum for the firſt man, and for the other riſe as the queſtion importeth. Therefore ſeeing you ſet the 1. man his debt to bee 20 l', the ſecond man cannot owe 30 l': for the declaration is, that their debts added together did make 47 l', ſo muſt the ſecond man his debt be but 27 l'. Now this ſecond debt with the third muſt make 88: therefore ſubtract 27 out of 88, and there will remaine 61, as the third man his debt. Then ſaith the declaration, that the firſt & third mens debts do make 71: but by this ſuppoſition they make 81, that is 10. too much, which I muſt ſet for the firſt errour. Now worke you the ſecond poſition.

Scholar. I ſuppoſe the firſt mans debt to be 24 l': then muſt the ſecond mans debt (by you your declaration) be but 23 l', ſeeing both they make but 47 l'. And the ſecond man his debt with the third, do make 88 l', and the ſecond man oweth but 23: therefore the third man muſt owe 65 l'. Now the third mans debt with the firſt ſhould make by the declaration 71 l', and they doe make 89 l', that is 18 l' too much, and that is the ſecond errour, which I ſet downe with the firſt, and their poſition in this forme, and then I doe multiply in croſſe waies 20 by 18, and it is 360. And 10. by 24 maketh 240. Alſo becauſe the ſignes of the errours bee like, I muſt

20 24

10 18

worke

worke by Subtraction: therefore I ſubtract 240 out of 360, and there reſteth 120, which is the Dividend: then do I ſubtract 10 out of 18 by the ſame reaſon, and ſo is the Diviſor 8, which is found 15 times in 120: therefore I ſay, that the firſt man did owe 15 l', & then the ſecond man muſt owe 32 l': for thoſe 2 do make 47 l', and the third man his debt is 56, for ſo much remaineth if I bate 15 out of 71, or if I take 32 out of 88.

Maſter. For the fourth example take this eaſie queſtion for the variety in worke. Two men having ſeverall ſummes which I knowe not, doe thus talk together: The firſt ſaith to the ſecond, If you give me 2 s of your money, then ſhall I have three times ſo much money as you. The ſecond anſwereth, It were more reaſon that our ſummes were made equall, and ſo wil it be if you give me 3 s of your money. Now gheſſe what each of them had.

Scholar. I imagine that the firſt had 9 s.

Maſter. Conſider evermore in your imagination that you take a likely ſumme, as in this queſtion, take ſuch a ſumme that having 2 added vnto it, may be divided into 3 parts even.

Scholar. Why? I remember you ſaid before, it forceth not how fondly ſoever I gheſſed.

Maſter. As for the poſſibility of the ſolution, it is truth; but for eaſineſſe in worke, the aptеſt

aptest numbers are most conuenient.

Scholar. I thought no lesse, and therefore I toke 9 as an apt number to be parted into three: but I perceiue I should haue considered the aptnesse of that partition after the addition of two vnto it, and then 7 had beene more meet.

Master. That is truth, and then should the second man his summe be 5: for although he haue now but the third part of 9, that is 3, yet you must remember that he lent the first man 2, and so had he 5.

Scholar. Then to go forward: if the second man had three of the first man, then should hee haue 8, and the first man but 4: so hath hee double to the first man: yet he said in the question they should haue equall: wherefore it appeareth that he hath 4 too much.

Therefore I note that error with his supposition, and ghesse againe that he hath 10 s: whereunto I adde 2 shillings borrowed of the second man, and then he hath 12 shillings: so the second man hath remaining but fowre, whereunto if I adde the 2 that hee lent to the first man, so had he but 6 s at the beginning. Then take 3 shillings from the first man, and giue to the second, then hath the first man but 7, and the second hath 9, which are not equal, but there are two too many wherefore I set down both

7 10
4 2

the

the positions with their errore, as here you see, and multiply acrosse, so commeth there 40 & 14: and because the signes be like, I take 14 out of 40, & so resteth 26 to be the dividet: then likewise I take 2 out of 4, and there resteth 2, by which I divide 26, and the quotient will bee 13, which is the summe that the first man had. And so appeareth, that two being added thereto, the summe will bee 15, so hath the second man now but 5, and before he had 7: then take three from the first, and put to his 7, and so haue each of them 10, and that is equall, as the question would.

A question of Lambs. The fifth example.

Master. For the fifth example take this question. One man said to another, I think you had this yeere two thousand Lambs: So had I, said the other: but what with paying the tithe of them, and then three seuerall losses they are much abated: for at one time I lost halfe as many as I haue now left, and at another time the third part of so many, and the third time ¼ so many. Now ghesse you how many are left.

Scholar. Because here is mention made of certaine parts, I must take a number that may haue all these parts, that is to say, ½, ⅓, and ¼, which will be 24, howbeit 12 hath the same parts. Therefore I take first 12 to bee the number that doth remaine, so hath he lost 6, 4, and 3, that is, 13, and in the whole 25, but it should be 2000.

Master

Master. Ye are deceiued yet stil: you haue forgotten the 10 part, which must bee defalked, that is 200, so there remaineth but 1800 and now goe on againe.

Scholar. Then to finde the errour, I take 25 out of 1800, and there remaineth 1775 to few, which I set for the first errour. Then for the second position I take 24, whose halfe is 12, the third part 8, and the quarter 6, whereby riseth 50, which is to little by 1750, therefore I set downe both the positions wt their errours, thus:

12	24
1775	1750

And multiply in crosse waies 1775 by 24, whereof commeth 42600. Also I multiply 1750 by 12, and there ariseth 21000. And because the signes are like, I doe subtract the one from the other, and so remaineth the Diuident 21600. Then doe I subtract 1750 out of 1775, and there resteth 25, by which I diuide 21600 and the Quotient is 864, whereof the halfe is 432, and the third part is 288, the quarter is 216, which all being added together will make 1800: And if you adde thereto the tenth which was abated before, then will the whole summe be 2000. And now doth there come a question to my memory which was demanded of me, but I was not a-

864
432
288
216
1800

ble

able to answere to it: and now me thinketh I could solue it.

Master. Propone your question.

A question of sheepe & tillage. The sixth example.

Scholar. There is supposed a lawe made, that (for furthering of tillage) euery man that doth keepe sheepe, shall for euery tenne sheepe eare and sow one acre of ground: and for his allowance in sheepe pasture, there is appointed for euery fowre sheepe one acre of pasture. Now is there a rich sheepe master which hath 7000. acres of ground, and would gladly kéep as many sheepe as hee might: by that Statute, I demaund how many sheepe shall hee keepe?

Master. Answer to the question your selfe.

Scholar. First I suppose he may kéep 500 sheepe, and for them hee shall haue in pasture after the rate of fowre sheep to an acre, 125 acres, and in arable ground 50 acres, that is 175 in al: but this errour is too little by 6825 Therefore I ghesse againe that he may keepe 1000 sheep, that is in pasture 250 acres: and in tillage 100 acres, which maketh 350 that is too little by 6650.

500 1000
6825 6650

Both these errours with their positions I set down as you sée, and multiply them crosse 6825 by 1000 and it maketh 6825000 also I multiply 6650 by 500 and thereof commeth 3325000, which summe

I subtract out of the former, and there remaineth 3500000 for the diuident; likewise I subtract the lesser error out of the greter, and there resteth 175, by which I diuide 3500000 (the diuidents aforesaid) and the quotion wil be 20000, so that by this rate, hee that hath 7000 acres of ground may keep 20000 sheep.

Master. You haue done well notwithstanding, both this last question, and the next before might be wrought without the second position by the rule of proportion, as this. When in this question yee found in the first errour, that for 500. sheepe, there must bee 175 acres, then might you reduce it to ẏ golden rule, thus.

An other way of working.

175	Z	500
7000		2000

If 175 acres will admit in allowance 500 sheepe, then 7000 wil haue 20000. And so by one position, with the helpe of the Golden rule, may you answere that question. Likewise for the question of Lambes, when you had found that 12 came of 25, you might haue set the figure thus as you see, and haue sayd:

25	Z	12
1800		864

If 25 doe leaue but 12, what shall 180 leaue? and it would appeare to be 864.

Scholar. Sir, I thanke you for this ayde, for it doth much shorten the worke of this rule.

Master

Maſter. Yet againe, I will ſhewe you an other way to aunſwere to this laſt queſtion without the rule of falſe poſition, and that by the rule of fellowſhip, for it appeareth in the propoſning of the queſtion, that 10 ſheepe muſt haue in paſture 2 acres and ½, and for thē muſt there be eared but one acre: ſo it followeth, that for two acres eared, there muſt be 5. ſet to paſture. And if you put them both into one ſumme, they wil make 7. Therefore lok what proportion 7 being this total, doth beare to 5 and to 2, ſuch proportions ſhal any totall in this queſtion beare to the paſture ground, and the eared ground. An other way yet.

Scholar. This ſerueth wondrous aptly. Therefore to proue it, I demand this by the former ſuppoſition: If a man haue 300. Acres, how much ſhall he leaue in paſture, and how much ſhall he torne to tillage? You ſay that as 7 is to 5, ſo ſhall 300 be to the acres of paſture: and as 7 is to 2, ſo is 300 to the acres of tillage, whereof for both I haue ſet examples here following, wherby appeareth that of Paſture, there ſhall bee 214 ⅔ Acres, and of Tillage 85 ⅓, which both ſums added together doe make 300.

7 Z 5
300 Z 214 ⅔

7 Z 2
300 Z 85 ⅓

Maſter. Now take an other example: A man hath three ſiluer cuppes with one couer, An other queſtion.

the

the couer wayeth 18 ounces, the second cup wayeth euen halfe the waight of the first and the third. Now if the couer bee put to the first cuppe, they wey iust as much as all the three cuppes do wey: and if the couer be ioyned with the second cuppe, they wey as much as the second twice, and the third: and if the couer be put to the third cup, they will make twice as much as the first and the second cup. Now trie you what was the iust weight of euery cup.

Scholer. I doe set the weight of the first cup to be 9 ounces: then in as much as these two (that is to say, the couer and the first cup) doe wey the weight of the three cuppes, I see that the three cups must wey 27 ounces for so much is 18 and 9. Also because the first and the third doe wey double so much as the second, therefore is it the third part of that weight, that is 9. and then would it followe, that the third cup also should wey 9. ounces, but then the question saith, that the couer being ioined to the second cuppe, they wey as much as the second twice, and the third once, that should be 27, and so it doth: then beeing ioined with the third cuppe, they should wey twice as much as the first and the second, that should be 36, and they wey but 27, so is that error 9. too little. Then begin I againe, and say, that the first cuppe doth wey 12 ounces, which I ioine with the couer, and they make

30. ounces

30 ounces: then seeing the second is of that weight, it must néeds wey 10 ounces, and the third must wey 8 ounces, seeing the first and the 3 must wey 20 ounces. Now put I the couer to the second cup, & they wey 28 ounces, which should be euen so: then ioine I the couer with the 3 cup, and so should it wey twice the first and the second, that is 44 ounces, and they doe wey but 26, that is eighteene to little: those errors with their positions I set downe, and multiply in crosse waies 9 by 12, whereof commeth 108. Also 9 by 18, and that yéeldeth 162: and in as much as the signes bee like, I abate the lesser out of the greater, and there doth remaine 54. Then doe I also abate the lesser error from the greater, and so remaineth 9, by which I diuide 54, and the quotient is 6: which I take for the true weight of the first cup: which being ioined with the couer must wey as much as the three cups, so do they wey but 24 ounces. Then seeing the second cup is the third part of that weight, for the other two cuppes (you say) must wey double his weight, the weight of the second cuppe is 8 ounces, and so the weight of the third must be 10 ounces. Now put the couer to the second cup, and it will make 26 ounces: that must be the weight of the second twice, and

9 12
9—18—

the third once, that is twice 8, and once 10, and so is it. Again, put the couer to the third cup of 10 ounces, and they must wey twice as much as the first and the second, that is 28 and so is all agreeable.

Master. Then answere to this question.

There is a Cesterne with foure cockes, containing 72 barrels of water: and if the greatest cocke be opened, the water wil auoid cleane in 6 houres: at the second cocke it will aske eight houres: at the third cocke it will auoide in no lesse then nine houres: and at the smallest it will require twelue houres: Now I demaund in what space will it auoid, al the cockes being set open?

Scholer. First I imagine it will auoid in two houres.

Master. Then must there auoide by the first cocke $\frac{1}{3}$ of the water, that is 24 barrels, and by the second cocke $\frac{1}{4}$ that is 18, and by the third cocke $\frac{1}{5}$ that is 16 barrels, and by the smallest cocke $\frac{1}{6}$, that is 12 barrels, all which summes put together, do make 70, as by their additiō it doth appeare, but it should be 72, therefore the error is 2 to few.

Scholer. Then I begin again by your fauor, because I think I vnderstand the worke, and put three houres for the due time: so shall there run out at the greatest cocke $\frac{1}{3}$,

	24
	18
	16
	12
	70

that

that is 36 barrels, & at the second hole ⅓ that is 27, and at the third cocke ¼ that is 24, and at the smallest hole ⅕, that is 18 barrels, which altogether doe make 105, and should be but 72, so is it too much by 33, therefore do I set the errours in order of the figure with their positions, and worke by multiplication, in crosse, saying: 2 times 3 is 6, & two times 33 maketh 66: and because the signs are vnlike, I must adde those two totals together, which make 72: also I adde the two errors, and they make 35, by which I diuide 72 and the quotient riseth $\frac{2}{37}$, whereby I see that all the cockes being set open, the water will auoide in 2 houres and $\frac{2}{37}$ of an houre.

2 3
2 — 33

Master. This exercise maketh you to grow expert in the Rule. Therefore I will inure you somewhat more with a question or two.

A question of partners.

There were two men that had been partners, and had in account betweene them 300 duckets: whereof the one should haue for his part 180, and the other 120: but in the parting of them they fel at variance, so that each of them catched as many as he could: yet afterward being reconciled, they agreed that he which had gotten most part of them, should lay downe ⅔ of them againe, and hee that had

gotten leaſt, ſhould lay down ⅓ of thoſe which he had taken, and then parting them into two equall parts, each man to haue halfe thereof, and ſo had they their iuſt portions, as they ought: now I demaund of you what each of them had gotten by the skambling?

Scholar. I ſuppoſe he that had leaſt, got 108 buckets, then the other had 192: wherfore in laying downe againe of the 192, there was put downe ¼ that is 114, and ſo had hee left but 48. Alſo of the 108: there was laide downe 36, that is ⅓, and ſo hee had left 72. Then I put together 144, and 36, and it maketh 180, which I part into two parts euen, and ſo cõmeth 90 to be giuen to each of them: which ſum put to 72, maketh 162, and ioyned to 148, it maketh 238: and now I doubt how I ſhall go forward.

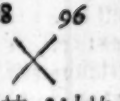
Maſter. You néede not to take but one of them which you liſt, the greater or the ſmaller, for all commeth to one purpoſe: and ſo may you compare it that you take to anie of the other ſums, remembring that you make compariſon to the ſame in the ſecõ worke: as for example of the firſt part. If you compare 138 with the leſſer ſumme due, that is 120, ſo is it 18 too much: and if you compare it with the greater ſumme, then is it 42 too little. Againe, if you compare 162 to the greater ſumme, the error will be 18, as it was in the other: but it will haue a contrary ſigne:

and

and if you compare it with the leſſer ſumme, it will be 42 too much: ſo that the erroꝛ both waies is either 18 oꝛ 42 and as foꝛ the ſignes it little foꝛceth, foꝛ in them is nothing conſidred here, but likneſſe and vnlikeneſſe, which in this caſe, doth neither further noꝛ hinder: But now go on with the woꝛke.

Scholar. If it be ſo, then am I out of my greateſt doubt. Then I ioyne that 90 (which I found as the halfe of the latter partition) vnto 48, which is left with the one man, and ſo hath he 138, which (I may ſay) is 18 too many, foꝛ the leaſt ſhould be but 120: that erroꝛ do I note, and then make a newe poſition, ſuppoſing the one man to haue 204, and the other to haue 96, wheꝛefoꝛe of the 204, there muſt be layd downe 153, and ſo remaineth with him 51. Alſo of the 96, there muſt be laid downe $\frac{1}{7}$, that is 32, and ſo reſteth with that man 64. Now of the 153 and 32 I make one ſumme as 185, which I muſt diuide into two equall parts, and ſo each man ſhall haue $92\frac{1}{2}$, wheꝛeunto if I adde their foꝛmer poꝛtions reſerued, then the one ſhall haue $156\frac{1}{2}$, and the other hath $143\frac{1}{2}$. Wheꝛefoꝛe take the leſſer ſum now againe, as I did befoꝛe, that is $143\frac{1}{2}$, and finde that he hath too many by $23\frac{1}{2}$, foꝛ he ſhould haue but 120, ſo haue I foꝛ my two poſitions two erroꝛs, which I ſet downe, as here may bee ſeene, each erroꝛ vnder his poſition, and

then by the rule I doe multiply in croſſe waies 108 by 23 ⅓ and there riſeth 2538, which I note, then again I multiply 96 by 18, and therof amounteth 1728.



Now becauſe the ſignes are both like, that is both too many, I muſt worke by ſubtraction, and ſo abating 1728 out of 2538, there will reſt for the dividend 810: then for the diviſor I ſubtract 18 out of 23 ⅓, and there remaineth 5 ⅓, by which I divide 810, and the quotient will be 247 $\frac{1}{11}$, which is the iuſt portion of him that had the leaſt ſumme. And if I doe ſubtract it out of 300. being the totall ſumme, then will there remaine 152 $\frac{10}{11}$ as the portion that the other did get.

Maſter. For the proofe of this worke, you may chooſe whether you will examine thoſe numbers according to the forme of the queſtions, or elſe worke by two other poſition (for to finde the ſecond number: and if thoſe poſitions bring the ſame numbers that did amount by the firſt two poſitions, then doth each worke confirme other.

Scholar. By your patience, I will proove both waies, not only to ſeek their agrément, but alſo to accuſtom my mind to thoſe workes for I perceiue it is exerciſe that muſt bee the chiefe engrauer of theſe rules in my memory.

Maſter

Maſter. You conſider it well: then goe to.

Scholar. Firſt I will by two other poſitions trie to finde the portion of him which had moſt.

Maſter. Although you may do it with any poſitions, yet to ſee the agrément of your work the better, take the ſame poſitions that you did before, comparing them now to the greater, as you did before vnto the leſſer.

Scholar. Then I ſuppoſe, that he that had moſt, had 192, ſo had the other 108. Now if I take $\frac{1}{4}$ out of 192, that will be 144, & there will reſt to that man but 48. And from the ſecond which had 108, if I take $\frac{1}{3}$, that is 36, there will remaine to him 72: then ioyning 144. with 36, it will make 180, the halfe whereof beeing 90. If I adde to each of thoſe two mens portions remaining with them, the one ſhall haue 138, and the other 162, of which two I take the greater (that is 162) and ſée it to be 18 too fewe, for it ſhould be 180, that error I note vnder this poſition. Then for the ſecond poſition I take (as I did before) 204, for the one and ſo reſteth 96 for the other: then take I $\frac{1}{4}$ of 204, and it will bee 153, and there reſteth to him 51. Alſo of the 96 I take $\frac{1}{3}$ that is 32, and there remaineth to him 64. Now put I that 32 to 153, and it yéeldeth 185: which beeing parted in equall values, maketh 92 $\frac{1}{2}$ to be added to each mans

remainder, and so the one hath 143 $\frac{1}{17}$, and the other 156 $\frac{1}{17}$: wherefore I take the greatest sum me, and it is 23 $\frac{1}{17}$ to little, that do I note also, and set both these errours vnder their positions, as in this example following doth appeare.

And then multiplying 192 by 23 $\frac{1}{17}$, there doth arise 4512.

Againe, I multiply 204 by 18, and it maketh 3672, which I do subtract out of 4512, because the signes be like, and there resteth 840 for the dividend: then subtracting 18 out of 23 $\frac{1}{17}$, there will remaine 5 $\frac{1}{17}$ which I must take for the divisor. And so dividing 840 by 5 $\frac{1}{17}$, the quotient will be 152 $\frac{1}{17}$, whereby I haue found an agreeable sum to that which I found by the former positions, for him that had most, which I doe subtract out of 300, that is the totall, there will rest 147 $\frac{16}{17}$, which was ỹ portion of him that had the least part.

192	204.
18—	23 $\frac{1}{17}$

Maister. So by diuers positions you see, that one doth confirme the worke of the other. Now examine those two numbers by the forme of the question, and so shall you proue your worke good also.

Scholar. If that he which gate most, had 152 $\frac{1}{17}$, then must he lay downe $\frac{1}{4}$ of this sum: that is 114 $\frac{4}{17}$, and so shall remaine with him but onely 38 $\frac{1}{17}$, The other which had least,

that

that is 147 $\frac{1}{11}$ must put downe of his sum $\frac{5}{9}$, that is 49 $\frac{1}{11}$, and so doth there remaine with him yet 98 $\frac{3}{11}$. Then do I adde together 114 $\frac{6}{11}$ and 49 $\frac{1}{11}$, ad it wil make 163 $\frac{7}{11}$, which I must part into two equall parts, and that wil be 81 $\frac{7}{11}$ to be giuē to each of them: so putting 81 $\frac{7}{11}$ vnto 38 $\frac{3}{11}$, there doth amount 120 iust, which is the true portion of him that shoulde haue the lesser summe: and adding 81 $\frac{7}{11}$ to 98 $\frac{3}{11}$, the totall will be 180, the true portion of the other. And so is the worke by this proofe also tried to be good. And this I marke by the way, that in their scambling, hee gate most (as it chanceth often) that ought to haue had least by iust partition.

Master. Let your study be to learne truth and iust art of Proportion, and to distribute and part according thereunto, as often as occasion shall be ministred. And here would I make an end of this rule, saue that I remember one pleasant question which I cannot ouerpasse, which I wil declare somwhat largely, because you shall as wel vnderstand some reason in the pleasant inuention, as apt proceeding in the nittie working thereof.

An example of mixture of gold and siluer.

Hiero. king of the Syracusans in Sicilia, had caused to be made a crowne of Gold of a wonderfull waight, to be offered for his good successe in wars: in making whereof, the Goldsmith fraudulētly tooke out a certain portion of gold, and put in siluer for it, so that there was

was nothing abated of the full waight, although there was much of the value diminished. Which thing at length being vttered, (as no euill can alwaies lie hid) the king was sore moued: and being desirous to trie the truth, without breaking of the Crowne, proponed the doubt to Archimedes, vnto whose wit nothing seemed vnpossible, which although presently he could not answere vnto, yet hee had good hope to deuise some policy for that inuention, and so musing thereon, as he chaunced to enter into a baine full of water to wash him, he obserued that as his body entred into the baine, the water did runne ouer the Tub: whereby his ready wit of such small effects coniecturing greater works, conceiued by e by a reason of solution to the Kingsquestion, and therefore reioycing exceeding more then if he had gotten the Crowne it selfe, forgate that he was naked, and so ran home, crying as he ran, εὕρηκα εὕρηκα, I haue found, I haue found. And thereupon caused two massie péces, one of golde, and another of siluer, to bée prepared of the same weight that the saide crowne was of: and considering that gold is heauier of nature then siluer, and therefore gold of like weight with siluer must needes occupie lesse rome, by reason it is more compact and sound in substance, hée was assured, that putting the masse of gold into a vessell brim full of water, there would not so much

water

water run ouer, as when he ſhould put in the ſiluer maſſe of the like weight. Wherefore he tried both, and noted not only the quantities of the water at each time, but alſo the differēce or exceſſe of the one aboue the other, whereby he learned what proportion in quantitie is betweene gold and ſiluer of equall weight. And then putting the crowne it ſelfe into the veſſell of water brim full (as before) marked how much water did run out then, and comparing it with the water that ranne out when the gold was put in, noted how much it did excēde that: and likewiſe comparing it to the water that ran out of the ſiluer, marking how much it was leſſe then that, & by thoſe proportions found out the iuſt quantity of gold that was taken out of the crowne, and how much ſiluer was put in ſtead of it: But ſeeing Vitruuius which writeth this Hiſtory, doth not declare the particular worke of this triall, it ſhall be no inconuenience to ſuppoſe an example for declarations ſake, wherein although the true and iuſt proportions be not expreſſed, yet the forme of triall ſhall be truly ſet forth. And for an example, I ſuppoſe the weight of the crowne to be 8 pound, & ſo of each the other two maſſes. And when the maſſe of gold was put into the water, I imagine that there ran out 2 pound of water: & when the maſſe of ſiluer was put in, I ſuppoſe there ran out 3 pound ½. Againe, when the

crowne

crowne was put in, there ran out 2 pound $\frac{2}{7}$: Now to know what quantity of siluer was in the crowne, worke by the rule of false position, and imagine that there was 2 pound of siluer, then must there be 6 pound of gould, then say thus by the rule of proportion: if 8 pound of gold do expel 2 pound of water, what shall 6 pound expell? and it will be 1 pound $\frac{1}{2}$. Againe for the siluer: if 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 2 pound of siluer put out? it will be $\frac{7}{8}$. now adde those two weights of water together, & they will make 2 pound $\frac{3}{8}$ and it should be by the supposition 2 pound $\frac{1}{2}$, so is it too much by $\frac{3}{8}$.

Scholer. Now doe I vnderstand the worke as I thinke, therfore I pray you let me worke the rest of the question. And because this first supposition did erre, I note that position, and his errour, and take a new position, esteeming the siluer to be but one pound, so must there be in gold 7 pound. Then say I: if 8 pound of gold doe yelde 2 pound of water, what shall 7 pound yelde? and it will be 1 pound $\frac{3}{4}$. Again if 8 pound of siluer expell 3 pound $\frac{1}{2}$ of water, what shall 1 pound expell? and it will be $\frac{7}{16}$: Now must I adde those two summes together, and they make 2 pound $\frac{3}{16}$, & they should make 2 pound $\frac{1}{2}$, so is it too little by $\frac{1}{16}$. Therefore I set the posessions with their errours in order, as here followeth. And then I multiplie in crosse waies by 2 $\frac{3}{16}$, and it maketh $\frac{1}{2}$:

like

likewiſe 1 multiplied by $\frac{1}{7}$: maketh $\frac{1}{7}$. And becauſe the ſignes be vnlike, I muſt adde thoſe two ſummes which make $\frac{1}{7}$, and that is the diuident.

Againe I muſt adde $\frac{1}{8}$, to $\frac{1}{12}$, and it will be $\frac{1}{24}$, that is the Diuiſor. Now I ſhall diuide $\frac{1}{7}$ by $\frac{1}{24}$, and the quotient will be $\frac{24}{7}$, that is, 1 $\frac{17}{7}$: whereby I know that there was put 1 and $\frac{17}{7}$ of ſiluer into the Crowne, and ſo much gold taken out for it.

Maſter. Proue it now by examination according to the queſtion.

Scholler. If there were 1 pound $\frac{1}{7}$ of ſiluer then was there of gold 6 pound $\frac{6}{7}$. Now ſay I by the Rule of proportion:

If eight pound of Gold expell two pound of water, what ſhall 6 pound $\frac{6}{7}$ expell.

8 Z 2 / 6 $\frac{6}{7}$ Z 1 $\frac{5}{7}$

It will be 1 pound $\frac{5}{7}$. Againe,

8 Z 3 $\frac{1}{2}$ / 1 $\frac{7}{12}$ Z $\frac{7}{12}$

If eight pound of ſiluer expell three pound $\frac{1}{2}$ of water, what ſhall 1 $\frac{7}{12}$ expell? It will be $\frac{7}{12}$.

Now muſt I adde together 1 pound $\frac{5}{7}$, and $\frac{7}{12}$, and they will make two pound $\frac{2}{7}$ that is 2 pound $\frac{1}{24}$, according to the ſuppoſition of the queſtion, whereby I perceiue the worke to be well done: And as I cannot but much reioice of this excellent inuention, ſo my deſire is kindled vehemētly to be perfectly inſtructed

in

in euery part thereof, & namely in this point, whether the portion betwéene water & gold be such, that for 8 poũd of gold put into a vessel full of water there shall run out two pound of water, and for as much siluer, whether thrée pound $\frac{1}{3}$ of water would auoid?

Master. I perceiue your meaning, and coniecture your imagination to be thus: that if you knew the exact proportion betwéene gold and siluer, & water both in their waight and quantities, then could you easily finde out the mixtures of them, which thing I haue reserued for another worke that intreateth of such matters especially. And at this time you must consider that you learne Arithmeticke, which intreateth of the maner to solue doubtfull questions touching number, without regard what matter is signified by that number; else were it necessary in Arithmeticke to teach all arts, séeing in it may be moued questions of all arts. But séeing you are so desirous to know this things, I will tell it you in such a sort, that you shall practise your art in finding it, & propound it in forme of a question. Gold beareth greater proportion to water than siluer doth, & their two proportions be in proportion together, as $\frac{48}{25}$. But to helpe you somwhat in this riddle, you shal note that the proportion of quick-siluer vnto water, is the iust middle number proportional in Progression Geometrical, betwéen the proportions of

gold

sold and siluer vnto water. And this proportion is $\frac{125}{41}$. Now if you will know the iust numbers of these two proportions, then must you find out 3 numbers in progression Geometrical, whereof the middlemost must be $\frac{125}{41}$, and the first must be vnto the last, as 25 to 48. And thus I will leaue you to finde those numbers when you be at leasure.

Scholer. Yet Sir, I thanke you hartily for thus much, for now I see the possibility to finde them out. Howbeit, because this question seemeth strange, if it might please you to instruct me somwhat in the order of working for it, I should the more easily find the true working.

Master. You desire too much ease if you will study for nothing: therefore to occasion you to study the better, I will leaue this doubt wholy to your owne search. But as touching the generality of the rule, Archimides needed not to take two masses of gold and siluer equall in weight with the crowne, for the proportion might as well be found in any other waight, yea although the masse of gold were of one waight, and the masse of siluer of another. As for example: if the crowne were of 8 pound waight as I did suppose, and I haue not so much other fine Golde, but onely one pound, and trying that by water and finding that it doth expell but $\frac{1}{2}$ of an ounce of water yet then by it I may inferre, that 8 pound of

Gold

Gold would expell fiue ounces of water. And likewise of siluer: wherof if I had but 2 pound, and finde that it doth expel 3 ounces of water, then might I affirme that 8 pound would expell 12 ounces, that is, one pound waight: and so is it as good as if the 3 masses were al of one waight. And thus for this time I will make an end of this other part of Arithmetike.

Scholar. Although I cannot sufficiently thanke you for this, yet your promise made me to looke for the art of extraction of rootes, whereof hitherto I haue learned nothing.

Master. I will not breake my promise, but intend (God willing) to performe it within these three or foure monthes, if I perceiue this my paines to be well taken in the meane season. And you shall not repent the tarying for it: for it shall be increased by the tarying. And in the meane time, you shall take this Addition, not for the second part of Arithmetike which I promised, but for an augmētation of the first part, vnto which I would haue annexed the extraction of rootes, square & cubike, namely for examples of the Statutes of Assise of wood, but that in the second part I must write of diuers other roots, and thought it best to reserue those Rules also with their examples vnto the same second part.

Scholar. Sir, although I cannot recompence your goodnes, yet I shall alwaies doe mine indeuour to occasion you not to repent your

your benefit on me thus imployed.

Maſter. That recompence is ſufficient for your part.

THE THIRD

*part of Addition to this Booke, en-*treateth of briefe Rules, called Rules of Practiſe, of rare, pleaſant, and commo-*dious effect, abridged into a briefer* Method then hitherto hath beene publiſhed.

With diuers other neceſſarie Rules, Tables, and Queſtions not onely profitable for Merchants, but alſo for Gentlemen, and all other Occupiers *whatſoeuer, as by the con-*tents of this Booke may appeare.

Set forth by IOHN MELLIS Schoole-Maſter.

The first Chapter of Addition

entreateth of briefe Rules, called Rules of Practise, with diuers necessarie Questions, profitable not onely for Merchants, but also for all other occupiers whatsoeuer.

THe working of Multiplication in Practise, is no other thing the a certain maner of multiplying of one kind by another: whereupon is brought forth the product of the proponed number, which is accõplished by the meanes of Diuision in taking the halfe, the third, the fourth, the fift, or such other parts of the sum, which is to be multiplied.

And for the better vnderstanding of such conuersions, you shall vnderstand that in the maner and vse of these rules of Practise, you ought first to know the euen or aliquot parts of a shilling, which in this Table following doth appeare.

Item d	6 4 3 2 1	in the	1/2 1/3 1/4 1/6 1/12

of a s.

Where-

Wherein as you see according to the order of these rules of Practise at 6 d the yard of any thing: You must take ½ of your number which is to be multiplied, & the product that commeth therof shal be shillings, if any vnite doe remaine it is 6 d.

For 4 d take the ⅓ of the number that is to be multiplied, and the product also produceth shillings, if any vnites doth remaine, each one shal be worth in value 4 pence The like is to be vnderstood of the other 3, &c.

Example.

At 6 pence the yard what 379 yards?

189—6 d

II

At 4 pence the yard what 104 yards?

34 s—8 d

III

At 3 pence the yard what 5014 yards?

1253—6 d

IIII

At 2 pence the yard what 522 yards?

88 s—8 d

V

At 1 peny the yard what 409?

34 s—1 d

Where you may see in the first example, that 379 yards at 6 d the yard are worth 189 s—6 d, in taking the ½ of 379. And in the second example the 104 yards at 4 pence the yard, are

worth 34 s 8 d in taking the $\frac{1}{3}$ of 104. Likewise in the third example 5014 yards at 3 d the yard, bringing forth 1253 s 6 pence, in taking the $\frac{1}{4}$ of 5014. Also in the fourth exãple at 2 d the yard, maketh 88 s 8 d.

And lastly in the fifth example: 409 yards at 1 d the yard, amounteth to 34 s 1 peny, in taking the $\frac{1}{12}$ of 409: and so is to be done of all other questions the lik, when the nũber of the pence is any of the euẽ or aliquot parts of 12 d. Item to bring the products of these shillings & all other the like into pounds is very easie in diuiding of it in your minde by 20, for it is to be vnderstood that as often as 20, is found in that product, so many poũds doth it containe: which with facility to performe, alwaies strike off the figure toward your right hand, with a right downe dash of your pen, for the o that appertaineth to the 20. And then begin at the left hand, in taking the $\frac{1}{3}$ of the rest. And if at the last any vnite do remain the same shal be ioined with the figure that is cut off, which shal represent the odde shillings contained in that worke.

As for example, in your third question at 3 d the yard, which amounteth to 1253 s 6 pence, the product whereof maketh 62 l 13 s 6 d, as here you may see, is easily performed by this example.

	1
125	35
62	13 6

Item also for the working of one peny the yard

yard, it is something harsh and hard to take the $\frac{1}{11}$ of some products: therefore to ease that hard worke, you shall first bring your deliuered sum into groats, by taking the $\frac{1}{4}$ part of the product, & if any vnits remaine of that $\frac{1}{4}$ part, as sometimes there may, they are pence: and must bee signified with a line from the groates with their title of pence; and because that 60 groats maketh a pound or twenty s, strike off the first figure toward your right hand, for the o that appertaineth to 60 (as you did euen now for the o that belongeth to 20:) the in taking the $\frac{1}{4}$ of that product, if there do remaine any vnits, the same shall you ioine with the figure that you cut off, esteeming the as groats, which keepe in your minde, and by taking the $\frac{1}{3}$ part of them, you shal turne into shillings: and so haue you done; as for example by a question or two hereafter proponed shal more plainly by the worke appeare:

At 1 d the yard what 54368 yards?

1359 | 2 groates.

$\frac{1}{4}$ 226 10 s 8 d.

Here in taking the $\frac{1}{4}$ part of 1359: in comming to the last worke, the $\frac{1}{3}$ part of 39 being taken, the remainder is 3, which ioined with the two that was cut off, maketh 32 groats, which conuerted into shillings by taking the $\frac{1}{3}$ part, maketh as appeareth ten s 8 d. Many other waies there are, but none

more apt for a yong learner to vnderstand the this: wherefore this one way well impressed in memory is better then 20 waies doubtfully vnderstood.

At 1 peny the yard, what 4533 yards?

113 | 3 groats—1 d

1 18——17——9 d

At 1 peny the yard, what 64768 yards?

1619 | 2 groates.

1——299——17——4 d.

NOw followeth also to be vnderstood, that if the number of pence be not an aliquot part of 12, you must reduce them into some aliquot part of 12: and after the aforesaid maner, you shall make of them two or three products, as nede shall require, and adde the together into one summe. And here for thy furtherance appeareth a note of the order of their parts, as they are to be taken.

For pence	5. take. 3 and 2	or	4 and 1
	7. take. 4 and 3		6 and 1
	8. take. 4 and 4		6 and 2
	9. take. 6 and 3		4. 4. and 1
	10 take 6 and 4		4. 4. and 2
	11 take. 6. 4. & 1		4. 4. and 3

Here is the first note of this Table at 5 d, you shall first take for 3 d the ½ of the number that

that is to be multiplied: and likewise for 2 d the $\frac{1}{6}$ of the same number, adding together both the product s: But if you will worke by 4 and 1, you must for 4 d first take the $\frac{1}{3}$ of the number that is to be multiplied: and for 1 d take the $\frac{1}{12}$ of the whole sum, or rather, which is more better, for 1 peny you may take the $\frac{1}{2}$ of the product which did come of the 4 pence: because that 1 d is the $\frac{1}{4}$ of 4 pence. The totall sums of these two numbers shall be the solution to the question. And in like maner is to be done of all others, as by those examples following shall appeare.

I

At 5 pence the yard what 758 yards?

3 d	189—6 d
2 d	126—4 d
	315—10 d

Otherwise.

At 5 pence the yard what 758 yards?

4 d	252—8 d
1 d	63—2 d
	315 s—10 d

II

At 7 pence the elle what 763 Elles?

4 d	187—8 d
3 d	140—9 d
	328 s—5 d

III

At 8 d the pound what 112 pound?

4 d	37——4 d
4 s	37——4 d
	74 s—— 8 d

Otherwise.

At 8 pence the poũd what 112 pounds?

6 d	56—0
2 d	18—8
	74 s—8 d

1111

At 9 pence the Elle what 356 elles?

6 d	178—0
3 d	89—0
	267 s—0 d

V

At 10 pẽce the pẽce what 795 pẽces?

6 d	397—6
4 d	265—0
	662—6

VI.

At 11 d the pound what 7576 pounds?

6 d	3788——a
4 d	2525——4
1 d	631——4
	6944—8 d
	347-4s-8d

Here in this first example where it is demaunded (at 5 d the yard) what will 758 cost? First of 3 d ℔ I take the $\frac{1}{4}$ of 758: thereof commeth 189 s 6 d: then for 2 d ℔ I take the $\frac{1}{6}$ of the same product, which amounteth to 126 s 4 d, these two sums added together, doe make 315 shilling 10 d: and so much are the 758 yards worth at 5 d the yard.

Item also for the same againe; First for 4 d ℔ I take the $\frac{1}{3}$ of 758: and therefore commeth 252 s 8 d: then for 1 penp ℔ I take the $\frac{1}{4}$ of the same product: that is to say, of 252 s 8 d, and it yeldeth me 63 s 2 d: which both added together

gether maketh 315 s—— 18 d, as before.

Item for 7 d there is taken the $\frac{1}{7}$ and the $\frac{1}{4}$ of the whole sum which is to be multiplied, and adde them together, that is to say, first for 4 pence there is taken $\frac{1}{7}$ of 563: which comes to 187 s—8 d, as appeareth by the worke: and for 3 d there is taken the $\frac{1}{4}$ of the whole sum, which amounteth to 140 s—9 d. Both with products added together, maketh 328 s—5 d: and so much comes 563 elles to at 7 d the elle.

Item for the first 8 d there is taken for 4 d, the $\frac{1}{3}$ of the whole summe, and another $\frac{1}{7}$ for the other 4 d, which added together, as in the example doth evidently appeare, amounteth to 74 s—8 d.

Againe, for the second worke of 112 l, there is taken first the $\frac{1}{2}$ of the whole summe for 6 d which comes to 56: s then for that 2 d you have to take $\frac{1}{7}$ of the whole summe, or if you will, the $\frac{1}{7}$ of the product that came of 6 d, either of which maketh 18 s—8 d. These two summes being added together, doe make 74 s 8 d, as in the third example appeareth.

Item, for 9 d there is taken for 6 pence the $\frac{1}{7}$ of the whole summe: & the $\frac{1}{4}$ of the whole sum for 3 d, or otherwise for the 3 d you may take the $\frac{1}{2}$ of the product that came of 6 d because 3 pence is the $\frac{1}{2}$ of 6 d: which added together, as plainly appeareth in the fourth example, amounteth to 267 s 0 d.

Item

Item for 10 d, first there is taken for 6 d the $\frac{1}{2}$ of the whole sum, which amounteth to 397 s–6 d. Then for 4 d there is found 265 s both which added together, make 662 shillings 6 d, as appeareth in the fift example. It may also bee wrought, as appeareth by the second note in the table, by 4 d twice taken, and the $\frac{1}{2}$ of the product of 4 pence: or else by the $\frac{1}{6}$ of the whole sum, &c.

Item, for 11 d there is first taken the $\frac{1}{2}$ for 6 pence: then the $\frac{1}{3}$ of the whole sum for 4 d: lastly, that $\frac{1}{4}$ of the last product for 1 d. All which 3 summes added together, maketh in s 6944 8 d, and in pounds 347 4 s 8 d.

3 Rule.

ITem likwise by the same reason when you will multiply (by shillings) any number that is under 20 s, you shall haue in the product pounds, if you know the euen or aliquot parts of 20, which are here in this little Table set downe to sight.

Item s.		is the		of one pound.
	10		$\frac{1}{2}$	
	5		$\frac{1}{4}$	
	4		$\frac{1}{5}$	
	2		$\frac{1}{10}$	
	1		$\frac{1}{20}$	

So that for 10 s, which is the $\frac{1}{2}$ of a pound, you may take the $\frac{1}{2}$ of the number which is to be multiplied, and you shal haue in your product

duct pounds: if an vnite doe remaine, it shall be worth ten shillings.

Likewise for 5 shillings you must take the $\frac{1}{20}$ of the number which is to be multiplied: and if there doe remaine any vnites, they shall bee fourth parts of a pound, euery vnite being in value 5 shillings.

For 4 s take the $\frac{1}{5}$ of the number which is to be multiplied: and if there do remaine any vnites, they shall be fift parts of a pound, each vnite being in value 5 shillings.

For 2 shillings you must take the $\frac{1}{10}$ of the number to be multiplied wherfore to take the $\frac{1}{10}$ of any number, you must cut off the last figure of the same nũber (which is neareſt your right hand) from all the other figures with a small right downe line or dash with a pen, & so haue you done: for all the other figures which doe remaine toward your left hand from the same figure that you doe separate, shall be the said $\frac{1}{10}$ of a pound: and that figure so separated towards your right hand, shal be so many pẽces of 2 s the pẽce: the which figure you must double to make thereof the true number of s, as by the example shall appeare.

Finally, for 1 shilling needeth small worke, for it is so many shillings as be proponed in the summe, which to bring into pounds hath bẽn already taught in the first rule.

Example.

At

At 10 s the pæce
What are 6543 pæces worth?

½ l 3267 —— 10 s

At 5 s the Elle
What comes 4373 — Elles to?

¼ l —— 1093 —— 5 s

At 4 s the yard
What —— 7839 yards?

⅓ l —— 1567 —— 66 s

At 2 s the pound waight
What are —— 752|7 pound waight worth?

⅒ l —— 752 —— 14 s

At 1 s the pæce
What comes —— 757|3 pæces 2?

¼ l — 378 —— 13 s.

NExtly, now followeth (in order to be understood, that if the number of shillings be not some euen or odde aliquot part of 20, you must then conuert the same number of shillings into the aliquot parts of twēty, and thereof make two or three products, as nēde shall require: which done, adde them together, and bring them into pounds. And here for thy furtherance I haue set downe a note of the order of their parts, as they are to bee taken.

For

s				s		
3		2 & 1		13		10. 2. & 1.
6		4 & 2	5. & 1.	14		10. & 4
7		5 & 2		15		10. & 5
8	of	4 & 4	02 5.2.1	16	of	10 5.1
9		5 & 4	4.4.1	17		10.5.2
11		10 & 1		18		10.4.4
12		10 & 2		19		10.5.4

For 3 s according to the tenor that you see is expressed in the Table, you must first take for 2 shillings the $\frac{1}{2}$ of the number that is to be multiplied. Then for 1 shilling you must take the $\frac{1}{3}$ of the product which did come of the same $\frac{1}{10}$. part: which two sums added together produceth the effect desired.

Item, for 6 shillings according to the note set forth in the Table, first for 4 s I take the $\frac{1}{2}$ of the number that is to be multiplied: Then for 2 s the $\frac{1}{3}$ of the product that came of 4 s, & adde them together.

Or else, as appeareth also in the Table, for 5 shillings you may take the $\frac{1}{2}$ & the $\frac{1}{10}$ part of the product that came of 5 shillings, and adde them together.

Item, for 7 s, first take for 5 s take $\frac{1}{4}$ of the product that is to be multiplied, then for 2 s, take the $\frac{1}{10}$ of the number that is to be multiplied, and adde them together, &c.

Item, for 8 s, according to reason, and the intent of the Table, for the first 4 s take the

$\frac{1}{2}$ of

$\frac{1}{2}$ of the product, & the same number again for the other 4 shillings: and adde them together.

Item, for 9 shillings: first for 5 shillings take the $\frac{1}{4}$: then for 4 shillings take the $\frac{1}{5}$: and adde them together.

Otherwise as you see by ỹ intent of ỹ table, worke twise for 4 shillings, as was taught even now for 8: & then take the $\frac{1}{4}$ of the last product for the 1 shilling. But 5 & 4 is the shorter

Item for 11 s, first dispatch 10 s: for which you must take the $\frac{1}{2}$ of the product, then lastly for 1 shilling take the $\frac{1}{10}$ part of the sum produced of the $\frac{1}{2}$ of the product & adde thē together.

Item, for 12 shillings where I wil end with the first part of my table. I first take ỹ $\frac{1}{2}$ for 10 shillings. And then for 2 shillings take the $\frac{1}{5}$ of the sum that came of 10 s, take and adde them together: or else if you please, for 2 shillings you may take the $\frac{1}{10}$ of the whole given number.

To write more of the maner of taking the true parts, I omit. The desirous practitioners wil (no doubt) conceiue it. Also the Table is some aide to helpe the vnperfect, whereupon by & by I will set down three or foure of these notes in examples, and the rest I will leaue to thine own industry & practise to labor vpon. This is the order most commonly vsed in practise, when the number of shillings is not an aliquot part of a pound. But (louing Reader) after I haue touched the euen or aliquot parts

of

of a pound that falleth out in pence and shillings, I will deliuer two new rules that shall drowne this common order quite and cleane: wherein shall be comprehended in one line or working both euen & odde parts of shillings vnder 20 without regard whether it be an aliquot or not an aliquot part: which two rules when they come in place, I commit to thy friendly iudgement in working.

Now follow the examples vpon the notes before said.

At 6 shillings the yard.

What— 3215 —yards?
4 shillings 643
2 shillings 321—10
£964--10 shillings

Otherwise by Multiplication of 6.

3215
6 shillings 19290
£—964—10 shillings.

At 7 shillings the Elle

What——4563 Elles?
5 shillings 1140—15
2 shillings 456—6
£ 1597—1 shilling.

Other

Otherwiſe by multiplication of 7.

4. 563

7s	3194|1

At 8 s the pèce, what 7563 pèces?

4 s	1512. . 12
4 s	1512. . 12
	l 3025 . . 4 s

Otherwiſe by multiplication.

7563

8s	6050|4
	l 3025 4 s

At 13 s the pèce, what 401 pèces?

10 s	200 . . 10
2 s	40 . . 2
1 s	l 260 . . 13

Otherwiſe by multiplication.

401

136	1203
	401
	521|3
	l 260 13 s.

These and such like Questions of compound numbers, which I haue here in this fourth rule for orders sake set downe, for that it hath been heretofore a common course of worke, I account but superfluous. For in the eigth and ninth Chapters of this my simple Addition shall appeare, that the giuen price of any euen or odde number of shillings either vnder or aboue 20. shall be wrought at one or two workings at the most, how difficult soeuer the Question be.

ITem, there resteth yet a kind of Practise, how to bring pence into pounds at the first working: whereupon you must understand, that 240 pence maketh one pound, or 20 s. In consideration whereof I cut off the last figure or 0, and there remaineth but 24 (of which 24) 8 d is the $\frac{1}{3}$ part thereof, 6 d is the $\frac{1}{4}$ part, 4 d the $\frac{1}{6}$ part: & 2 pence is the $\frac{1}{12}$ part thereof.

Whereupon if it were demaunded what 1486 yards or pounds of any thing commeth to, at 8 pence the yard, in picking or cutting off the first figure towards your right hand, for the 0 that appertaineth to 249: There is remaining of the said summe 148, whereout I taking the $\frac{1}{3}$ part, and it commeth to 49 l, and there resteth 1, which 1 I put to the 6, that I picke or cut off, and it maketh 16. peeces of 8 pence, which I double to make into groates,

and they make 32, whereof the $\frac{1}{4}$ part maketh 10 s, and there remaineth $\frac{1}{4}$ s, which is 8 pence, whereby it followeth, that the 1486 yards at 8 pence the yard, maketh 49 l 10 s 8 d, as by the example shall appeare.

Item for 6. pence, take $\frac{1}{4}$ part of the number from the prickt figure: and if any vnites doe remaine they are so many fiue pences, whereof taking the $\frac{1}{5}$ they are shillings, if there doe remaine yet one, it is in value fiue pence.

Item for 4 pence, take the $\frac{1}{4}$ part of the number from the prickt figure: If any vnites remaine, they are so many groats, which to conuert into shillings, take the $\frac{1}{3}$ part: And if any yet thing remaine, they are thirds of shillings, each one in value being worth 4. pence.

Item for 3 pence, take the $\frac{1}{4}$ part from the prickt figure, if any vnites remaine, they are so many peeces of 3 pence, whereof in taking the $\frac{1}{4}$ part, maketh shillings: If any thing yet remain, they are fourth parts of shillings each one being in value 3 pence.

Item for 2 pence, as appeareth also by the table, take the $\frac{1}{12}$ part of the number from the prickt figure: if any thing remaine, they are so many peeces of 2 pence, which by taking $\frac{1}{6}$ part, you shall turne into shillings: and if any vnites remain, they are so many first parts of shillings, or peeces of 2 d, whether you will.

If

If one cost 8 pence,
What are 148|6 worth?

maketh pounds 49—10———6 pence

If one cost 6 pence,
What 786|5 worth?

maketh pounds 196——12—6 pence

At 4 pence the yard
What are 873|6 yards worth?

maketh pounds 145—12—0 D

If one cost 3 pence,
What are 987|4 worth?

maketh pounds 123—8—6 pence,

At 2 pence the Elle
What comes 780|4 Elles to?

maketh pounds 65—15—8 D

But if your number of pence be not an aliquot or even part of 24: then must you bring them into the aliquot parts of 24, and make thereof diuers products which must be added together, as by the question hereafter following shall appeare.

Item for 5 D, first take for 3 D, then for 2 D, and adde them together according to the instruction of the second rule: Or else first take

for 4 pence, then for one pennie:

Item for 7 d, first take for 4 d: then for 3 d, and adde them together.

Item for 9 d, first take for 6 d, then for 3 d, and adde them together.

Item for 10 d, first take for 6 d, then for 4 d, and adde them together.

Item for 11 d, first take for 8 d, then for 3 d, and adde them together: as by these examples following both appeare.

Example.

If one yard cost 5 pence,
What are 759|6 worth?

4 pence	126	12
1 penie	31	13
maketh pounds	158	5 s

Otherwise.

1 5 759|6

3 pence	94	19
2 pence	63	6
maketh pounds	158	5 s

If one cost 7 pence,
What are 98|7 worth?

4 pence	16	9	
3 pence	12	6	9
maketh pounds	28	15	9 pence.

Other-

Otherwise.

	1	7	98|7
6 pence	24	13	6
1 peny	4	2	3
maketh pounds	28	15	9 pence,

If one coſt 9 pence,
What are 78|9 worth,

6 pence	24	13	6
3 pence	12	6	9
maketh pounds	37	0	3

Otherwiſe.

1—9—987

3 pence	12	0	9
maketh pounds	37	0	3

If one coſt 10 pence,
What are 98|7 worth?

6 pence	24	13	6
4 pence	16	9	0
maketh pounds	41	2	6

If one coſt 11 pence,
What are 98|7 worth?

8 pence	32	18	0
3 pence	12	6	9
maketh pounds	45	4	9

But if you haue any ſhillings and pence to be multiplied together: Then are you to take for the ſhillings according to the inſtructiō of the third rule. And for the pence according to the firſt rule before mentioned: vnleſſe you can ſpie the aduantage thereof, and thereby helpe your ſelfe: as appeareth in this ſecond example, where firſt I worke for 6 d: which is to be rebated out of the giuen number, and I haue 7 19 l. 11 s my deſire.

At 19 s —— 6 d the yard
what 738 yards?

	738	
10 s	369	
5 s	184 —	10
4 s	147 —	12
6 d	18 —	9
l.	719 —	11 s

Otherwiſe by Rebating.

	738	
6 d	18	— 9 s
l.	719 —	11 s

The like againe is done by rebating, as by theſe 2 examples appeareth.

At 18 s the Elle what 418 Elles?			At 16 s the Elle what 517 Elles.		
2 s	41	16	4 s	103	8
l.	376	48	l.	413	12 s

And

And now I will touch a little the euen parts of a pound, that falleth out in pence and shillings, whereof for those parts you shall take such like part of the giuen number that is to be multiplied, as the price of that giuen number beareth in proportion to a pound, which also for their better aide is here set downe.

1 s	8 d		
2	6	is the	part of a l.
3	4		
6	8		

Item, first for 1 shilling 8 pence take the $\frac{1}{12}$ part of the giuen number, and if any thing do remain, they are twelue parts of a pound, each one being in value 1 shilling 8 pence.

Item for 2 shillings 6 pence take the $\frac{1}{8}$ part of the number that is to be multiplied. And if any thing do remaine, they are eight parts of a pound, each one being in value 2 shillings 6 pence.

Item for 3 shillings 4 pence, as appeareth by the Table, you must take the $\frac{1}{6}$ part of the giuen number, and if any thing doe remaine, they are 6 parts of a pound, each one being in value 3 shillings 4 pence.

Item for 6 shillings 8 pence take the $\frac{1}{3}$ part of the number that is to be multiplied: And if any vnites doe remaine, they are thirds of

a pound, euery one being worth 6 shillings 8 pences:

Other infinite numbers there are, that may be reduced by abbreuiation into the proportionate parts of a pound: as 16 shillings 8 pence maketh ½: which 16 shillings 8 pence is easily reduced into groats by multiplying 16 by 3, and thereto adde 2, which maketh 50 groats: Then set 60 the groats of a pound vnder 50, cutting off the 2 ciphers, as is here performed in the margent. And then haue you brought 16 shillings — 8 pence into the knowne parts of a pound, which maketh ⅚:

16—8
3
50
60

But yet gentle Reader, for thy further instruction, I haue hereunto annexed in a Table, how pence and shillings beareth proportion to a pound, which I commit to thy friendly benevolence; it will be some aide vnto the vngrounded practitioner: but I count him the best workeman that can presently reduce his giuen price into the knowne and proportionate parts of a pound.

Here follow 4 examples vpon the 4 notes deliuered.

At 1 s 8 d the yard
What are 3884 yeards worth?

maketh pounds—323—13—4 pence.

At 2 s—6 d the yard
What are 4563 yards worth?

maketh pounds 570——7——6 pence.

At 6 s—8 pence the Elle
What comes 7562 Elles to?

maketh pounds 2520——13——4 pence.

Now by custome you are able to worke by all sorts of summes being deliuered in shillings and pence, as one shilling one peny, 2 shillings two pence, 3 shillings 3 pence, and so of all other: wishing you to haue some considerations of your questions, when they are set downe, for there are many subtill abbreuiations, and great aduantages to be gotten, & easily to be perceiued.

As 3 s—8 d of 2 s and 1 s 8 d.

4 s—2 d, of 3 s—4 d, and 10 d, which 10 is $\frac{1}{4}$ of 3 s—4 pence.

5 s—8 d of 4 s—1 s 8 d.

5 s 10 d, of 5 s and 10 d, which 10 d is $\frac{1}{2}$ of 5 shillings.

And by this meane when you haue taken one product, you may oftentimes vpon the same

same take another more briefly then vpon the sum which is to be multiplied, &c.

NOw (gentle Reader) that you haue seene the vertue of the euen or aliquot parts of a pound in shillings alone, & also in the aliquot parts of shillings and pence: according to my promise hereafter following a briefer and easier method for any euen number of shillings, either vnder or aboue 20, then euer yet hath bene published: Notwithstanding Maister Humfrey Baker, whose trauel is worthy commendation, and whom for knowledge sake I reuerence, hath in some part touched this first part, though not in this method. The worke of the rule is both pleasant, ready and briefe, as by the variety of the examples deliuered thereupon shall appeare. And first I will set forth a question, thereby the better to expresse or teach you the order thereof: which is this.

If one cost 6 s, what 8574?

1——6——8574

maketh pounds——2572——4 s

To the vnderstanding of this example, after you haue set downe your giuen number in forme of the rule of 3, with a line drawne vnder it, you shall presently sent a pricke vnder your first figure 4, towards your right hand, draw-

drawing from the pricke, as heretofore hath bene practised, a little short line, thereto set downe the shillings anon, which done, multiply the first figure 4 by 6, the value of your price, (which here you see standeth in sight aboue the line) it maketh 24. which is 1 pound 4 shillings. The 1 pound kepe to carry to the next place, and the foure shillings set downe at the end of the prescribed line towards your right hand. Thus haue you done now with fire aboue the line, and also with 4 in the first place (for the pricke vnder foure doth represent that 4 hath done his office.) Then secondarily for a generall rule take but the $\frac{1}{3}$ of the giuen price which here is 3, which 3 is the number that shal now continue the rest of the multiplication and end the worke, whereupon I multiply three into seuen, standing in the second place it maketh 21, and with the one I kept in minde 22; set downe 2 and kepe 2 in mind working according to the rule of multiplicatiō, deliuering the tents in mind in their due place, which done, the product from the pricke to your left hād representeth the pounds & the other at the end of the line the shillings, as appeareth by the examples.

If one yard cost 2 s, what 7536?

1 2 7563

maketh pounds ——— 753 ——— 12 s.

If

If one yard cost 4 s, what 8792?
1 4 8792

maketh pounds 1758 8 s

If one peece cost 6 s, what 9537
1 6 9537

maketh pounds 2861 2 s

If one cost 8 s, what 7509 ?
1 8 7509

maketh pounds 3003 12 s

If one cost 12 what 5794
1 12 5794

maketh pounds 2593 10 s

If one cost 14 s what 37052
1 14 30705

maketh pounds 3476 8 s

If one cost 18 s, what 57032
1 18 5703

maketh pounes 5132 14 s

If one cost 22, s what 9532
1 22 953

maketh pounds 1048 6 s

Let

Let these suffice (gentle Reader) for an entrance into euen numbers. And now I will shew the like rule for any odde or vneuen part of a pound.

TO helpe you to the vnderstãding of these other questions that hereafter follow: where in my first example the giuen number is 6487. at 3 shillings the yard: I multiply 3 aboue the line into 7, it maketh 21. The one shilling is set downe, & the 1 I keepe. Now am I to take the ½ of three, which because it is an odde number I cannot. Therefore I shall keepe and continue my multiplication by three still, and worke by the ½ of the rest of the giuen figures or number, to wit, 648. And first the ½ of 8 which is 4. multiplied into three maketh 12, thereto ioyne the 1 I in minde, it maketh 13: set downe 3, keepe one. Then againe multiply by two the ½ of foure, it maketh fiue, and with one in minde it maketh 7. Then lastly take the ½ of five, which is three, saying 3 times three is nine, which nine set downe, & so is the question answered, as appeareth by the practise, and the examples following.

At 3 s the yard what 6487 ?

1 - 2 6487

maketh pounds— 973—15

If one yard cost 5 s, what 4269?
1 5 4269

maketh pounds 1067 5 s

At 7 s the elle, what 6489?
1 7 6489

maketh pounds 2271 3 s

If one elle cost 9 s, what 2807?
1 9 2807

maketh pounds 1263 3 s

If 11 s the Pistolet, what 8263?
1 11 8263

maketh pounds 4544 13 s

If one perce cost 13 s, what 4629
1 13 4629

maketh pounds 3008 17 s

But now note (gentle Reader) when the giuen price falleth vpon an odde number, as 3,5,7,11,13, &c. then it is to be presupposed that the giuen summe to be multiplyed must be a summe made of euen numbers, as 2,4,6 8,10, &c. els cannot that question be wrought at one line of working.

Prouiding alwaies that it may beare an odde figure in the first place towards your right hand, as appeareth in these 6 examples, which last were wrought, and such like, &c.

which

which may beare an odde number for the price, and be done at one line or working very well.

But if the giuen price be an odde number, and the summe to be multiplied odde numbers also: then can it not be done at one working, but requireth the aid of two workings, for odde with odde will not agree, which notwithstanding to bring to passe, take this for a generall rule. First worke for the euen number, contained in that question: or giuen price according as you haue learned, and then afterwards for the 1 odde shilling, take the $\frac{1}{2}$ of the summe giuen to be multiplied, omitted the first prickt place, as was taught for the working of one shilling in my first rule of Practise, and adde those 2 together, and you shall haue your desire.

Example.

At 3 s the yard

What are 7539 yeards worth?

2 s	753 — —	18
1 s	376 — —	19
maketh pounds	1130 —	17 s

At 7 s the elle, what 7539?

1	7	7539
6 s		2261 — — 14
1 s		376 — — 19
maketh pounds		2638 — 13 s

At

At 13 s the yard, what 7534?

1 —— 13 —— 75534

1 2 s	4520—8
1 s	376—14
maketh pounds	4897—2

And thus haue I abridged into these two rules, how to bring any number of shillings whatsoeuer they be into pounds, with a briefer method then euer yet hath bene published, which I commend vnto thy friendly censure and iudgement in the vse & practise thereof.

If one coll 6 s 5 d, what 1231?

1 —— 6 s —— 5 d —— 1231

6 s	369—6
4 d	20—10—4
1 d	5—2—7
maketh pounds	394—18—11

At 14 s 2 d, what 2825?

1 14 2 2825

14 s	1977—10
2 d	23—10—10
maketh pounds	2001—00—10

At 16 s 4 d, what 2531

1 16 4 2531

16 s	2024—16
4 d	42—3—8 d
maketh pounds	2066—19—8

At 3 s the Pistolet, what 8325?

1	3	8325
maketh pounds—	1248—	15 s

At 7 s the crowne, what 6529?

1	7	6529
maketh pounds—	2285—	3 s

At 9 s the peece, what 6567?

1	9	6567
maketh pounds—	2955	3 s

These three last questions may seeme something hard, yet they are easie enough if you marke them well, if I should explaine them, then are they too easie: therefore I leave them to whet the minds of the desirous.

Item, when any one of the summes which is to be multiplied, is composed of many denominations, and the given number but of one figure alone: then shall you multiply all the denominations of the other sum by the same one figure, begining first with that sum which is least in value towards your right hand, & bring the product of those pence into shillings, and the product of the shillings into pounds, as by this example appeareth.

At 3 l—70—5 s—4 d, what are 9 worth?

maketh pounds—30—6 s—0 d

But if in any of the summes that are to be multiplied there be a broken number. First worke

work for the whole according to the instructions that you haue learned, and then take such part of the giuen price, as that broken number beareth in proportion to the price, as in the example. After you haue wrought for 3 s and for 6 d, then are you to take the ½ of 3 s 6 d for the ½ yard, and adde that to the sum: so adding all 3 products together, which maketh 43 l. 2 s 9 d, the iust price of 246 ½ elles: and thus must you doe of al other:

At 3 s 6 d the elle, what 248 ½

1	3	6	246 ½
3 s			36 — 18 —
6 s			6 — 3 —
½			1 — 9
maketh			43 — 2 — 9

At 16 s — 4 d the pece, what 14 ¼

1	91	4	14 ¼
16 s			11 — 4 —
2 d			0 — 4 — 8
¼			12 — 3
maketh pound			12 — 00 — 11

If one pece cost 4 l. — 3 — 6 ½ d
what are 12 peces worth?

4			48 — 0 — 0
3 s			1 — 16 — 0
6 d			0 — 6 — 0
½			0 — 0 — 6
maketh pounds			50 — 2 — 6 d

The proofe.

If 12 peeces coſt 50 l. 2 s 6 d
What one peece?

50——2——6

maketh pounds 4——3——6 1/2

Item, touching the manner how to vnderſtand the order of this queſtion, and others the like, firſt ſeek how many times 12 is contained in 50 which is 4 times, and ſo reſteth 2 pound, which 2 pound conuerted into ſhillings, and ioyned with the other 2 ſhillings, maketh 42 ſhillings, wherein is found 12. 3 times: reſteth 6 ſhillings which turned into pence, putting thereto the 6 d in the firſt place, it maketh 78: wherein 12 is found 6 times: reſteth 6 d, which containeth 12, but 1/2 a time: put that 1/2 to the 6 d: and then the ſolution is 4 l. 3 s 6 1/2, as appeareth by the practiſe thereof.

Item, the like is to be done of any thing that is bought or ſold after fiue ſcore to the hundreth, or the Quintall. As for example.

If

If 100 pound coſt 27 l. 13 s 4 d what one pound?

27	13	4
	20	
s	5	53
		12
	1	10
	5	3
d	6	4 0
	10	0 3/7

maketh 5 s 6 3/7 d,
I haue wrought this at length for the aide of the yong learner, becauſe hee ſhould vnderſtand how all the multiplication is ſet downe.

27	13	4
	20	
s	5	53
		12
d	5	40
		100 2/7

maketh 5 s 6 3/7 d
But to worke it more neately, it is by a little vnderſtanding ended thus.

Item, to the vnderſtanding of this and ſuch like queſtions, the right downe line is all the guide. which is pulled downe cloſe by 20 as you ſee in the example, where 27 pound 13 ſhillings is reduced all into ſhillings, and maketh 553 ſhillings.

The 5 towards the left hand being ſepa-

rated with the hanging or right downe line, is the iust number of shillings that answereth to the question. Nertly, 53 shillings is multiplied by 12 to reduce them to pence, putting to the 4 d, it yeldeth for the multiplication of the first figure two, 100: the one beyond the line towards the left hand, is 1 peny towards the rest of the price: then 53 also multiplied by 1 yeldeth 53: but the 5 behinde the line toward the left hand, is also 5 pence more, towards the price, which 1 and 5 I adde together vnder the line: it maketh 6 d. So is there found now as appeareth by the Titles of shillings and pence, 5 shillings 6 pence.

Finally, I come now on this side the line towards the right hand, & vnder 12 I finde first 10, & then 3, which added together maketh 40, vnder which 40 you must put the 100, and it maketh $\frac{40}{100}$, which abbreuiated commeth to $\frac{2}{7}$. So the iust price of one pound after 5 score to the hundreth maketh 5 s 6 d. $\frac{2}{7}$ d.

One example more, and so I will leaue this this rule.

If 100 cost $10\frac{3}{4}$ d

What 9874?

100 — $10\frac{3}{4}$ — 9874.

6 d	246	17	0	
4 d	164	11	4	
$\frac{3}{4}$ d	20	11	5	
$\frac{3}{4}$ d	10	5	8	$\frac{7}{8}$
l 4	42	5		
	20			
s 8	45			
	12			
d 5	$45\frac{3}{4}$			

maketh

$\frac{91}{100}$ parts of a d.

100

Also the like may bee done of the vsuall weights here in England (which is 112. for euery hundreth weight) in case you know the aliquot parts of a hundreth weight, which are these, 56 l, 28 l, 14 l, and 7 l: For 56 l, is the $\frac{1}{2}$ 112. pound 28 l, is the $\frac{1}{4}$ of 112 l, 14 l is the $\frac{1}{8}$ and 7 l is $\frac{1}{16}$ part.

Therefore for 56 l, take the $\frac{1}{2}$ of the summe of money that 112 weight is worth.

For 28 l, take the $\frac{1}{4}$ of the summe of money that 112 l. weight is worth.

For 14 l, take the $\frac{1}{8}$ of the summe that 112 l, is worth.

And for 7 l, the $\frac{1}{16}$ of the summe of money that C is worth.

As for example: At 17. l 19 s the hundreth

pounds weight, that is to say, the 112. pound what shall 3. quarters and 7. pound cost?

1	17	19	3	7
2 quarterns	8	19	6	
1 quarterne	4	9	9	
7 pounds	1	2	$5\frac{1}{2}$	
maketh pounds	14	11	$8\frac{1}{4}$	

The second Chapter intreateth of the Reduction of diuers measures to others value by Rules of Practise.

NOw will I shew a few examples of practise in reducing of measures, as Elles, yards, Braces, Pawnes of Genes, &c. Much more would I haue touched, but that I feare the booke will rise to too great a volume.

In 864. elles of Antwerpe, how many yards of London?

864	864
432	216
216	648

maketh 648 —— yards of London.

Item in these and such like questions of Flemmish measure to bee brought into yards english: still take the $\frac{7}{9}$ of the giuen number,

as appeareth in the first example towardes your left hand. Then take the $\frac{1}{3}$ of that product, or the $\frac{1}{3}$ of the giuen number, and adde those 2 products together, they shall be yards English, as by the example you may perceiue.

The second example towards your right hand is yet briefer then the first, whose worke is this: take the $\frac{1}{3}$ of the deliuered number, and that product subtract out of the giuen number, and the rest sheweth your desire. Of these two wayes vse which you thinke best.

The Proofe.

In 648 yards of London.
How many elles of Antwerpe.

648

216

maketh 864 Elles of Antwerpe.

Item for the vnderstanding of this worke: first take the $\frac{1}{4}$ part of the yards of London, which found, adde that $\frac{1}{4}$ part and the yards together, as apeareth by the practice, and the product sheweth the elles of Antwerpe.

In 320 yards of London,
How many elles of Antwerpe,

maketh

maketh 426 2/7 Elles.

320 yards	Proofe.
1/3 106	426 2/7 Elles
426 2/7	106 1/3
	320 yards

Other Reductions.

Item, you shal vnderstand, that forasmuch as fiue braces of Hillane make fiue Elles of Antwerpe, wherupon according to the Rules of Practice, you may reduce the one into the other by the like reasons aforesaid, in taking the 1/2 part, & then subtract the same to make Elles of Antwerpe. And againe by the contrary, taking the 1/7 part with adding the giuen number, to turne the elles to Braces, as for example,

In 876 Braces how many els of Antwerp.

876 Braces	The contrary.
(146	730 Els Flem.
Els 730 Antwerpe	146
	876 Braces.
Els 730 Antwerpe.	

182 1/3

Yards 547 1/3 English.

Thus

Thus appeareth, that 876 Braces by practice, make 730 Elles Flemmish: which elles Flemmish reduce into English yards.

So againe vpon the same first question of Braces, I would know how many yardes English they make.

After the rate that 100 Braces are worth 62 $\frac{1}{2}$ yardes.

876 Braces.
438
109 $\frac{1}{2}$
I answere 547 $\frac{1}{2}$ yards.

Item, to the vnderstanding of this worke, and such like, first take the $\frac{1}{2}$ of the giuen braces, and after take the $\frac{1}{2}$ of that halfe, or the $\frac{1}{4}$ of the giuen number and adde them together, and the products are also yards English.

Item three elles of Rochell make 5 elles at Lisbone. So likewise three elles at Lions make 5 elles at Antwerpe.

To worke these and such like, double the elles of Lions, and the elles of Rochell, and from their products subtract the $\frac{1}{7}$: and the rest shall be the elles of Antwerp, or the elles of Lisbone.

Example,

Example.

In 63 elles of Lions how many elles of Antwerpe?

63	
63	
126	
1/6 21	

Answ. 105 elles Ant.

In 100 Elles of Rochel, how many Elles of Lisbone?

100	
200	
2/3)	33 1/3

Ans. 166 1/3 els of Lisb.

Touching the proof or returne of these and such like questions (as a generall rule), you shall first take the 1/3 of the giuen number: and adde that 1/3 and the giuen number together, and the 1/3 of that product shall be your desire.

Example.

In 105 elles of Antwerpe, how many elles of Lyons?

105	
1/3 21	
126	

In 166 2/3 elles of Lisbone, how many els of Rochell?

166 2/3	
1/3)	33 1/3
	200

An 63 els of Lions. Ans. 100 els of Roch.

The

The third Chapter teacheth of the order and work of the Rule of 3. in broken number after the trade of Merchants, digressing something from M. RECORDS, which is comprehended in three RVLES.

NOw that I haue somewhat intreated of the Rules of Practice, I will giue a few instructions, after my simple order, for the working of the Rule of three in broken numbers, wherein I shall nede to say the lesse, because I hope the studious learner, that hath trauel'd any thing in the Ground of Arts, is not vnfurnished of knowledge capable to vnderstand me. But before I deliuer any instructions for broken numbers, I will propone a question, which shall be wrought three sundry waies, thereby to shewas it were three degrees of cõparison: how farre the rule of three in broken, for more spede of worke, differeth from the whole, which I rather set downe for a view, that the studious herein may be more desirous to attaine broken: leauing any more to discourse in dialogue forme, but onely to giue instructions where nede is: and in the rest to put forth the questions with their answeres

My

My first question is this.

If one yard cost 6s 8d, what are 789 worth at that rate?

1—6—8——789
12 80
80 63120 d

Here the product of the summe are pence, according to the nature of the midle number

11
137 | 1
63120 | 5260
12222 | 2220 (263
|111 |

I answer 263 l'

1——$6\frac{2}{3}$——789
3 20 20
15780

Here the product of the summe are s, according to the nature of the middle number.

1
15750 | 5260 | 263 l'
3333 | 2220 |
1——$\frac{1}{3}$——789
3 1 1
789

Here

Where the product is pounds, according to title of the second number.

x
789
333 (263
I aunswere 263 l

Now that you haue seen the 3 former vertues of the rule of three, whose products haue first brought forth d, next s, & lastly pounds, I will deliuer three notes in order following; and with them a dozen questions that shall shewe the worke of the Rule of 3 in broken numbers or Fractions.

1 The first foure shall be sundry questions of a Fraction comming in the second place.

2 The second fowre shal be of two Fractions comming in the second or third place.

3 The third foure of Fractions in al three places.

My first question is this.

If one yard cost me 3s 4d, what are 756 worth at that price?

In setting downe the question to performe the worke, I turne 4 pence into the Part of a shilling, which is $\frac{1}{3}$, and then the question standeth thus:

1 —— $3\frac{1}{3}$ —— 756

To

To the ready working of this question, and all such other like, my first note is this, which take for a generall rule: that when any one Fraction shall come either in the second or third place, that the Denominator of that Fraction or Fractions, must alwaies bee brought vnto the number or Numerator of the first place: and therby multiply the one into the other.

And this benefit is alwaies gotten by the vertue of bringing the Denominator of the second numbers Fraction vnto the first place For the Fraction in the middle number is now released: and the product that commeth of the multiplication, is of the nature & like Denomination of the whole number in the second place which here are shillings.

Whereupon now to worke the question, I bring three the Denominator of the fraction in the second place, vnto my first number one with a line set vnder 1 thus: and the three vnder it $\frac{3}{1}$ thus: saying, once 3 is 3 my Diuisor: that done, reduce three $\frac{3}{1}$, saying: 3 times 3 is 9, and the 1 ouer 3 make 10: my second number in the rule of 3: by which 10 I doe multiply my last number 756, as appeareth by the worke thereof, and it yeeldeth 7560 my diuident.

Then diuiding 7560 by 3 my diuisor, it yeldeth in quotiēt 2520 shillings, which maketh 126 pounds, as appeareth here most plainly

both

both by the example and the worke.

At 3 s—4 d the yard, what 756 yards?

1——3	1	756
3	3	10
	10	7560 s

x	1	
7560	2520	126: I answere 126 l.
3333	2220	

Yet otherwise vpon the same question, altering the price now into ỳ proportion it beareth to a pound, for the 3 s 4 d is $\frac{1}{6}$ part of a pound: which example first standeth thus, as appeareth on the left hand, and afterwards wrought as appeareth on the right hand.

1—$\frac{1}{6}$—756	$\frac{1}{6}$——$\frac{1}{6}$——756
	1 1
	756 pounds.

As soon as I haue caried 6 the denominator of my middle number vnto my first place, as before hath bẽn taught, I pull downe one, the numerator of 6, with a line vnder 6 thus, $\frac{1}{6}$. and that one in custome I pull downe in sight, beeing the figure that I will multiply my third or last number by, according to the tenor of the rule of three. And because one can

neither multiply nor yet diuide (though here it is set down in forme of multiplication, the rather for your vnderstanding) the product of the multiplication according to the declaration of this my first rule or note is conuerted into the title of my second number, which here are pounds. Now followeth the diuision performed by my diuisor 6; to make an end of that question.

~~13~~
~~756~~ (126: which maketh 126 l', as before.
~~666~~

And thus much for the variety in working that question.

And now followeth another.

If one yard of cotten 8 d, $\frac{1}{4}$, what 859?

1	$8\frac{1}{4}$	859
4	33	33
		2577
		2577
		28347

2	~~1~~1	l' s 6
~~28347708~~ (6	~~590~~	29—10—6—$\frac{1}{4}$ d.
~~444412~~	~~220~~	

This

This question was also wrought like the 1, and bringeth forth 29 l' 10 s—6 $\frac{3}{4}$ d the price of 859 yards.

If 7 pounds of any thing cost 3 l'— 10's, what comes 987 pounds to?

7——3 $\frac{1}{2}$——987

$\frac{2}{14}$ 7 $\frac{7}{6909}$

6909|493 $\frac{7}{14}$ I answere 493 l'—10 s 14.

Notes vpon my second rule for two *Fractions comming in the second* and third place.

My first question is this.

If one El cost 13 s—4 d: what half a quarter or $\frac{1}{2}$ of an Ell?

Answer. First bring 13 s— 4 d into the parts of a pound, which is $\frac{2}{3}$, and then wil the question stand thus:

1—$\frac{2}{3}$—$\frac{3}{3}$

Item for the performance of this work, do as before was taught in the first Rule: first

bring 3 the denominator of the second fraction vnto your first number 1: setting a line vnder thus: Saying, once 3 is 3: that done, bring 8 the denominator of the third fraction setting it vnder 3, and multiply them together, saying, 3 times 8 maketh 24, which 24 is your deuisor: (Now haue you done with the denominator 3, and also with the denominator 8.) Therefore you shall put a line vnder 3 thus. And the little line also vnder 8, setting or pulling downe vnder them their owne numerators, that is 2 vnder 3, and also 1 vnder 8, as apeareth in the example, which numerators for a generall rule are euermore to be pulled down of custome in sight, to multiply the one by the other, according to the tenor of the rule of 3. Then I multiply the one by the other, saying: once 2 is two, which signifieth 2 l, being of the nature and like denomination of the middle number, which 2 l is to be reduced into shillings, otherwise it cannot be diuided by my first number 24.

Then diuiding 40 by 24, the quotient bringeth forth 1 $\frac{1}{3}$ s. So much is $\frac{2}{3}$ of an elle worth after that rate. Otherwise although 2 pound could not be diuided by 24: yet it might haue been abbreuiated to $\frac{1}{12}$ of a pound, which is worth 1 s - 8 d, as before.

1 —— 2 —— $\frac{1}{8}$

3 2 1

8 20 16

—— 40 (1 $\frac{2}{5}$ s

24 40 24

Second question.

If one pound of any weight cost 13 shillings 4 pence, what are $\frac{7}{8}$ of the pound worth after that rate.

Answer. Reduce the 13 shillings 4 pence into the parts of a poūd: which is $\frac{2}{37}$ and then will the question stand thus:

1 —— $\frac{2}{3}$ —— $\frac{7}{8}$

Item for the vnderstanding of this, if you marke well the last example, this and the rest lieth open, and need small instruction. For as you did last, so now againe bring the denominator of the second & third fraction vnto the first figure 1, multiplying the one into the other, which maketh also 24 your deuisor.

Then making a line vnder 3 thus, and a line vnder 8 thus, and pulling downe their Numerators vnder each figure, that is, 2 vnder 3, and 7 vnder 8, which as I said before for a generall rule I pull down of custome in sight, to be the two numbers that of duety

ought to be multiplied together, which done I bring 2 being the lesser figure vnder 7: multiplying them together it maketh 14: which are of the nature of the middle number: that is to wit pounds: which 14 cannot aptly be diuided among 24, therefore are reduced into shillings, as is plainely to bee seene in the example, then 280 shillings parted among 24 yeeldeth for his quotient 11 s 8 d your desire, and the iust price of $\frac{7}{12}$ of an ell. Otherwise 14 though it could not be diuided by 24, might by mediation or diuision in broken numbers haue beene diuided or abbreuiated to $\frac{7}{12}$, which in effect being reduced to his known parts, maketh 11 s 8 d as before. But my good will and meaning is to aid young beginners: therefore haue I reduced the 14 l into s, which is the easier way.

Now followeth the Example.

1 **	2 **	7		(1 4	
8	8	8	46		
8	2	7	280	11 $\frac{7}{12}$ s 8 d	
		2	144		
24		14		2	
		20			
		280 s			

The third Example.

If one yard coſt me 2 s —6 d, what 345 ¼ yards?

Anſwer. Firſt put 6 d into the parts of a ſhilling, and then the queſtion ſtandeth thus:

1 ——— 2 ½ ——— 345 ¼.

Item, to the ready vnderſtanding of this and all ſuch like, according as before hath been declared, bring the Denominators of the ſecond and third fractions vnto the firſt place, multiplying them the one into the other, all which make 8 from the diuiſor common. Then next reduce your ſecond number: ſaying, two times 2 is 4, and 1 is 5, as was taught in the example aforeſayd. Laſtly, reduce your third number 345 ¼ all into fourths, and they make 1381: which 1381 is to be multiplied by 5, according to the tenor of the rule of 8: which done, maketh 6905 s: and diuided by 3 your Diuiſor, yeelded in quotient 863 ½ s, which maketh in pounds 43 l —— 3 9 —— 1 d ½: and ſo much are the 345 and ¼ yards worth at that price.

The ſame queſtion wrought again by two ſhillings 6 pence, is now conuerted into the parts of a pound, and ſtandeth thus:

1 ——— ¼ ——— 345 ¼

Item, after I haue brought here my ſecond and third fraction vnto my firſt place, and found 32 to bee my diuiſor; hauing thus furniſhed

shed my first place with all things vnto him belonging (which is meant of bringing and multiplying the Denominators of the second and third Fractions into him) I then goe in hand to see what is to doe in my second place, where presently of custome I put downe my Numerator 1 vnder 8: being the figure in sight that shall multiply my third number.

Then lastly I reduce 345 $\frac{1}{4}$ al into fourths as afore was practiced, which maketh 1381, the which 1381 I am to multiply by 1 my second number, they are nothing increased, but by the Metamorphosis of my worke they are now 1381 pound, being of the nature of the middle number, as I haue often shewed you, which diuided by 32 my diuisor, yeeldeth 43 pound and $\frac{5}{32}$ which $\frac{5}{32}$ of a pound reduced into knowen numbers, make 3 shillings, 1 d. $\frac{5}{8}$, as before.

Example.

| | | |

Item first for the finding out of your Diuisor : for you shall take this for a most certaine and generall rule ; That you must multiply the numerator of the first number in the question by the denominator of the second : And also that againe by the denominator of the third: and the totall therefore shall be your diuisor

Secondarily, for a generall rule to find out your dividend , multiply the denominator of the first number by the Numerator of the second , and the whole thereof by the Denominator of the third. And the totall thereof shall euermore be your dividend.

Now for an example I propone this question , thereby to make my meaning the more plaine , and to shew you as I haue done in the rest, the manner and order of the worke.

If $\frac{3}{7}$ of any waight or measure cost $\frac{5}{6}$ of a pound or 20 s, what are $\frac{7}{8}$ of the like waight or measure worth after that rate ?

Example.

$\frac{3}{5}$ — $\frac{5}{6}$ — $\frac{7}{8}$

Item for the more plainer vnderstanding hereof, and all other the like, in broken numbers: First you shall pull downe two the numerator of the first number or fraction, with a line vnder $\frac{3}{5}$ thus: that done , according as you

haue

haue learned before, bring 6. the denominator of the second fraction, and set it vnder 2, multiplying the one into the other, which maketh 12. Then lastly bring 8. the denominator of the third fraction, and set it vnder 12, multiplying that 12: by 8, which amounteth to 96. or els for more briefe, multiply 6, by 8. saying 6. times 8. makes 48 which 48, set vnder two, and multiply the one into the other, it maketh 96. as before. And this 96. is the first number in the rule of three. That shall alwaies for a most generall rule be your deuisor.

Secondly to worke for your diuidend, you shall, as hath beene sufficiently declared afore, pull downe 5, the numerator of your second fraction, and set it vnder 6. with a line vnder thus. That done, as you know, you are to pull downe 3. the numerator of the third fraction, and set it vnder 8, with a line vnder thus, multiplying the one into the other according to the tenour of the rule of three, which maketh 15. Then according to my note, forget not to bring the Denominator of the first Fraction which is 2, vnder 15. and multiply them together, which maketh 45, which 45 is your diuidend. Which 45 are of the nature or denomination of the middle number, as I haue oft taught you before. And therefore are 45. l. which aptly cannot bee diuided by 96. Therefore you shall reduce the 45 l. into s. as you see perfourmed in the example, which

amoun-

amounteth to 900s, which diuided by 69 your diuisor, it yealdeth 9 s, and $\frac{36}{66}$ of a shilling, which in lesser termes is $\frac{3}{8}$, which $\frac{3}{8}$ in money maketh $4\frac{1}{2}$ d: and so much will the aforesaid $\frac{3}{8}$ cost, as by the work following shal appeare.

The example.

2	5	3	
3	6	8	
2	5	3	
48		15	36
96		3	23 0 $\frac{39}{96}$
			96
		45	
		20	
		900	

Otherwise though 45 could not be diuided by 66, yet by diuisiõ in broken numbers it might haue béene abbreuiated to $\frac{15}{23}$ of a l', which reduced into knowen parts will make 9s $4\frac{1}{2}$ d, as before.

Now my second example shall be the proofes of this question.

If $\frac{3}{8}$ ysrds cost $\frac{15}{32}$ of a pound or 20 shillings what shall $\frac{2}{3}$ cost?

Answer. Work as was taught you before, and you shall haue your desire.

Héere

$\frac{3}{4}$ —— $\frac{15}{32}$ —— $\frac{2}{3}$

3 15 2
32 2
96 30
3 8
288 240

Heere, as appeareth by the worke, the multiplication being ended, 240 is to be reduced by 288, which to some perchance may seeme hard, yet notwithstanding is the worke good. Therefore abbreuiate 240 by 288, as you see heere is practised: and the end of your abbreuiation shall come to $\frac{5}{6}$ your desire $\frac{240}{288}|\frac{120}{144}|\frac{60}{72}|\frac{30}{36}|\frac{15}{18}$.

Otherwise. 240|120|60|30|5
288|144|72|36|6

Otherwise. 240|40|5
288|48|6

The third question.

If $\frac{1}{4}$ elles cost 13 s—4 d, what $156\frac{1}{2}$ elles?

Answer. To worke this question the shortest way: reduce 13 s—4 d into the parts of a pound, which is $\frac{3}{5}$.

Then as you did afore, after you haue set down the question, the numerator of the first fraction 3 is pulled downe vnder 4, and the

Deno-

Denominators of the other 2 Fractiõs multiplied into him which maketh 18 your deuisor.

Then the numerator of the second fraction two is pulled downe vnder three of custome now in sight, ready to multiply my third nũber by: which is performed as sone as the last number 156 ½ is reduced into halfes.

Then lastly, I multiply that product by 4, the denominator of my first fractiõ: it yeeldeth 1504, which I diuide by 18, & my quotient is 139 ½, and ½ of a pound remaining, which is worth 2 s 2 ½ d: And so much will 156 ½ els cost, as by the worke following doth appeare.

¾	⅔	156 ½	x
6		313	47
3		2	x76 (2
18		626	2504x | 139 ½
		6	1888
		2504	xx

The fourth example.

If ⅔ elles cost 1 ⅓ l, what commeth 29 ¼ elles to?

Item, to the workmanship of this question: first reduce your second number to one direct Number, in saying three times 1 is 3, and 2 is 5: Then bring the multiplication of the Denominators of the second and third fractions,

ons, which maketh 12: and multiply that 12 by fiue your first numerator, it maketh 60, which is your diuisor.

Then the reduction of the second number, which is 5 multiplied by 117 the product of the last numbers reduction, make 585, which 585 yet resteth to be multiplied by two, the denominator of the Fraction in the first place yeldeth 1170: which diuided by your diuisor 60: yeldeth 19 pounds 10s, as appeareth by the worke thereof.

Thus hauing now touched the 12 questions wherof I first pretended, which with diligēce and oft practise, I trust are sufficient to aide the desirous vnto the working of any broken numbers, I will now intreate of diuers necessary rules incident vnto trafficke, as hereafter followeth.

The 4 Chapter treateth of Losse and gaine in the trade of Merchandise.

IF one yard cost 6 s——8 d: and the same is sold againe for 8 s 6 pence: the question is, what is gained in 100 pounds laying out on such commodities.

Answer. The rule of three direct, applieth two manner of wates to doe the same, the one is, to say: If fiue $\frac{5}{5}$ giue $\frac{1}{1}$, what giueth 100?

Vult

Multiply and diuide, & loke what your quotient bringeth forth aboue your laying out, is the neare gaine, and the solution to your question: If you follow the worke, your quotient will bring forth 1741 l — 10 s, which is 27 l 10 s more then your principall, and so much is gained in the 100 pounds laying out.

Item, to worke it the other way, which I take the neareſt, ſeeke the difference betwixt the iuſt price and the other price, which is one shilling-10 pence. Then say by the rule of 3. If $6\frac{3}{4}$ s gaine $1\frac{5}{4}$ s, what shall 100 l gaine? Multiply and diuide, and you shall finde 27 l — 10 s, and so much is gained in 100 l laying out.

Vse which of these two waies you thinke good.

The proofe.

If a yard of cloth be delinered for 8 s, 6 d, whereupon was gained after the rate of 27 l 10 s in 100 pounds laying out. The question is, what the yard cost at the first hand?

Answer. Put your gaine to 100 l, all maketh 127 l 10 s: then say, if 127 l 10 s gine but 100 pound, what gineth $8\frac{1}{2}$ s? Worke and you shall finde 6 s, 8 d, the true solution to your question.

Yet another or proofe vpon the same first question.

If one yard coſt 6 s—8 d, the queſtion is what price the ſame is to be ſold againe for to gaine 27 l——10 sin 100 l laying out.

Anſwer. Say by the rule of three, if 100 l gaine 27 l——10 s, what giueth 6 $\frac{2}{7}$ s. Multiply and diuide, & you ſhall finde 8 s——6 d your true ſolutions.

If one elle coſt 7 s——8 d, and ſold againe for 8 s—Queſtion. What is gained in 20 l laying out in ſuch commodities.

Anſwer. Make the difference betwixt the iuſt price & the other price, which is 10 pence and then apply the Rule of three, as before is taught, ſaying, If 7 $\frac{2}{3}$ s giue $\frac{2}{3}$ shillings, what giueth 20 l? Multiply and diuide, & you ſhall finde 2 l. 3 $\frac{11}{23}$ s. and ſo much is gained in 20 l laying out.

The proofe alſo by an example of loſſe.

A Merchant hath bought Holland cloth at 8 s—6 pence the elle, which proueth not to his expectation, whereupon he is content to loſe 2 l. 3 s $\frac{11}{23}$ s in 20 pounds laying out. The queſtion is what price ought to be made of the elle abating this loſſe.

Anſwer. Doe as before is gaines hath bene taught, putting 2 l—3 $\frac{11}{23}$ s to your 20 l, all together maketh 22 l—3 $\frac{11}{23}$. Then ſay by the rule of three: If 22—3 $\frac{11}{23}$ s giue but 20 l, what

what shall come of 8 ½ s. worke and you shall finde 7 s—8 d, the iust price that the ell ought to be sold for after the rate of this losse.

That it appeareth euidently, as in company the Rule is applicable as well to gaine and losse.

If 20 ¼ yards cost 26 l—10 s, how shall I sell the same againe to gaine ⅓ of the principall, or to make of 3,4: which is all one.

Answer. By the rule of 3, if 3 doe giue 4 what will 39 ½ giue? Multiply and diuide, & you shall finde 48 l—⅓. Then say againe, if 20 ¼ yards do giue 48 ⅓ pounds, as well principall as gaine, what will one yard be worth at that price? Multiply and diuide, & you shal finde 2 l 8 $\frac{4}{27}$ s.

If one Elle of cloth cost me 8 s—8 d, and afterwards I sell 10 ½ Elles thereof for 5 l, 13 s—4 d, I would know whether I winn or lose: and how much vpon the 100 l of money.

Answer. See first at 8 s—8 d the elle what 10 elles ½ comes to, and you shall find 4 l 11 s, & I sold the same for 5 l—13 s—4 d, so that I did gaine vpon the 10 ½ yards, 1—12—4 d. Then if you could know how much is gained in the 100 l, I say by the rule of three, if 4 l—11 s did gaine 22 s 4 d, what will 100 l

gaine? Multiply and diuide, & you shall finde 24 pound—10 s—10 d $\frac{10}{11}$. And so much is gained in the 100 pound of money.

If 12 $\frac{1}{2}$ yards cost me 11 pound—5 s, & I sell the yard againe for 16 s, the question is, whether I doe winne or lose, and how much in or vpon the pound of money.

Answere. Looke what the 12 $\frac{1}{2}$ yards come to at 16 s the yard, & you shall finde 10 pound. But they cost 11 pound 5 s—. So there is lost vpon the whole 1 pound 5 s. Then to know how much is lost in the pound, say by the rule of 3, if 11 $\frac{1}{4}$ pound do lose 1 $\frac{1}{4}$ pound, what will 1 pound lose? Multiply and diuide, and you shall finde 2 s—2 d $\frac{2}{9}$, and so much is lost in the pound of money.

If I sell the C weight of any commodity for 4 pound, whereupon I do lose after 10 pound in the 100 pound, I demand how much I shal lose or gaine in the 100 pound, if in case I had sold the same for 4 pound—10 s.

Answere. Say, if 90 pound yéld 100, how much will 4 giue? Multiply and diuide, & you shal finde 4 $\frac{4}{9}$. Then say againe, if 4 $\frac{4}{9}$ giue me 4 $\frac{1}{2}$, what will 100 come to? Multiply and diuide, and you shall find 101 pound $\frac{1}{4}$, which is more then 100 pound by one pound 5 s. And so much is gained in the 100 pound.

A Merchant hath sold Currants for the summe of 430 pound, & he hath gained therin after

after 10 pound in the 100 pound. The question is to know how much he gained in all.

Answer. Say by the rule of three. If 100 pound do gaine 10 pound, what wil 430 poũd gaine? Multiply and diuide, & you shal find 43 and so much hath he gained in all.

If one yard be worth 28 ½ s, for how much shall 10 yards be sold to gaine after 8 l——6 s 8 d in the 100 pound.

Anſ. Firſt adde 8 l——6 s, 8——d to 100. Then say, if 100 l do giue 108 ⅓ for principall and gaine, what will 28 ½ s principall yelde? Multiply and diuide, and you shall finde 30 ¾ s. Then say againe by the rule of three, if 1 yard doe gine 30 ¾ s, (which is as well the principall as the gaine) what shall 10 yards giue? Multiply and diuide, and you shal finde 15 l—8 s—9 d. And for the same price shall the 10 yards be sold, for to gaine after the rate of 8 l——6 s——8 d vpon the 100.

A Branch or proofe out of this Question.

A Merchant hath sold Clothes for 15 l——8 s——9 d, and he hath gained in the whole, the sum of 1 l——3 s——9 d. The question is to know how much he hath gained in the 100 pound.

Answere. To know this, first rebate the gaines from the price, and there will remaine

14 l—5 s—od Then say by the rule of 3 direct, if 14 l $\frac{1}{4}$ giue me 1 l—3 s—$\frac{1}{4}$ s, what wil 100 l giue? Multiply and diuide, and you shall find 81—6 s–8 d, the effect desired. the proofe is apparant in the question before.

Yet another Branch or Proofe of the first Question.

If 10 yards be deliuered for 15 l—8—9 whereupon was gained after the rate of 816 s—8 d vpon the 100 l, the question is, what the yard did cost at the first hand.

Answer. First, say by the Rule of three, if 10 with principall and gaine yeeld 19 l—8 $\frac{1}{2}$, what shall 1 yeeld? Multiply and diuide, and you shall finde 30 $\frac{7}{8}$ s. Then say againe by the rule of three, if 108 $\frac{1}{3}$ principall and gaine giue but 100, what shall 30 $\frac{7}{8}$ s of principall and gaine yeeld? Worke and you shall finde 28 $\frac{1}{3}$ s. And so much did the yard cost at the first penie.

If one yard cost 36 s, how much shall 12 yards be sold for to gaine after the rate of 10 l in the 100?

Answer. First say, if 100 giue 100 pound principall and gaine, what will 36 s giue? Multiply and diuide, and you shall finde 39 $\frac{3}{11}$ s. Then say againe by the rule of three. If 1 yard of principall and gaine yeeld 39—s

what

what shall 12 yeares gaine? Multiply and diuide, and you shall finde 23 l—15 $\frac{1}{7}$ s, which $\frac{1}{7}$ s in knowen number is 2 $\frac{1}{7}$ d. And for the same price shall the 12 yards be sold, to gaine after the rate of 10 in the 100.

The proofe.

If 12 yards be sold for 23 l—15—2 $\frac{1}{7}$ d, whereupon is gained after 10 l in the 100 l, The question is, what the yard cost at the first penie?

Answer First say, if 12 giue 23 l—15 $\frac{1}{7}$ s, what one yard? Multiply and diuide, and you shall finde 39 $\frac{3}{7}$ s. Then say againe by the rule of three, if 100 pounds giue but 103 what shall 39 $\frac{3}{7}$ s. giue? Worke, and you shall finde 36 s, the iust price of the yard at the first hand.

Item, when one Merchant selleth wares to another, & he giueth to the buyer 1 l—16 s 8 d vpon the score, or 20 l: The question is, how much shall the buyer gaine vpon the 100 pound after that rate.

Answer. First adde 1 l—6 s—8 d vnto 20 l, and they are 21 $\frac{1}{3}$. Then say, if 20 pound giue 21 $\frac{1}{3}$ what shall 100 giue? Multiply and diuide, and you shall find 106 $\frac{2}{3}$. So the buyer

getteth after the rate of $6\frac{2}{7}$ l' vpon the 100 pound.

Gentle Reader, other necessary questions appertaining to Losse and Gaine, you shall haue in the eight chapter of this Treatise.

The fifth Chapter entreateth of Losse and Gaine vpon time, wrought by the double Rule of three: or by the rule of three composed, which is contained in 4 speciall selected branches or questions of diuers formes, each one of them springing from the first question, and each one of them also being a proofe to other, &c.

IF one yerd cost me 25—8d ready money, and after I sel the same againe for 25—10d, to be paid for it at the end of 3 moneths, The question is, what I gaine vpon the 100 l in 12 moneths.

Answere. First say, if $2\frac{2}{7}$ gaine $\frac{1}{4}$, what shal 100 l' gaine? Multiply and diuide, and you shal finde $9\frac{1}{4}$ l'. Then say againe by the rule of three: if three moneths gaine $6\frac{2}{4}$ pound what shal 12 moneths gaine? Worke and you shall finde 25 l', and so much shall I gaine in 12 moneths after that rate.

Item, you may also worke it at one working by the first part of the rule of 3 composed saying

saying, if 2 ½ d in three moneths do gaine ½ of a shilling (which is 2 d) what will 100 l' gaine in 12 moneths: which for thy further encouragement the worke of this one example I haue here put downe, to verifie that I affirm in the first part of this Ground of Arts, that this rule, and so all others more reioyceth in Broken then in Whole.

s	moneths.	s	l'	moneths.
2 ½	3	½	100	12
8				100
3	2			1200
24	7			3
6	168			3600
144	36000 | 25			
	4444			
	44			

Where the multiplication & the Diuision being ended, maketh 25 l your desire.

If a yard be deliuered for 2 s—10 d to be paid at 3 moneths, whereupon was gained after the rate of 25 l' in the 100 for 12 moneths, the question is now, what the yard cost at the first hand.

Answere. First say, if 12 Moneths gaine 25 l', what shall three moneths gaine? worke and you shall finde 6 ¼ l'. Then say againe the

second time, if 106 ¼ l giue but 100, what shall 2 ½ s giue? Worke, and you shall finde 2 s 8 d which is the iust price that the yard cost at the first hand.

If one yard of cloth cost me 2 s—8 d ready money, for what terme shal I sell the same againe for 2 s—10 d, so that I might gaine after the rate of 25 pound vpon the 100 pound in 12 moneths.

Answere First, say, if 2 ⅓ gaine ¼: what shall 100 pound gaine? Multiply and diuide, and you shal finde 6 ¼ pound. Then say again for the second worke, if 25 pound become of 12 moneths, what shall come of ¼? Worke, and you shall finde 3 moneths the iust terme of time that the cloth ought to be deliuered at 2 s, 10 d, to gain 25 pound vpon the 100 pound in 12 moneths.

If one yard cost me 2 s–8 d ready money, for what price shall I sell the same againe to be paid at the end of three moneths, so that I may gaine after the rate of 25 pound in the 100 pound for 12 moneths.

Answer. First say, if 12 gaine 25 l, what shall 3 moneth gaine? Multiply and diuide, & you shall finde 6 ¼ l. Then say for the second worke, if 100 l giue 106 ¼, what gineth 2 ⅓ s? Worke, and you shall finde 2 s—10 d, and for that price must the yard bee sold to gaine after 25 poūd in the 100 poūd for 12 moneths.

Many

Many other of these questions I might here haue deliuered but for feare the book would rise to too thick a volume, and so to make the price so much the dearer, wherby it might not be so portable to my country-men as I wish it But these 4. I haue of purpose framed in this order, hauing relation one to another, assuring you that what question soeuer may bee proposed within the compasse of this Rule, you shall finde by one of these 4, to make a solution. And moreouer, diuers other are yet to be deliuered: where the Creditor giueth diuers dayes of payment, which can neuer bee well wrought, nor yet vnderstood, vnlesse you can first find by art the iust times that all those paiments, how different soeuer they be, ought to be paid at once: wherupon first I thinke good heere to giue some instructions into such a rule, for it is the only ayde for the finishing of such questions as hereafter shall follow.

The

The sixt Chapter intreateth of Rules of Payment, which is a right necessary Rule, & one of the chiefest hand-maids that attendeth vpon buying and selling, &c.

Example.

A Merchant doth owe a summe of money, whereof the ¼ is to be paid at 6 moneths, the ⅓ at 8 moneths, and the rest at a yeere. If he would pay at one payment, the question is what time ought to be giuen him.

Answer. I haue omitted the quantitie of the summe, for you shall vnderstand, the rule is appliable, and yeelds a true solution to what sum soeuer shall be proponed. But now for orders sake in teaching, I do imagine the sum to be 60 pounds, whereupon the maner of this worke is to multiply the proportionate part of the mony by the time, as in company. Then 20 being the first payment, and the ¼ of 60, which multiplied in broken numbers, by 6, his time of payment maketh ½, which in whole numbers, as appeareth by the example in the margent, maketh two moneths: nert 20 which is the ⅓ multiplyed by his term 8, yeelds 4 moneths, then the rest which

¼ by 6/1 = 6/4		2 moneths
⅓ by 8/1 = 8/3		4 moneths
5/12 by 12/1 = 12/12		2 moneths
		8 moneths

is 10 l' must nedes bee abbreuiated into the proportion it beareth to 60 which is $\frac{1}{6}$, which multiplied by his time 12 moneths, produceth $\frac{12}{6}$, maketh two moneths. All which added together, as appeareth in the margent, maketh eight moneths, which is the iust time that al those paiments ought to be paid at once.

A Merchant hath 800 l' to pay, the $\frac{1}{4}$ therof ready money, the $\frac{1}{4}$ are two moneths, the $\frac{1}{3}$ at foure moneths, and the rest at a yeere. The question is, if hee would pay all at one payment, what time ought to be giuen him.

Answere, the ready money is neuer multiplied: thē $\frac{1}{4}$ multiplied by 2 months as you did before, maketh $\frac{1}{2}$, thē $\frac{1}{3}$ by 4 produceth 2 months as appeareth here in the margent. But now for the rest of the money, you cannot multiply it vntill you haue sought what proportion it beareth to 800 pounds. Therfore you must subtract the ready mony, thē $\frac{1}{4}$ & the $\frac{1}{3}$ out of the principall. The rest will be $66\frac{2}{3}$ l', which you must seek what part it beareth to the principall, which you shall finde to be $\frac{1}{12}$, the same you must also multiply by his time 12 moneths, and it yeeldeth 1 moneth, so all make $3\frac{1}{2}$ moneths, as appeareth in the margent.

A Merchant is to pay 1600 l' in three terms, that is to wit, 400 l' at two weekes and 600 l' at fowre monthes, lastly, 200 l' at 5 moneths. The question is, in what time they ought to be paid at once.

Answere. Proportionate the parts, & you shall finde $\frac{4}{12}$, and likewise 200 is the $\frac{1}{12}$ part, which multiply by their times as before, and you shal haue $\frac{4}{12}$ weeks, which together maketh lastly three $\frac{1}{12}$ weekes, which together maketh 12 weekes or three moneths, your desire.

A Merchant is to pay 600 l' in three tearms, shall find that 405 is $\frac{1}{7}$ part, and for 600 you whereof 100 l' is paid present, more 300 l' at whereof 100 l' is paid present, more 300 l' at 20 daies, and the rest at 5 moneths, accounting 30 daies to a moneth. The question is, what time ought these payments to bee paid at once.

Answere Worke, and you shall finde two moneths.

The

The seauenth chap. intreateth of buying and selling in the trade of Merchandise, wherin is taken part ready mony, and diuers daies of paiment giuen for the rest, and what is won or lost in the 100 l' forbearance for 12 moneths more or lesse, according to the quantitie of money, or proportion of time, &c.

A Merchant hath bought Sattens which cost 8 s the yard ready money. And hee selleth the same againe to another man for 10 s the yeard, but he giueth 2 dayes for the paiment, that is to say three moneths for the one halfe, and fiue months for the other half. The question is to know how much the seller doth gaine vpon the 100 pound in 12 moneths after that rate.

Answere Make first by the rules of payment, at what time those two payments ought to be paide at once, and you shall finde foure moneths, at which time the second merchant ought to haue paid the whol entire payment. And therefore say by the first part of the Rule of three composed: 8-4—2—100—12

If 8 shillings in 4 moneths doe gaine 2 s, what will 100 l' gaine in 12 moneths

4		12	
32		1200	2400 | 75
		2	232
		2400	

mul-

Multiply and divide, and you shall finde 75 pounds, as appeareth in the example, and so much doth the first merchant gaine upon 100 pounds in 12 moneths.

A Merchant hath sold fittie clothes at 9 l. the peece, to be paid the one ⅓ at foure months the ⅓ at fiue moneths, and the ⅓ at 7 moneths, and the sellers minde is to take no more but after 8 pounds in the 100 for 12 moneths: The question is now what the first merchant gaineth in the sale of these cloathes after that rate.

Answere. First loke what the 50 cloathes come to at that price, and you shall finde 475 pounds. Then secondly, according to your direction in the rules of payment, seek at what time all the paiments are to be performed at ones. And you shall finde 4 ⅔ moneths. Then thirdly say by the first part of the rule of three composed: If 100 l in 12 moneths gaine 8 l, what will 475 l gaine in 4 ⅔ moneths? Work and you shall find 15 l 16/20 of a pound, which is the neat gaines that the first merchãt hath after the rate aforesayd.

A Merchant hath bought Holland at 7 s 3 d the elle ready money, and hee selleth the same again for 8 s 4 pence the elle, to be paid ¼ part in ready money, more ⅓ part at 2 moneths, and the rest at 4 moneths. The que-

tion

tion is now to know how much the first merchant doth gaine vpon the 100 pounds in 12 moneths after the rate.

Answer. According to the direction delivered you in the rule of payment, the ready money is not to be multiplied. Then working for the other two paiments, to find out ỹ true proportiõ at what time they ought to be paid at once, you shal find for $\frac{1}{3}$ at 2 moneths $\frac{1}{3}$ of a month. And the rest of the mony which is $\frac{2}{3}$ multiplied by his term 4 moneths yeeldeth 1 $\frac{1}{3}$ months, both which added together make 2 $\frac{1}{3}$ months, the iust time ỹ both the paimẽts ought to be performed at once. And therefore say by the 1 part of the rule of 3 cõposed, if 7 $\frac{1}{2}$ in 2 $\frac{1}{3}$ moneths doe againe $\frac{17}{140}$ of all: what shal 100 pounds gain in 12 months after that rate? worke and you shal find 76 pounds, 16 shillings 11 d $\frac{71}{201}$ pounds. And so much doth he gain vpon 100 l' in 12 moneths.

A Merchant hath bought 30 cloathes at 6 pounds the peece for ready mony. Afterward hee selleth 10 of them for 7 pounds the peece, for 3 months terme. And the other 20 he selleth for 8 pounds the peece for fower moneths term. The question is now what he gaineth vpon 100 pounds in 12 moneths.

Answere. First finde the value of the 30 cloths, which amount to 180 pounds. Secondarily, seek what the ten peeces come to at 7 l', and what the twenty peeces come to at 8 l'

the

the one comes to 70, and the other to 160: both which together make 230, which is 50 l' more then they cost. Thirdly, as I haue taught you in the rule of paiment, proportionate the first and second prices vnto the proportion they beare vnto 230, the product of their two prices, you shall finde $\frac{7}{23}$ for the first, and $\frac{16}{23}$ for the latter. Then fourthly, multiply those parts by their times, and you shall haue $\frac{21}{23}$ and $\frac{64}{23}$: both which together maketh 3 whole moneths, and $\frac{19}{23}$ of a moneth, which is the iust time that both those payments are to be paide at once.

Then say by the first part of the rule of 3 composed: If 180 l' in $3\frac{19}{23}$ moneths do gaine 50 l', what shall 100 gaine in 12 moneths? Multiply and diuide, and you shall finde $90\frac{10}{23}$ pounds. and so much doth he gaine vpon 100 pounds in 12 moneths.

A Merchant hath bought Cinamon which cost him 9 s the l' ready mony. The question is now at what price hee ought to sell the 100 waight. To wit 112 pounds, to be paid the $\frac{1}{4}$ at two moneths, and the residue at the end of 3 moneths, so that he may gaine after the rate of 10 l' vpon 100 l' for 12 moneths.

Answere. Seeke first by the rules of payment what terme both the payments ought to bee paide at once, where the $\frac{1}{4}$ multiplied by his terme 2 moneths, making $\frac{1}{2}$ moneths.

Like

Likewise the nert payment which is $\frac{1}{4}$ multiplied by this terme 3 moneths, maketh $2\frac{1}{4}$ moneths, both which added together, maketh $2\frac{1}{4}$ months, which is the time, that both the paiments ought to be paid at once. Then say by the Rule of 3, if 12 moneths do giue me 10. pound, what will $2\frac{1}{4}$ moneths giue? Multiply and diuide, and you shal find $2\frac{7}{32}$ pound. The say againe by the rule of three. If one pound cost me 9 s, what will 112 pound cost? Multiply and diuide, and you shall finde 50 l—8 s. Then say once againe: If 100 pound do giue $102\frac{7}{32}$, what will 50 l giue? Multiply and diuide, and you shall finde 51 l—11 s—$1\frac{1}{4}$ d: And for that price ought I to sell 112 pound of Cinamon to be paide at two seuerall payments aforesaid, to gain therby after the rate of 10 l vpon the 100 l in 12 moneths.

Briefe Rules for our hundreth waight here at London, which is after 18 pound for the C.

Item, who that multiplieth the pence that 1 pound waight is worth by 7, and diuideth the product by 15, shal find how many pounds in money the 112 pound waight is worth.

And contrariwise, he that multiplieth the pounds, that 112 l waight is worth by 15, and diuideth the product by 7, shall find how

many pence in money the 112 pound weight is worth.

Example.

At 10 pence the pound weight, what is 112 pound weight worth?

Answer. Multiply ten by 7, and thereof commeth 70, the which diuide by 15, and you shal finde 4 ⅔. And thus the 112 l is worth 4 l—13 s—4 d, after the rate of 10 pence the pound aforesayd.

At 6 l the 112 l waight, what is one pound worth?

Answere. Multiply 6 l by 15, and thereof commeth 90: the which diuide by 7, and you shal find 12 d 4. So much is one pound worth when the 112 pound did cost 6 l.

The eight Chapter intreateth of Tares and allowances of Merchandise sold by weight, and of losses and gaines therein, &c.

AT 16 pound the 100 suttle, what shal 795 pound suttle be worth in giuing 4 pound weight vpon enery 100 for treat?

Answere Adde 4 vnto 100, and you shall haue 104. Then say by the rule of three: If

104 bee worth 16 l', what are 895 l' worth? Multiply and diuide, and you shall find 237 l' 13—$10\frac{1}{11}$ d, and so much shall the 895 pound weight be worth.

Item at 3 s 4 d the pound weight, what shall $745\frac{1}{2}$ be worth, in giuing 4 l weight vpon euery 100 for treat.

Answer. See first by the rule of three what the 100 l' is worth, saying: If one cost $3\frac{1}{2}$ s, what 100? Multiply and diuide, and you shal find 16 l, $\frac{2}{7}$. Then adde 4 vnto 100, and they are 104. Then say againe by the rule of three: If 104 be sold for 16 $\frac{2}{7}$, for how much shal $745\frac{1}{2}$ be sold for? Multiply & diuide, and you shall find 120 l 18 s $3\frac{3}{7}$ d. And for so much shall the $745\frac{1}{2}$ be sold for at 3 s. 4 d the l' in giuing 4 vpon the 100.

Other necessary briefe rules there are for the finding of treats, or casting vp of chests of Sugar, &c. which for that it is a mystery, I omit: If any lacke instruction that way, they shall finde me ready to pleasure them.

Item if 100 l be worth 36 s 8 d, what shall 800 l' bee worth in rebating 4 l' vpon euery 100 for tare and cloffe.

Answer. Multiply 860 by 4, and thereof commeth 3440, the which diuide by 100, and you shall haue 34 $\frac{2}{5}$ l, abate $34\frac{2}{5}$ from 860, and

there will remaine 825 $\frac{5}{7}$. Then say by the rule of three. If 100 l cost 36 $\frac{2}{3}$ s, what wil 825 $\frac{5}{7}$ cost after that rate? Multiply and diuide, & you shal find 15—2—6 $\frac{16}{21}$. And so much shal the 860 cost, in rebating 4 l vpon euery 100, for tare and cloffe.

Item, whether doth he lose more that giueth 4 l vpon the 100: or he that rebateth 4 l vpon the 100.

Answer. First note that he that giueth 4 lon 100, giueth 104 for 100. And he which rebateth 4 l vpon the 100, giueth the 100 for 96. Therefore say by the rule of three, If 104 be deliuered for 100, for how much shall the 100 be deliuered? Multiply and diuide, and you shal find 96 $\frac{4}{13}$ and he which rebateth 4 in the 100, maketh but 96 of 100, so that he loseth 4 in the 100, and the other which giueth 4 vpon the 100 loseth but 3 $\frac{9}{13}$ vpon the 100. Thus you may see that he which rebateth 4 in the 100, looseth more by $\frac{4}{13}$ in the 100 l, then the other which gaue 4 vpon the 100, for tare and cloffe.

If 100 l of any thing cost me 3 s — 4 d: the question is, how I shall sell the pound to gain after the rate of 10 l, vpon the 100 l.

Answere. Say by the rule of three, if 100 l giue 110 l, what shall 3 s $\frac{4}{12}$ s giue? Multiply and diuide, and you shal find 1 $\frac{17}{20}$ l. Then say

again,

againe, If 100 pound be worth 1 $\frac{17}{60}$ l, what is one pound worth? Multiply and diuide, & you shall find 3 d $\frac{3}{17}$. And so much is the pound worth in gaining, 10 l, vpon the 100.

Item a Grocer hath bought a C. weight of commodity, for 6 l, 10 s. The question is now to know how many pounds thereof he shall sell for 3 2 s 4 d to gain 20 s in the C. weight.

Answere. Adde 20 s vnto 6 l — 10 s, and they make 7 l — 10 s. Then say, if 7 $\frac{1}{2}$ yelde me 112 pound, what shal 1 $\frac{1}{2}$ l yelde. Multiply and deuide, and you shal find 24 pound $\frac{8}{10}$. And so many pound ought he to sell to gaine 20 s in his C. weight.

If one pound weight cost 3 s — 4 d, and I sell the same againe for 4 s, which is gained 100 pound of money laid out in that commoditie?

Answere. You may say, If 3 $\frac{1}{3}$ s giue 4 s, what will 100 l giue? But then when you haue found, you must subtract 100 l, out of y product, the rest is your neat again, or else to produce, the neat again in your worke at the first. Then subtract the iust price out of the ouerprice, as I taught before in the first beginning of losse and gaine, and your conclusion shall bee all one. Multiply and diuide by which of the two waies you thinke good, and you shall finde that hee gayneth 20 l, in

in the 100 pound.

Item, if the pound weight which cost 4 s, be sold againe for 2 s—4 d, I demaund what is lost in the 100 l' of money?

Answere. Say, if 4 s lose $\frac{1}{7}$ s, what shall 100 lose? Multiply and divide, and you shall find 16 l—12 s—4 d, and so much is lost vpon the 100 l of money.

Item, if C wright of any commodity cost 45 l. and the buyer repenting, would lose 5 l in the 100 l' of money, I demaund how the l' may be sold: his losse to bee neither more nor lesse then after the rate aforesayd of 5 by the hundreth?

Answer. By the rule of 3, if 100 l lose 5 l, what shal 45 l' lose? Worke, and you shal find $2\frac{1}{2}$ pound, which rebated from the principall 45 resteth 42 l'—15 s. Lastly say, if 112 l' resteth but 42 l—15 s, what one pound? Multiply and divide, and you shall finde 7 s, 7 d $\frac{7}{23}$ And so much is the pound worth after that losse.

A Grocer hath bought 3 pecks of raisins, weying 175 $\frac{1}{2}$ l 182 $\frac{1}{4}$ l: 191 l': tare for each trayle 2 $\frac{1}{4}$ l, at 25 $\frac{1}{2}$ s the C weight. The question is, what they amount to in money.

I answere 6 l—3 s—4 d $\frac{11}{12}$.

A Grocer hath bought 3 sacks of Almonds weighing 267 ½ pound, tare 2 pound at 257 ½ l, tare 2 ½ pound 252 pound tare 3 pound at 28—10 ½ the pound, what amount they to in money?

I answere 110l—12s—3 ½d.

The ninth Chapter intreateth of lengths and breadths of Arras and other clothes, with other questions ncident vnto length and breadth.

IF a peece of Arras be 7 elles and ¼ long, & 5 elles and ½ broad, how many elles square doth the same peece containe?

Answer. Multiply ỹ length by the bredth, that is to say 7 ¼ by 5 ½. And therof will come 43 ⅛ elles: so many els square doth the same peece containe.

Item more, a peece of Arras doth containe 22 elles square, & if the same were in length 3 ¼ elles, I demaund how many elles in bredth the same peece doth containe?

Answere. Divide 22 elles by 3 ¼, & therof commeth 6 ⅘. So many elles doth the same containe in breadth.

Item more, a Merchant hath 3 ¼ elles of

Arras, at 1 $\frac{1}{4}$ elles broad, which he wil change with an other man for a peece of Arras, that is $\frac{3}{4}$ elles square. The question is how many elles of that squarenesse ought the first Merchant to haue.

Answere, Multiply the first Merchants peece his length by the breadth, and you shall finde it containeth 5 $\frac{1}{12}$ elles, which $\frac{1}{12}$ elles you shall diuide by $\frac{3}{4}$ and you shall finde 6 $\frac{4}{12}$ elles, so many elis of that squarenesse ought the latter merchant to giue the first.

Item a Student hath bought 3 $\frac{1}{2}$ yardes of broad cloth, at 7 quarters broad, to make a Gowne, and should line the same throughout with Lambe, at a foot square each skin: the question is now how many skins hee ought to haue.

Answere. Seek first the number of yards square that his cloth containeth, which to doe multiply 3 $\frac{1}{2}$ his length, by $\frac{7}{4}$ his breadth, and you shall finde 6 $\frac{1}{8}$ yardes square: then say by the rule of three, if one yard square giue 9 foote, what shal 6 $\frac{1}{8}$ worth and you shal find 55 $\frac{1}{8}$ skinnes.

Item more, a Lawyer hath a rich peece of seeling come home, which is 24 foot and three Inches long, and 7 foot and 7 $\frac{1}{2}$ Inches high the Joiner is to be paide by the yard square: the question is how many yards this peece containeth?

An-

Answere. Multiply his length by his breadth, that is, to wit $24\frac{1}{4}$ foote by $7\frac{5}{24}$ foote, and you shall finde $174\frac{77}{96}$ foote square, which 174 you shall diuide by 9 (for so many foote make a yard square) & you shal finde 19 yards the foote, and $\frac{77}{96}$ of a foote, and so many yards doth this peece hold.

Item, bought a peece of Holland cloath containing $36\frac{1}{3}$ elles Flemish. The question is, how many elles English it makes?

Answere. You must note, that fiue Elles Flemmish doe make but 3 elles English.

Therefore say by the rule of three, if 5 elles Flemmish make but three elles English, how many elles English wil $36\frac{1}{3}$ elles Flemmish make? Multiply and diuide, and you shall finde $21\frac{4}{5}$, and so many elles English doth $36\frac{1}{3}$ elles Flemish containe. The like is to be done of al others.

Item more, I haue bought 342 elles Flemmish, of Arras worke at two elles broad Flemmish, and I would line the same with elle broad Canuas of English measure. The question is, how many elles English will serue my turne?

Answere. For as much as three elles English are worth fiue elles Flemmish, therefore put 3 elles English into his square, in multiplying 3 by himselfe, which maketh 9. Likewise

wise multiply the English elle, which is fiue quarters, euery way into himselfe squarely, & you shall find 25. Then multiply 342 which is the length of the peece, by two which is the breadth, and thereof commeth 684, then say by the rule of three, as before: if 25 elles square of flemish measure be worth 9 elles square of English measure, what are 684 of flemmish measure? multiply and diuide, and you shall find 246 $\frac{6}{25}$ elles English.

The same is also wrought by the backer rule of three, in seking the squares contained in the Flemish elle of two elles broad (which are 18:) and also in seking the squares contained in the English elle (which are 25) then say by the rule of 3 backward, if 18 quarters require 342 elles, what shal 25 quarters giue? Multiply and diuide by the rule of three Reuerse, and you shal finde as before 246 $\frac{6}{25}$ elles English.

Item more, at two shillings foure pence the Flemish elle, what is the English elle worth after the rate?

Answer. Say if three quarters giue 3 $\frac{1}{3}$ s, what giueth fiue quarters? Multiply and diuide, and you shall finde 5 shillings 6 $\frac{4}{9}$ d.

Item more, at 8 s 4 d the Flemish Elle square, what is the English Elle worth after that rate.

Answer.

Answer. According to the reason of the last question, consider that a flemmish elle square is equall to 9 quarters of a yard English, and an English elle square is equall to 25 quarters of a yard Therefore say by the rule of 3, if 9 quarters giue 8 $\frac{1}{9}$ s, what 25 quarters? Worke and find 23 s 1 $\frac{7}{9}$ pence. And so much is the English elle worth.

Item more, at 6 s 8 d the elle square: what shall a peece of cloath cost that is 7 $\frac{1}{2}$ elles long and three $\frac{1}{4}$ elles broad.

Answer. Multiply the breadth by the length, and you shall finde 24 $\frac{1}{2}$ elles square cost 6 $\frac{2}{3}$ s, what 24 $\frac{1}{2}$: Multiply and diuide, and you shall finde 8 pounds, two s 6 pence, and so much the same peece of cloath cost.

Item more, a Mercer sold 3 peeces of silke, To wit, 24 $\frac{1}{4}$ 23 $\frac{1}{7}$, and 25 yards at 9 $\frac{1}{2}$ s the yard, and was glad to receiue in part of payment againe, a cloth containing 34 $\frac{1}{2}$ yards at 7 $\frac{1}{3}$ shillings the yard. The question is now, what the Debtor is in the Creditors debt. Worke and you shall find he oweth the Mercer two and twenty pounds—3 s—2 $\frac{1}{4}$ d.

The

The tenth Chapter intreateth of the reducing of Pawnes of Geanes into English yards.

NOte, that 100 Pawnes doe make 26 yeards, whereupon 3 Pawnes $\frac{12}{13}$ doe make one yard and one pawne after that rate and proportion is $\frac{13}{100}$ of a yard.

In 4563 pawnes of Geanes, how many yards English?

Answere. Say by the rule of three, if 100 Pawnes doe make 26 yards, what will 4563 Pawnes make? Multiply and diuide, and you shal finde 1186 yards $\frac{13}{100}$, So many yards doe 4563 Pawnes make.

Otherwise, take some other number at your pleasure, as 10 Pawnes, which is the $\frac{1}{10}$ part of 100, then to find his proportion, take the $\frac{1}{10}$ part of 26, which is $2\frac{3}{5}$, and then say also by the rule of three, if 10 pawnes giue $2\frac{3}{5}$ yards, what will 4563 pawnes giue? Worke, and you shal finde $1186\frac{13}{100}$ yeards, as before.

Note, at 2s 6 pence the pawne of Geanes, what will the English yard be worth after the rate?

Answere. Say by the rule of three, if $\frac{13}{100}$ of a yard cost $2\frac{1}{2}$ s, what one yeard? Multiply and

and diuide, and you shall finde 9 s $\frac{11}{15}$ d.

More, if 346 $\frac{1}{2}$ Pawnes cost 30 l — 13 s — 4 d sterling, what is that the English yeard after the rate?

Answere. Say by the rule of thrée, If 346 $\frac{1}{2}$ Pawnes cost 30 $\frac{2}{3}$ pounds, what are three $\frac{11}{13}$ Pawnes worth (for so many Pawnes make a yeard:) multiply and diuide, and you shall finde, $\frac{9200}{87039}$ parts of a pound, which in known numbers is worth 6 s 9 pence $\frac{5272}{90092}$.

The eleuenth Chapter intreateth of Rules of Loane and Interest, with certaine necessary Questions and Proofes incident thereunto, &c.

ITem, lent my friend 326 l for 5 $\frac{1}{2}$ moneths simply without any Interest, vpon condition, to haue the like curtesie againe when I néede. But when I came to borrow, he could spare me but 149 l — 8 s — 4 d. The question is now, how long time I ought to haue the vse thereof, to counteruaile my friendship before time shewed him.

Answere. Say by the backer rule of three, if 326 giue 5 $\frac{1}{2}$ moneths, what time will 149 pounds $\frac{5}{12}$ giue? Multiply and diuide, and you shall find 12 moneths, & so long time ought I

to

to vse his money.

The proofe.

Item, lent my friend 149 pounds—8 s—4 d, for 12 moneths. The question is now, how much money he ought to lend me againe for 5 ½ moneths to recompence my friendshippe shewed him.

Answere. Say by the backer or reuerse rule of 3: if 12 moneths giue 149 ½, what shall 5 ½ moneths giue. Worke and you shall finde 326 pounds; and so much ought he to lend me to requite my gentlenesse or good turne.

Two other Branches yet more for proofe out of the same question.

Item lent my friend 149 l—8 s—4 penie for 12 moneths, to haue the like friendshippe againe when I need. And comming to borrow of him, he very courteously tolde me 326 l (for that he could well then spare the same.) The question is now, how long I ought to occupy it, not vsurping friendship, but in his due time to restore it againe,

Answere. Say by the rule of three Reuerse, if 149 ½ giue 12 moneths, what shall 326 pounds giue. Multiply and diuide, and you shall finde, that at 5 ½ moneths terme, I ought to restore it againe.

Proofe

Proofe.

Item, lent my friend 326 l for 5 ½ moneths The question is now, how many pounds he ought to lend me for 12 months to recompence this pleasure againe.

Scholar. Worke by the rule of three Reuerse, as you haue done before, and you shall find 1491—8 s—4 d.

Againe, foure other selected questions of Loane and interest, all out of one branch, and each one also a necessary Question, and a particular proofe to other.

Item, lent my friend 430 l at interest for three moneths to receiue after the rate of 8 l in the 100 for 12 moneths. The question is what the interest commeth to.

You may if you please worke it at two workings by the rule of three direct, in saying: If 12 moneths giue 8 l, what giueth three moneths: multiply and diuide, and it giueth 2 pound.

Then for the second worke say, If 100 l yeelde two l, what yeeldeth 430 l: multiply and diuide, add you shal finde 8 l 12 s, and so much comes the loane of 430 l to for 3 moneths

neths, after the rate of 8 pounds in the 100 pounds of 12 moneths.

Otherwise wrought thus by the rule of 3 at twice also.

If 100 l giue 8 l, what giueth 430 l? multiply and diuide, and you shal find 34 pounds $\frac{2}{7}$. Then againe for the second worke say: If 12 moneths giue 34 pounds $\frac{2}{7}$, what giueth three moneths? Worke & find 8 l 12 s as before.

Otherwise yet at one working: By the first part of the rule of 5 numbers forward, in saying if 100 pounds in 12 monthes gaine 8 l, what shall 43 pounds gaine in 3 moneths? Multiply the first by the second for your diuisor, and the other three the one into the other for the diuidend, and you shall find 8 pounds 12 shillings, as afore said.

Proofe.

Item a friend of mine receiued of me eight pounds 12 shillings for the Interest and vse of 430 pounds for three moneths termes: the question is now, what hee take in the 100 pounds for 12 moneths after that rate.

Answer. For most brief, say by the first part or rule of 5 numbers forward: If 430 pounds in three moneths did pay 8 l 12 s, what doth 100 pounds in 12 moneths take after the rate Worke, & you shall finde 8 pounds & so much he take vpon the 100 pounds for 12 moneths.

A third queſtion and proofe alſowrought by the Backer Rule of 5 Numbers.

Item, lent my friend 430 pound to receiue for the Intereſt thereof, after the rate of 8 l in the 100 for 12 moneths. The queſtion is now how long time my friend ought to giue the vſe thereof, that it may be returned with 8 l — 12 s gaines.

You may worke it if you pleaſe by the rule of three direct at twice, in saying: If a 100 l yeld 8 l, what yeeldeth 430 l? Multiply and diuide, and find 34 l and ½.

Then againe for the ſecond worke ſay, if 34 ½ giue 12 moneths, what giueth 8 ½ pounds Multiply and diuide, and you ſhall finde three moneths, and ſo long time ought my friend to vſe it to returne with 8 l — 12 s gaine.

Otherwiſe at one working by the backer rule of 5 numbers, in saying: If 100 l in 12 moneths doe gaine 8 l, how long time ſhall 430 l be a gaining of 8 l — 12 s? Multiply the firſt and the ſecond into the laſt for your diuidend, and the third and fourth multiply together for your diuiſor, and then diuide, and you ſhall finde 3 moneths, the iuſt time that my friend ought to vſe it, to returne it with 8 l, 12 s gaine:

A Fourth deriued queſtion out of this Branch, which is a proofe of this laſt, and alſo of the other two going before.

ITem, how much money ought a Merchant to deliuer after 8 l in the 100 for 12 moneth, that in three moneths he may gaine 8 l 12 s?

Anſwer. You may alſo if you pleaſe worke it by the Golden Rule of three at twice, firſt ſaying, if 3 moneths gaine 8 l, what 12 moneths gaine? you ſhall finde 34 l. Then ſay againe, if 8 l become of 100 l, what ſhall come of 34 l 8 s? Worke and you ſhall finde the anſweere to the queſtion, which is 430 l, and ſo much ought the merchant to deliuer.

But moſt briefely it is anſwered by the Backer Rule of 5 numbers; where I argue thus, ſaying: If 100 l be 12 moneths a gaining of 8 l, then but for three moneths terme onely to take 8 l 12 s muſt needs bee a good round ſumme: to worke it, ſet your numbers thus, 100—12—8—3—8½: multiplying the firſt into the ſecond, and alſo by 43 the product of the fift, for your dividend, & the third and fourth together with 5 the Denominator of your fraction for your Diuiſor: then diuide and you ſhall finde as before, 430 l: the true ſolutions to your queſtion.

The

The twelfth Chapter intreateth of the making of Factors, which is taken in two sorts.

The first is, when the estimation of the factor, is taken vpon the sending of the merchant, as if the estimation of his person be $\frac{3}{4}$. it is vnderstood that he shall haue $\frac{1}{4}$ of the gain, and the merchant the other $\frac{3}{4}$.

The other sort is when the estimation of his making is out of the sending of the merchant, as if the order and agreament betweene them were such, that the merchant shal put in 800l, and the factor for his making shall haue $\frac{1}{4}$ neuertheleſſe he shall haue but $\frac{1}{5}$ of the gaine or profit, for the $\frac{1}{4}$ of 800 is 200 (for the estimation of his making) which with the 800 pound make 1000l, whereof the 200l, is $\frac{1}{5}$.

A Merchant doth put in 800 pound into the hands of his Factor, vnder such condition, that the said Factor shall haue $\frac{1}{5}$. And after certaine time, they finde in profit 124l 6 s 8 d, I demaund how much the merchant shall haue hereof, and how much ought the factor to haue?

Answer. When the estimation of the Factor is out of the sending of the merchant it maketh.

l	s	d	
99	9	4	for the Merchant.
24	17	4	for the Factor.

But if that his estimation bee at the sending of the merchant, then it maketh but,

l	s	d	
93	5	0	for the Merchant.
31	1	8	for the Factor.

For the Merchant is then to haue $\frac{3}{4}$, and the Factor $\frac{1}{4}$.

A Merchant doth put into the hands of his Factor 800l, and the Factor 400l to haue the $\frac{1}{3}$ part of the profit: I demaund now for how much his person is estéemed, when the same is counted vpon the sending of the Merchant?

Answer. According to the tenor and order before prescribed in the first rule, that is, if his estimate be $\frac{1}{4}$ he shall haue the $\frac{1}{4}$ of the gaine. Therefore say by the rule of three direct, If $\frac{1}{4}$ taken put in 400l, what is the estimate, or putting in of $\frac{1}{7}$ taking? Multiply and diuide, and you shall finde 320l, and so much is the person of the Factor estimated.

Otherwise.

To finde the estimation of the person of the

the factor, you shal consider, that seing it was agréed betweene them, that the factor should take the $\frac{1}{5}$, then the Merchant shall haue the residue, which are $\frac{4}{5}$: wherefore the gaine of the merchant vnto that of the factor is in such proportion as 5 vnto 4. Thē if you will know the estimation of the person of the factor: say, if 5 gine 4, what will 400 giue? Multiply and diuide, and you shall finde 320 l. And so much is the person of the Factor estéemed to bee worth.

Other conditions then these aforesaid, may also be betweene Merchants & Factors without respect either of sending or not sending of the Merchant, where most commonly the estimation of the body of the Factor is in such proportion of the stocke which the Merchant layeth in, as the gaine of the said factor is vnto the gaine of the merchant As thus, If a merchant doe deliuer into the hands of his factor 400 pound, and he to haue halfe the profite. The person of the said factor shal be estéemed to bee worth 400 pound, and if the factor doe take but $\frac{1}{5}$ of the gains, he should haue but $\frac{1}{5}$ so much of the gain as the merchant taketh, which must haue $\frac{4}{5}$, wherefore the person of the factor is estéemed but the $\frac{1}{5}$ of that which the merchant layeth in, that is to say, 200 pound.

And if the Factor did take the $\frac{1}{4}$ of the gain,

then the merchant shal take the residue which are $\frac{2}{3}$, wherefore the gaine of the Merchant vnto the factor is then in such proportion as 3 vnto 2; whereupon if you will then know the estimation of the person of the factor, say, if 3 giue 2, what shall 400 giue? Worke, & you shall finde 266 $\frac{2}{3}$ pounds. And so much is the person of the factor esteemed to be worth.

And if the merchant should delined vnto his factor 400 pound, and the factor would lay in 80, & his person, to the end he might haue the $\frac{2}{3}$ of the gaine, I demaund how much shall his person be esteemed?

Answere, Abate 80 from 400, and there will remaine 320. And at so much shall his person be esteemed.

A merchant hath delinered vnto his factor 900 l. to gouerne in the trade of Merchandise, vpon condition that he shall haue the $\frac{1}{3}$ of the gaine, if any thing be gained, & also to beare the $\frac{1}{3}$ of the losse, if any thing be lost. Now I demaund how much his person was esteemed at?

Answere. Seing that the factor taketh the $\frac{1}{3}$ of the gaine, his person ought to be esteemed as much as $\frac{1}{3}$ of the stock, which the merchant layeth in: That is to say the $\frac{1}{3}$ of 900 pound, which is 450. The reason is, because $\frac{1}{3}$ of the gaine that ther factor taketh, is the $\frac{1}{3}$ of the $\frac{1}{3}$ of the gaine that the merchant taketh, and so

the

the Factor his person is estéemed to be worth 450 pounds.

A Merchant hath deliuered vnto his factor 600 pound, and the factor layeth in 250 pound and his person. Now because he layeth in 150 pounds and his person it is agréed betweene them, that he shall take the $\frac{3}{7}$ of the gaine. I demaund for how much his person was estéemed?

Answere. For as much as the Factor taketh $\frac{3}{7}$ of the gaine hee taketh $\frac{3}{7}$ of that which the Merchant taketh, for $\frac{3}{7}$ are the $\frac{3}{7}$ of $\frac{1}{7}$. And therefore the factors laying in, ought to bee 400 pound which is $\frac{2}{3}$ of 600 pound that the merchant layed in. Then subtract 250, which the Factor did lay in, from 400 pound, which should haue beene his whole stocke, and there remaineth 150 pound for the estimation of his person.

More, a Merchant hath deliuered vnto his factor 1800 pound, vpon condition that the factor shall haue the gaine of 160 l, as though he laid in so much ready money: I demaund what portion of the gaine the said factor shall take?

Answer. See what part the 160 (which the Factor laid in) is of 960, which is the whole stocke of their company, and you shall finde $\frac{1}{6}$. And such part of the gaine shall the factor take.

But in case, that in making their cou-

nants, it were so agréed betwéene them that the Factor should haue the gaine of 160 l of the whole stocke which the Merchant layeth in, that is to say of the 800 pound, then should the Factor take $\frac{1}{5}$ of the gaine: for 160 is $\frac{1}{5}$ of 800 pound.

The thirteenth Chapter intreateth of Rules of Barter, and exchanging Merchandise, which is distinct into 7 Rules with diuers other necessary Questions incident thereunto.

The first Rule.

Two Merchants willing to change their merchandise the one with the other. The one hath 24 broad cloathes at 10 l—10 s the peece. The other hath Pace, at 12 s the l. The question is, how many pounds of Pace he ought to giue for his clothe, to saue himselfe harmelesse and be no loser.

Answer. Seeke first by the Rule of three, what the 24 clothes cost at 10 l—10 s the peece, and you shall finde 252 l. Then to finde the quantity of Pace, say againe by the Rule of 3, if 12 s buy one pound, what shall 252 pound buy me? Worke, and you shall find 420 pound of Pace: and so many pound ought he to giue for his cloaths.

The

The Proofe.

Two barter, the one hath 420 pounds of Pace at 12 s the l', to barter or change broad clothes at 10 pounds – 10 s the peece. The question is, how many broad cloathes hee ought to giue for all his Pace.

Answer. First say, if one coſt 12 s, what 420? You shall find 5040 s. Then say again, if 10 ½ pounds giue 1 cloath, what shall 5040 shillings giue? Worke, and you shall find 24 cloathes your desire.

The second Rule.

Two change merchandise for merchandise. The one hath Pepper at 2 shillinge 4 pence the pound, to sell for ready money. But in barter he will haue no lesse than three shillings the pound. And the other hath Holland at 5 s 6 pence the elle ready money. The question is now at what price hee ought to deliuer the elle in the barter to saue himselfe harmelesse.

Answer Say by the Rule of three direct: if 2 ¼ s ready money giue 3 s in barter what shall 5 ½ giue in barter? you shall finde 7 $\frac{1}{11}$ s, and at that price ought the second Merchant to sell his Holland in barter.

The

The proofe.

Two barter. The one hath Holland at 5 s. 6 pence the elle to sell for ready money. And in barter he will haue $2\frac{1}{12}$ s. The other hath Pepper at 2 s 4 d the pound to sell for ready money. The question is now how he ought to sell in barter?

Answere. Say by the rule of 3 direct, if 5 $\frac{1}{3}$ ready mony giue $7\frac{1}{12}$ s in barter, what ought $2\frac{1}{3}$ s to take in barter? Multiply and diuide, and you shall find 3 s your desire.

The third Rule.

Two barter. The one hath cloth of Arras at 30 s the elle ready mony, but in barter he will haue 35 $\frac{1}{2}$ s. And the other hath white wines, which he deliuered in barter for 16 l the tunne. The question is now what his wines cost the tunne in ready money.

Answer. Say by the rule of three direct, If 35 $\frac{1}{2}$ s in barter giue but 30 ready money, what did 16 l in barter cost? Worke and you shall finde 13 l 10 s $\frac{10}{71}$. And so much cost his wines for a tunne ready money.

The proofe.

Two barter merchandise for merchandise: The one hath wines white at 13 l — 10 s $\frac{10}{71}$

the

the Tunne to sell for ready money. But in barter he deliuered it for 16l'. The other to make his match good and saue himself harmelesse, deliuereth Arras at 35 ½ s the elle. The question is now, what an elle of his Arras cost in ready money.

Answer. Say by the rule of three direct: If 16l' in barter giue but 12—10—$\frac{10}{7}$ s in ready mony, what shal 35 ½ s yeeld in barter? worke and you shall find 30 s your desire.

The fourth Rule.

Two barter. The one hath Kersey at 14 l' the peece ready mony. But in barter he will haue 18l'. And yet he will haue the $\frac{1}{7}$ part of his ouerprice in ready money. And the other hath ginger at eight groats the l' to sel for ready money. The question is how he ought to deliuer the ginger by the pound in barter to saue himself harmeleſſe, and make the barter equall.

Answere. Item for the working of this question, & such other the like, you must vnderstand, if the party ouersselling his wares require to haue also som portion in ready money, as $\frac{1}{3}$ $\frac{1}{4}$ &c. Then shall you first rebate the same demaunded part whatsoeuer it bee from the ouerprice, and also from the iust price. And those two numbers that shall remaine after the subtraction is made, shall be

the

the two first numbers in the rule of three. And the last price of the same merchandise shall be the third number, which by the operation of the rule of three direct shall yeld you a true solutiō how and at what price you shal ouer-sell that your merchandise, to saue your selfe harmlesse, and make the barter equall.

Example.

Take the $\frac{1}{4}$ (of eightéene) which is the ouer price of his cloath, which $\frac{1}{4}$ of eightéene is fiue, which as appeareth here in the margent, you must subtract from 18, there rest 12. And also a- bate it from 14, which is the last price of the cloath, and there re- maineth 8, which 8 and 12 are the two first numbers in the rule of three. Then take eight groats or two $\frac{1}{3}$ shillings for the third num- ber. Then say by the rule of three direct: If 8 I giue 12 pounds, what shal $2\frac{1}{3}$ s giue? Mul- tiply and diuide, and you shall finde 4 s. And for so much shall the second Merchant sell his Ginger, or his commodity in barter, to bal- lance the same equall.

14	18
6	6
8	12

The proofe.

Two barter: The one hath fiue Prieyes at 14 pounds the peice ready money. But in bar-

barter he wil haue 18 pounds, and yet he wil haue the $\frac{1}{3}$ part of his ouerprice in ready monie: And the other hath Ginger, which he hauing cunning inough to make the barter equall, deliuered in barter for 24 s the l'. The question is now, what his ginger cost him in ready mony?

Answer. After you haue made the subtraction, abating 6 the $\frac{1}{3}$ part of 18, both from 18 and 14, (as be for was taught you:) then wil there remaine 8 and 12 for your two first nũbers in the rule of 3. Then say, if 12 giue 8, what shall come of 4 the ouerprice of the l of Ginger? Multiply and diuide, and you shall finde two shillings 8 d your desire.

Two Merchants barter merchandise for merchandise. The one hath Denshire whites at 7 l—13 s—4 d the peece ready money: but in barter he doth them away for 8 l—3 s 4 d. and yet he will haue the $\frac{1}{3}$ part of his price in ready money. And the other hath Cottens at 3 l the peece ready money. The question is now, at what price he ought to sel or exchange his Cottens in barter to saue himselfe harmlesse, and make the barter equall?

7——13——4	8——3——4
2——14——$5\frac{1}{3}$	2——14——$5\frac{1}{3}$
4——18——$10\frac{2}{3}$	5——8——$10\frac{2}{3}$

An-

Answere. First seek the $\frac{1}{3}$ part of 8 — 3 s — 4 d, which is, 2 l — 14 — 5 $\frac{1}{3}$ d, which rebate from 8 — 3 — 4 d, there resteth; as appeareth by the example aboue sayd, 5 — 8 - 10 $\frac{1}{3}$ pence, which $\frac{1}{3}$ of 8 — 3 d also rebated from 7 — 13 — 4 d, there resteth 4 - 18 — 10 $\frac{2}{3}$, the two first numbers in the rule of 3, & the 3 l', which is the next price of the péece of Cotten, is the third number. Then say by the rule of three direct as was taught before: If 4 — 18 10 $\frac{2}{3}$ d did giue me 5 l' - 8 s — $\frac{2}{3}$ d, what shall 3 l' giue Multiply and diuide, and you shall finde 3 — 6 — 0 $\frac{72}{89}$ pence, the iust price that he ought to deliuer his Cottens in barter.

The fift Rule.

Two Merchants wil change merchandise for merchandise. The one hath Kerseis at 40 s the péece to sel for ready mony. And in barter he will sell them for 56 s 8 d, and he wil gain after 10 l' vpon the 100 l. And yet hee will haue the $\frac{1}{2}$ of his ouerprice in ready mony. The other hath flaxe at 3 d the l', ready money. The question is now, how he shall sell the pound of this flaxe in barter?

Answere. Sée first at 10 l' vpon the 100 l, what the 56 $\frac{2}{3}$ commeth to, in saying (by the rule of three direct) if 100 l, giue 110 l, what 56 $\frac{2}{3}$ s? Multply and diuide, and you shal find 3 l — 2 s — 4 d, of which the $\frac{1}{2}$ that he deman-

deth

deth in ready money, is 1 l——11 s—— 2 d the sum 31 s——2 d abated from 40 s, and also from 56 s—8 d: there will remain 8 shillings 10 d, and 25 s—6 d, for the two first numbers in the rule of three, 3 d, the price of the pound of flax for the third number. Then multiply and diuide, and you shall find $8\frac{11}{17}$ d. And for so much shall he sel the pound of flax in barter.

The sixt Rule

Two are willing to exchange merchandise. The one hath Norwich Grograins at 25 s the peece ready mony: and in barter he wil haue 30 s, & he wil haue the $\frac{1}{4}$ part of his ouer price in ready money. The other hath Norwich stockings at 40 s the dozen to sell for ready mony. But in as much as the first Merchats Grograines are no better, he would deliuer them so to ballance the barter, that hee may gaine after 10 l in the 100 l. The question is now, how he shal sell his hose the dozen in barter according to his request.

Answere. Say if 100 giue 100, what shall 40 s giue, which is the iust price of the dozen of stockings? Multiply and diuide, and you shall finde 44 s. Then take the $\frac{1}{7}$ of 30 s, which is 7 s——6 d. And subtract it from 25 s and also from 30 s, and there will remaine

17 s——6 d——£22 s——6 d, for the two first numbers in the rule of three, and 44 shillings, which is the iust price (with his gain in the dozen of stockings) for the third number. Then multiply and diuide, and you shal find 36 s 6 ½: and for so much he is to sel his dozen of stockings in barter.

The seuenth Rule.

Two Merchants will change their Merchādise one with the other. The one hath 720 Elles of Cambricke at 5 s the Elle to sell for ready money, but in barter he requireth 6 s —8 d. And yet notwithstanding hee loseth by it after 10 pounds vpon the 100 l, wherevpon he requireth one halfe of his ouerprice in ready mony, and the other Marchant hauing skill enough to make the barter equall, deliuereth English Saffrons at 30 s the pound. The question is now, what his Saffrons cost the pound in ready money?

Answer. You must first seeke what is lost vpon the 100 l, which to doe you may say (if you please.) If 100 l loose 10, what shall 6 ½ lose? Worke, & you shall find ½ s (or 8 d) which must be rebated from 6 s —8 d. So resteth 6 s still. Or you may say, if 100 l giue me but 90 pounds, what shall 6 s —8 d giue? Worke this way either, and you shall find also as before, directly in your quotient 6 s your de-

sire

fire. Then are you next to cast vp what the 710 Elles of Cambricke commeth to at 6 s 8 d the Elle, and you shall find 240 pounds: the ½ whereof the Cambricke Merchant will haue in ready money (which is 120 l:) nextly you must cast what the Cambrick commeth to after his losse in the 100 l, which as you see and is but 6 s an elle, and you shall find 216 pounds. Now must you subtract his ready money (which is 120 l) out of 240 l, and also out of 216 l, and there will remaine 120 pounds, and 96 pounds for your two first numbers in the rule of three and 30 shillings is the ouer-price of your Saffron for the third number. Then multiply and diuide, and you shal find 14 s. And so much did his Saffron cost in ready monie.

Two Merchants barter. The one hath 50 Clothes to put away for ready mony at 11 l' the cloth, and in barter putteth them away for 12 pounds, taking Holland cloth at 20 d the Flemmish Elle, which was worth no more but 18 d. The question is now, what Holland payeth for the Cloth? and what hee winneth or loseth by the bargaine?

Answer. 50 Clothes at 11 l' the Cloth commeth to 550 l, and put away at 12 l the peece maketh 600 l. Then to finde what Holland payeth for the Cloth, say by the rule of three direct, If 20 d buy 1 Elle, what 600 l? Work

and you shall finde 7200 elles. Now to find the estate of his gaine or losse, you must seeke what his 7200 elles commeth to at 18 d the Elle: Worke by the rule of proportion direct, and you shall finde 540 pounds, which is not so much as his clothes were worth in readie mony by 10 l: and so much lost the first Merchant by his exchange.

A Venetian hath in London, 100 peeces of silke, to put away for ready mony at three l the peece. But in barter he deliuered them for 4 l the peece, taking wools of a Felmonger 7 l 10 s the 100 weight, which was worth no more but 6 l the 100 ready money. The question is now, what wools payeth for the silks, and which of them winneth or loseth by the barter?

Answer. 100 peeces of silke at 3 pound, is 300 l, and 4 l is 400 l. Then to finde what wooles payeth for the silke, say by the rule of three direct, If 7 ½ buy me 100 waight, what 400 l? Worke, and finde 5333 1/3 100 weight of wool. Now to find the estate of their gaine or losse, cast vp his wool at 6 l the 100 (for so much they were worth ready money) and you shall finde 320 pound, which is 20 pound more than the silkes were to be sold for ready money, whereupon the Venetian gained 20 pounds by the barter.

A

A Merchant hath 53 1/3 C. waight of wool at 6 l. the C. to sell for ready mony, but in barter he will haue 7 pounds—10 s: and an other doth barter with him for silkes, which are worth 3 l. a peece ready money. The question is now, how he ought to deliuer his silkes the peece in barter, and how many payeth for the woles?

Answer. Say, by the Rule of Proportion, (or by the rule of three direct) If 6 l. for 100 weight ready mony yeld me 7 l.—10 s, what wil 3 pound yeld, which is the iust price of a peece of silke in barter, to make the Truck equal? Worke and find 3 pounds—15 s, the price of a peece of silke in barter: then say if 3 pound 15 s require 1 peece of silk, how many peeces of silkes are bought with 400 pound, which is the value of the 53 1/3 C. weight of wol at 7 l.—10 s? Worke by the rule of 3. direct, and you shal find 106 peeces of silke and 2/3 of a peece, and so many peeces of silk paieth for the wool, and neither party hath aduantage of other.

Two men wil change merchandise the one with the other. The one of them hath Beere at 6 s 8 d the barrell to sell for ready money. But in Barter he will sell the barrell for 8 s, and yet he will gaine moreouer after 10 l vpon the 100 l. And the other hath white Spanish wool at 20 s the Roue, to sell for readie monis. The question is now, how he shall

deliuer the Roue of wool in barter to saue himselfe harmelesse.

Answere. Say, if $6\frac{2}{7}$ s, which is the iust price of the barrel of Beere, be sold in barter for 8 shillings: for how much shal 10 shillings (which is the iust price of the Roue of wooll) be sold in barter? Worke by the rule of three direct, and you shall finde 24 s. Then for because the first Merchant will gaine after 10 l' vpon the 100 l he maketh his 100 l' —— 110 l. And therefore say by the rule of three, if the second Merchant of 110 l' doe make but 110 l', how much shall he make of 24 s? Multiply and diuide, and you shall finde 21 s —— 9 d $\frac{1}{11}$ of a peny. And for so much shall hee sel the Roue of wol to be deliuered in barter, to the end the first Merchant may gain 10 in the 100.

Two Merchants will change their commodities the one with the other. The one of them hath white paper at 4 s the reame, to sel for ready mony. And in barter he will do it away for 5 s, and yet he will gaine moreouer after the rate of 10 l vpon the 100 l. And the other hath Pace at 14 s — d the pound waight to sell in barter. Now I demaund what the pound did cost in ready money?

Answer. Say, if 5 s, (which is the ouer price of the paper in barter) become at 4 s the iust price, of how much shall come 14 — s $\frac{}{}$, which is the surprise of the pound of Pace in barter?

harter: Multiply & diuide, and you shall find 11 s ½. Then for because the first Merchant of Paper will gaine after 10 vpon the 100. Say, if 100 doe giue 110, what shall 11 ½ shillings giue? Worke, and you shall finde 12 s—9 d ⅓: and so much did the pound of Pace cost in ready money.

The fourteenth Chapter intreateth of Exchanging of money from one place to another.

Exchange is no other thing than to take or receiue money in one City to render or pay the value thereof in another City: or else to giue money in one place, and receiue the value therof in another, at tearm of certaine daies, monethes, or faires, according to the diuersitie of the place.

But this practice chiefly consisteth in the knowledge of the money or Coines in diuers places, of which, for thy benefite (after a few examples giuen to the introduction of this worke) I will set downe certaine notes of the diuersitie of the common and vsuall coines in most places of Christendome for traffique.

And first I will begin at Antwerpe, where they vse to make their accounts by Deniers de gros,

gros, that is to say by pence Flemmish, whereof 12 doe make 1 s Flemmish, and 20 s doe make one pound de gros.

Item, a Merchant deliuered at Antwerpe 400 pounds Flemmish to receiue in London 20 s sterling, for euery 23 s — 4 d Flemmish The question is now, how much sterling money is to bee receiued at London for the saide 400 pounds of Flemmish?

Answere: Say by the rule of three, if 23 $\frac{1}{2}$ Flemmish giue 20 s sterling, what 400 l Flemmish? Worke you shall find 342 l 17 s 1 $\frac{3}{7}$ pence, and so much sterling shall I receiue in London for the sayd 400 l Flemmish.

Otherwise also wrought by rules of Practise in taking the $\frac{1}{7}$ of the Flemmish money deliuered, & abating the same from the principall, the rest in English money sterling, as before.

400 " 0 " 0

57 " 2 " 10 $\frac{3}{7}$

342 " 17 " 1. $\frac{3}{7}$ sterling.

A Merchant at London deliuered 200 l sterling for Antwerpe at 23 s — 5 d Flemmish the l' sterling. The question is, how much he must receiue at Antwerpe?

Answere. Say by the Rule of three, if 1 l sterling

sterling, giue 23 s 5 d flemmish, what 200 l sterling? Worke, and you shall finde 234 l —3 s—4 d. So many pounds flemmish shal he receiue at Antwerp for the said 200 pounds sterling.

Otherwise by practice.

1	23	5	200			
3 s	4 d			33	6	8
1 d					16	8

maketh 234 l 3 4 sterling.

In London 200 pound sterling is deliuered by Exchange, for Antwerpe at 23 s 9 pence flemmish the l' sterling. The question is, at what rate the flemmish money ought to bee returned to gaine 4 pound vpon the 100 l' sterling at London?

Answere. First say by the Rule of thrée direct: If 1 pound sterling giue 23 $\frac{1}{4}$ flemmish, what 200 pounds sterling? Multiply & diuide and you shall finde 237 pounds —10 s. The which to returne to gain 8 pounds sterling in London, say by the backer rule, if 200 poūds sterling require the exchange 23 s—9 d flemmish, what the exchange to make 208 l' sterl? Worke by the rule, and finde 22 s—10 d $\frac{1}{26}$ d flemmish, the effect in the question required.

If I take vp money at Antwerpe after 19 s-4 d Flemmish, to pay for the same at London 20 shillings sterling, and when the day of paiment is come, I am forced to returne the same money againe in London, to pay my bil of exchange: so that for 20 shillings which I take vp here at London, I must pay 19 s 6d at Antwerp I demaund whether I do win or lose? and how much in or vpon the 100 l' of money?

Answer. Say by the Rule of three: If 16 $\frac{1}{2}$ giue 19 $\frac{1}{2}$, what wil 100 l' giue? Multiply and diuide, and you shall find 99 l-2 s $\frac{106}{117}$ which being abated from 100, there wil remaine 17 shillings $\frac{11}{117}$, and so much I do lose vpon the 100 l' of money.

If I take vp at London 20 shillings sterl. to pay at Antwerpe 21 s —4 d, and when the day of paimẽt is come, my Factor is constrained to take vp money againe at Antwerpe, wherewith to pay the aforesayd summe, and there he doth receiue 23 s —4 d Flemmish, for the which I must pay 20 s at London: the question is now, whether I do win or lose? and how much vpon 100 l' of money after that rate?

Answer. Say by the Rule of proportion. If 21 s giue 23 $\frac{1}{2}$ s, what will 200 pounds giue? Mul-

Multiply and diuide, and you shall finde 104 pounds 9 s $\frac{17}{17}$, from the which abate 100 l, & there will remaine 4 pounds 9 s $\frac{17}{17}$, and so much is there gained vpon the 100 pounds of money.

In Antwerpe is deliuered 200 pounds Flemmish by exchange for London at 20 s sterling for euery 23 s—4 d Flemmish. The question is, at what rate the same is to be returned to gaine 10 pounds vpon the 100 pounds Flemmish in Antwerpe.

Answere. First, say by the rule of three, if 23 $\frac{1}{3}$ flemmish giue 20 s, what shal 200 pounds gaine? Worke and you shall find 171 pounds ——8 s——9 $\frac{1}{7}$ d. Then say againe by the rule of three direct, if 171 l—8 s—6 $\frac{5}{8}$ sterling, giue me 110 l Flemmish, what shall 20 s sterling giue? Worke and you shall find 24 s 6 d Flemmish. And at the same rate ought the same to be returned at Antwerpe to gaine 10 l vpon 100 Flemmish.

A Merchant of Antwerpe deliuereth 234 l—3 s—4 d Flemmish, to receiue at London 200 pounds sterling: the question is now how the exchange goeth after this rate?

Answere. Say by the rule of three direct, If 200 giue 20, what doth one giue? multiply and diuide, and you shall finde 23 shillings ——5—d. And so much goeth the Exchange.

Item, the Exchange from London into France

France is not like as it is in Flaunders, but is deliuered by the French Crowne, which is worth 50 soulx Turnois the pece.

Whereupon also you must note, that in France they make their accounts by Francks soulx, & Deniers Turnois, whereof 12 Deniers maketh one soulx Turnois, & 20 soulx maketh 1 l Turnois, which they call a Liure or Franke. But the Perchants to make their accounts do vse French crowns; which is currant among them for 51 soulx Turnois. But by exchange it is otherwise, for it is deliuered but for 50 soulx Turnois in Crown, or as the taker vp of the money can agree with the deliuerer, And note that this ℔ character representeth the Crowne by exchange, and is euer 50 soulx Turnois for French money.

A Perchant deliuereth at London 240 l sterling after 5 shillings—6 pence the crown, to receiue at Paris 50 soulx Turnois for euery Crowne. I demand how much Turnois or french money payeth the bils for the said 240 pounds sterling.

Answer. Say by the rule of three, if 1 l sterling giue me 50 s Turnois, what shall 240 l sterling giue? Reduce the pounds into shillings, then multiply and diuide, and you shal finde 2181 Liures—16 soulx, 4—Deniers

niers, and $\frac{4}{17}$ Turnois, and so much payeth the bils at Paris for the 240 pounds sterling.

A Merchant deliuereth at Roan, or elswher in France 1430 pound or Franks, the which franck or pound is 20 soulz, or a pound Turnois, to receiue in London 6 s 4 d sterling for euery △ of 50 soulz Turnois. The question is, how much sterling money I ought to receiue at London for my 1430 pound Turnois.

Answere Say, if two $\frac{1}{7}$ pounds giue me $\frac{1}{3}$ s, what will 1430 giue me? Worke, and you shall finde 3 622 $\frac{2}{7}$ sterling, which maketh 18 $\frac{1}{2}$ pounds 2 s 8 pence, and so much money is to be receiued at Londō for the said 1430 Liures Turnois, after 6 s 4 pence euery △ of 50 soulz.

In London is deliuered 200 pound sterling by exchange for Paris, at 5 s—9 d the △ of 50 soulz Tournois. The question is, at what price the said △ is to bee returned to gaine six pounds vpon the C. pounds sterling at London.

Answer. First say (by the rule of three direct) if 5 $\frac{1}{4}$ s sterling giue 50 soulz Tournois, what shall 200 pounds sterling giue? Worke, & you shal finde 1739 Francks or Liures, two soulz $\frac{24}{77}$. Then, the which to returne and gain 6 pounds vpon the 100 l in London, say by the rule of 3 direct, if 1739 Francks, two soulz $\frac{24}{77}$

yeld

field 212 pounds, what the ^ of 50 Soulx? worke, and finde 6 s—1 d $\frac{7}{5}$. the effect required in the question.

A Merchant delivered in London 160 l sterling, to receive in Biskay for every 3 s 6 d, 1 Ducket of 374 Marueids. The question is, how many Marueids ought I to receive at Biskay?

Answere. Say, if 5 $\frac{1}{2}$ s sterling give 374 Marueids, what shall 160 l. sterling give? Multiply and divide, & you shall find 217600 Marueides, and so many I ought to receive at Biskay for my 160 pounds sterling.

A Merchant delivered in Bayon 40000 marueides to receive in London 5 s—8 d sterling for every Ducket of 274 Marueids. The question is now, how much sterling money paieth the Bils of Exchange for the said 20000 Marueids?

Answer. Say, if 374 Marueids make 1 Ducket, what 40000 Marueids? multiply and divide, and find 106 Duckets $\frac{178}{187}$.

Then say againe, if 1 Ducket give 5 $\frac{1}{3}$ s, what giueth 106 $\frac{178}{187}$ Duckets? Worke, and find 30 pounds——6 s, and $\frac{34}{561}$ s: which is worth $\frac{408}{561}$ parts of a peny.

Otherwise it is wrought more briefe at one working, as in the last question before, in

con-

considering that 5 s, 8 d containeth one Duckate, or 374 Marueides. Therefore say by the rule of three, if 374 Marueides giue 5 $\frac{1}{3}$ s, what 40000 Marueides? Worke, and you shall also finde in your quotients 30 pounds 6 s $\frac{14}{187}$. And so many pounds sterling is to be receiued for the 40000 Marueids.

In London 200 l deliuered by Exchange for Vigo, 374 Marueids the Duckate of 5 s, 10 d sterling, maketh 25647 $\frac{1}{7}$ Marueids: the which to returne and gaine 10 l vpon the 100 pounds in London, say by the rule of 3 direct, if 210 pounds require 25457 $\frac{1}{7}$ Marueids, what 5 s 10 d? Worke, & fine 340 Marueids, prices of euery Duckate in returne: which is the effect in the question required.

These may seeme sufficient for instructions.

NOtwithstanding for the further aid and benefit, hereafter follow 6 speciall & most briefe rules of practise for English, French, & Flemmish money.

1 how to turne Flem. to Eng. sterling.
2 how to turne Eng. sterling to Flem
3 tea. how to turne Flemmish to French.
4 cheth how to turne French into Flemish.
5 how to turne sterling into French.
6 how to turne French into sterling.

The

The fifteenth Chapter intreateth of the said six Rules of Breuitie, and of valuation of Englisſh, Flemiſh, and French money, *and how each of them may eaſily be brought to other* value.

How briefely to reduce pounds, ſhillings, and pence Flemiſh, into pounds, ſhillings, and pence Engliſh ſterling.

IT is to be noted, that 71 Flemiſh maketh but 6 pounds ſterling: 7 s flemiſh, maketh 6 s ſterling, and 7 d flemiſh 6 d ſterling, ſo that 7 yeldeth but 6. wherin is euident that there is loſt $\frac{1}{7}$. (if it may be ſo called) when it is reduced into Engliſh money. Wherefore to know how much 1351, 13 s, 4 d, flemmiſh maketh Engliſh, you muſt ſubtract from it $\frac{1}{7}$, beginning with the pounds, &c. & that which reſteth after this ſubtraction is the ſumme required: ſo that 2331, 13 s 4 d flemiſh maketh 200 pounds, 5 s 8 $\frac{4}{7}$ ſterling.

Example.

l	s	d
233	13	4
$\frac{1}{7}$ 33	7	7 $\frac{4}{7}$
200	5	8 $\frac{4}{7}$ ſter.

Another example

l	s	d.
311	0	0
$\frac{1}{7}$ 44	8	6 $\frac{4}{7}$
264	11	5 $\frac{1}{7}$

To

To reduce pounds, shillings, & pence sterling into pounds, shillings and pence Flemmish.

Note that a pound sterling maketh 1 l, 3 s, 4 d flemish: that is, 1 l $\frac{1}{6}$: 1 s sterling maketh 1 s $\frac{1}{6}$ flem and 1 ster. maketh 1 $\frac{1}{6}$ flem. So that there is gained (if it may be so called) $\frac{1}{6}$ of the sum being thus reduced to flem: for of $\frac{6}{6}$ is made $\frac{7}{6}$, which is one whole and $\frac{1}{6}$. Then to know how much 237 l, 7 s, 6 d ster. maketh flem. subtract from your ster. the $\frac{1}{6}$ of the whole sum, and adde it to the same sum, & it maketh 276 l, 18 s, 4 d, which is the sum required.

Example.

l	s	d	
237	7	6	
$\frac{1}{6}$ 39	11	3	
276	18	9	ster.

Another Example.

l	s	d
337		
$\frac{1}{6}$ 56	3	4
393	3	4

We shall note that the equality of flemmish and french money is this, that is to say, the pound flemmish maketh 7 pound $\frac{1}{3}$ french, or Tournois, 1 s flemmish maketh 7 s $\frac{1}{3}$ french, and a groat flemish, maketh 7 d $\frac{1}{3}$ french.

Wherefore to know how much 143 l 4 shillings 9 d flemmish maketh french, yee must multiply the whole number twice by 6, begin-

ning

ning at 8 d, and so forward, and the product of your second multiplication diuide by 5, so the worke is finished. Or multiply the said sum by 7, and take out of it $\frac{2}{5}$, adding it to the product of your multiplicatiō by 7, & that is your number required. So that as well by the one as by the other, 143 l, 4 s, 9 d Flemmish, maketh 103 l — 6 s — 2 d $\frac{3}{7}$ French or Turnois.

Example.

l	s	d	
143	— 4	— 9	flem
		6	
859	— 8	— 6	
		6	
5156	— 11	— 0	
$\frac{3}{7}$ 1031	— 2	$\frac{3}{7}$	fren

The same otherwise.

l	s	d	
133	— 4	— 9	
		7	
1002	— 13	— 3	
$\frac{2}{5}$ 28	— 12	— 11	$\frac{3}{7}$
1031	— 6	— 2 $\frac{3}{7}$	fren

Another example.

143 l	Flem	
6		
858		
6		
5148		
$\frac{2}{5}$ 1029 l	0 s	12 s Frē

Or thus.

143	
7	
1001	
$\frac{2}{5}$ 28	— 12
1029 l	— 12 — french.

A briefe reduction of l, s and d, French, into l, s, and d Flemmish.

Multiply 233 l—8 s 4 d French by 5, & diuide the product twice by 6, that is, the said number by 6, & the product againe by 6, & the quotient of this second diuision is the thing required. So that 233 l—8 s—4 d French, maketh 321, 8 s, 4 d $\frac{5}{6}$ Flemmish.

Example.			Another.		
l	s	d	l	s	d
233	—8	—4 fren·	753		Fren.
		5	5		
1167	—1	—8 $\frac{5}{6}$	2756		
$\frac{5}{6}$ 194	10	—3 $\frac{2}{6}$	$\frac{5}{6}$ 627	—10	
$\frac{5}{6}$ 52	—8	—4 $\frac{5}{6}$ Flem.	$\frac{5}{6}$ 104	—11	—8 flem

To reduce l, s, and d Sterling into l, s, and d French, or Tournois.

The l sterling maketh 8 l, 8 s French, that is to say, 8 l $\frac{2}{3}$: the s maketh 8 s $\frac{2}{3}$, and the peny 8 d $\frac{2}{3}$ French. Wherefore to know what 231 l 13 s, 4 d sterling maketh French, ye must multiply your whole summe by 42, that is, by 7, and the product of it by 6, and diuide this

second product by 5, and that is the summe required.

Otherwise, multiply the summe sterling by 8, and adde twice to the product $\frac{2}{7}$, and it shall produce the sum required. So that both waies 231 l, 13 s 4 d sterling, maketh 1946 pound french, as hereunder followeth.

Example.

l	s	d	
231	13	4	ster.
		6	
1390	0	0	
		7	
$\frac{1}{7}$ 9730	0	0	
$\frac{1}{7}$ 1946	0	0	fren.

the same otherwise.

l	s	d.	
231	13	4	ster.
		8	
1853	6	8	
46		6	8
46		6	8
1946	0	0	fren

Another example.

753	sterl.
6	
4518	
7	
31626	
$\frac{1}{8}$ 6325 — 4	Fren.

The same.

753	ster.
8	
6024	
$\frac{2}{8}$ 150 — 12	
150 — 12	
6325 — 4	fren.

To

To reduce pounds, shillings, and pence French into pounds, shillings and pence Sterl.

To know how much 1256 l. 12 s. 6 d fren. maketh in sterling money, multiply the sum by 5 and diuide the product by 76 at twice: and the last quotient shall bee the thing required, that is to say, 1256 l. 12 s. 6 d. maketh 149 pounds, 11 s 11 d $\frac{4}{7}$ sterling.

Example.

l	s	d	
1259	18	6	fren.
		5	
0283	— 2	— 6	
$\frac{1}{6}$ 1 047	— 3	— 9	
$\frac{1}{7}$ 149	11	— $\frac{4}{7}$	ster.

Another Example.

l	s	d	
2531			Frē.
		5	
12755			
$\frac{1}{6}$ 2109	— 3	— 4	
$\frac{1}{7}$ 301	— 6	— 2 $\frac{2}{7}$	ster.

Note, that when any money is giuen by exchange at London for Roan at 7 d $\frac{1}{2}$, or rather 71 $\frac{1}{2}$ for the crowne of 50 s french, there is neither gaine nor losse: for it is one money for another, accounting 818 s french for 2 pound sterling. So the giuer looseth the time of payment, which is about 5 daies, and he that taketh it, hath the gaine of the same:

They of Roan that put forth or take money by exchange for London, ought to haue like condestation.

Item when any man giueth at London 64 pence ½, or rather 64 d ½, to haue at one of the Faiers of Lyons a crowne de Marc, he that so giueth the money loseth the time, and he that taketh it, gaineth the same: for 62 pence ½ is equall in value to 45 s French. He that putteth or taketh money at Lyons for London, ought to consider the same.

Item, when any deliuer in Antwerpe 75 d, to receiue at Lyons a crowne of Marc, hee that putteth it forth, loseth the time, and he that taketh it, gaineth the same: For 75 groates Flemmish is equall in value to 45 s French

Thus for this time I make an end of the practise of exchange & the instructions thereunto belonging: & according to my promise, yet further to gratifie such as are desirous to know the common coines vsed for trafficke among Merchants in these Cities following, here followeth a briefe declaration of their moneys, and the reckonings, & accounts of them.

The

The sixteenth Chapter containeth a declaration of the valuation and diuersity of Coynes of most places of Christendome for trafficke; And the maner of exchange in those places from one Citie or Towne to another: which knowne is right necessary for Merchants, by meanes whereof they do find the gaine or losse vpon the exchange.

ITem for as much as the greatest diuersity of money of exchange is at Lyons, therefore I will begin only of the money of that place:

At Lyons they vse Francks, Soulx, and Deniers Tournoies. A Francke maketh 20 soulx, and one Soulx 12 Deniers. But the Merchants to kepe their bookes of accounts doe vse French Crownes of the marke at 45 soulx the peece, and doe diuide it into 20 s, 1 s and 12 d.

Item, a Marke of Gold maketh 65 ℔ of the Marke, which serueth for exchange, and diuide it into 8 ounces. The ounce into 24 pence or deniers, the denier into 24 graines, and so the summe or whole by imagination or ghesse.

Also at Lyons there are 4 faires in a yeere, at the which they doe commonly exchange, which are from three moneths to 3 moneths.

At Geanes they vse the soulr: one Ducke maketh l 3,

At Naples they vse Dnckets, Caries, and Graines, the Ducket maketh 5 Cares, and one Carie 20 graines: but they take 6 Duckets (which maketh thirtie Caries) for the ounce.

A Ducket maketh ten Carlins, and a Carlin ten graines, so that 2 Carlins make a Carie, and 100 graines mak a ducket.

At Rome they vse the duckets of the Chamber: one ducket is worth 12 Guylis, and one Guylis 10 soulr.

At Venice they vse duckets Currant at 124 soulr a pece or 24 deniers, and one denier maketh 32 picolis.

At Palerme and Messine they write, after ounce, carie, and graines, & 1 ounce is worth 6 duckets or 30 caries, and 1 carie is 20 grains, and 1 graine 6 picolis, 1 ducket is also worth 24 Carlings.

At Millan they vse l, s, d, of Ducket Imderials, and ^ of exchange is worth 4 l.

At Lucques, Florence and Ancone, they vse the ^ of Gold: in Gold the french crowne is worth l 7, but at Boloigne l 3. 10 s.

At Barselone they vse the soulr: the ducket of exchange is worth 22 soulr.

At Valence and Saragosse they vse the Liuer, Soulr and denier: the french crowne of exchange is worth 20 soulr, and 1 soulr is

12

12 deniers.

At the faires af Castill they vse the Marueides, the ducket is worth 375 Marueides.

At Lisbone they vse the Raies, one ducket of exchange is worth 400 raies.

At Noremburge, Franckford, and August in Germany, they vse the Krentzars, whereof 60 make a florin.

At Antwerpe they vse s, & d de Gros, and they exchange into the denier de Gros, to wit, our English peny.

At London they vse the 1 l sterling, and 1 d sterling, and they exchange it in 1 d sterling.

The exchange of Lyons at sundry places.

Item at Lyons there is exchange in three sorts, at the cities and townes following.

First they deliuer at Lyons one Mark to haue or receiue at Naples almost 41½ duckets at Venice 70 duckets currant, at Rome 63 duckets of the Chamber, at Lucques and Florence 65 ℔ of Gold, at Millan 82 ℔.

And contrariwise, at the said Cities aforesaid, they doe giue so much of money to haue a marke at Lyons.

Secondly they giue at Lisbone one ℔ of Marke of 45 soulx Tournois a peece, to haue at Geanes almost 68 soulx, at Palerme and Messine almost 24 Carlins, at Barselone 22 soulx, at Valence or Saragosse 20 soulx, at the faire at Castell 350 Marueides, at Lis-

bone 360 Raies, in Antwerpe 57 deniers de Gros, and at London 70 d sterling.

And contrariwise they giue in the said Cities almost as much of their money to haue a French crowne of the make of Lyons.

Thirdly, they doe giue at Lyons ∆ of the sunne to haue almost 93 Krentzers at Franckford, Ausburg, Noremberge, or other Cities in Almaine.

Also at Lyons onely they do pay the change the $\frac{1}{4}$ in gold, and $\frac{1}{2}$ in money, or else all in money, in giuing $1\frac{1}{4}$ for the hundreth.

Changes at Naples and other townes.

Item at Naples they giue or deliuer almost 112 duckets to receiue at Rome 100 duckets of the Chambes at the old value.

Through Lucques and Florence they deliuer 100 duckets Carlins, to receiue their almost 86 ∆ of gold.

Through Palerme and Messine one ducket of 5 Tary, to receiue there almost 164 grains

Through Millan one ducket to receiue there almost 60 soulz.

Through Geanes one ducket, to receiue there almost 65 soulz. The whole summe to be paid within 10 daies after the sight of the bill of Exchange.

Also at Naples they deliuer one ducket to receiue in Antwerpe almost 67 d or deniers de gros, within 2 moneths. At London almost

most 90 d sterling in three moneths. At Barselone almost 20 soulz within two moneths: At Valence almost 18 soulz within 2 moneths, at Lisbone 333 Raies, within three moneths, and at the faire at Castill, almost 340 Marueids at the same faire.

Change of Venice to other places.

At Venice they deliuer 100 Duckets currant to receiue in Almaine almost 140 Florenes at 60 Krentzers the pece.

At Lucques and Florence almost 108 ℔ of gold in 10 daies.

Likewise at Venice they deliuer a Ducket currant to receiue at Palerme and Mescine almost 21 Carlins, at Millan almost 93 soulz: at Geanes almost 62 soulz, the whole at tenne daies end,

Of the Pair or Pari.

As touching the exchange, it is necessary to vnderstand or know the Pair, which the Italians call Pari, which is no other thing then to make the money of the change of one City or Towne, to or with the money of another, by meanes whereof they doe finde the gaines or losse vpon the exchange.

Example.

Item hauing receiued letters of credite of one of Antwerpe, that the ℔ of the Sunne is there worth 7 soulz: The question is, what the same is worth at London, when the Pair

or

or exchange goeth to 23 shillings.

Answere. Say if 23 giue but 20, what giueth 7? Worke and find 6 s, $1\frac{1}{11}$ d, & so much is the ▲ of the Sunne worth at London.

The seuenteenth Chapter containeth also a declaration of the diuersity of the waights and measures of most places of Christendome for traffick. At the end of which discourse are two Tables, the one for waight, and the other for measure, proportionate and reduced to an equality of our English measure & waight, by the aide wherof the ingenious may easily by the rule of three, conuert the one into the other at their pleasure, &c.

AT London, and so all England through, are vsed two kinds of waight and measures, as the Troy waight, & the Haberdepoise from the Troy waight is derived the proportion and quantitie of al kind of drie & liquid measures, as Perks, Bushels, quarters, &c. wherwith is bought and sold al kind of grain and other commodities mette by the Bushel. And in liquid, Ale, Beere, wine, Oile, Butter Hony, &c. vpon these grounds and statutes is bread made, and sold by the Troy waight. And so is gold, siluer, pearle, precious stones, and Iewels. The least quantitie of this Troy waight

waight is a graine: 24 of these graines make a peny waight, 20 penny waights an ounce and 12 ounces a pound: 2 pounds or 2 pintes of this waight maketh a quart. And so ascending into bigger quantities is produced the measures whereby are sold our other natural sustinance: viz. Ale or Beare, with all other necessary commodities, as Butter, Hony, Herrings, Eeles, Sope, &c. All which last before rehearsed, though their measures (wherein they are contained) bee framed and derived from the Troy waight, yet are they in traffick with diuers commodities, as Lead, Tinne, Flar, Ware, with al other commodities, both of this Realme, and of other forraine Countries whatsoeuer, bought and sold by the Haberdepoise waight, after 16 ounces to the pound, 112 pound to the C. waight. And vnto euery C. is allowed but 12 l waight at the cõmon beame. From hẽce is also deriued the waight of Suffolk Cheese, which containeth 32 cloues, 8 l to a clouc, and waieth in all 256 pounds. And also the barrel of Suffolk Butter is or should be of like waight with the waight of cheese, viz. 256 pounds. Note, 14 of these pounds make a stone, and 26 stone containeth a sacke of English wool. Forrain wools, to wit French, Spanish, and Estrich, is also sold by the pound or C. waight, but most commonly by the Roule, 25 pound to a Roue: other commodities of Male are bought and sold by the C.

stone.

Auerdoze to the C. Except headed ware, to wit cattell, nailes, and fish, which are sold after fiue score to the 100. There is also two other sorts of measures, to wit, the elle, and the yard. By the Elle is vsually mette Linnen Cloth, as Canuas &c. And by the yard, silkes, woollen clothes, &c.

Antwerpe.

At Antwerpe are also 2 sorts of weights, their gold and siluer weight, and their common waight. Gold and siluer is weighed by the mark, the mark is 8 ounces, the ounce 20 Esterlings, and the Esterling 32, as our graines. The Goldsmiths diuide that into smaller, but not the Merchants: the proofe of gold is made by caracts, whereof 24 maketh a marke of fine gold: the carat is 24 graines, the proofe of the mony is made by Deniers, 12 deniers is 1 ℔ fine, that is, a marke of fine siluer; the denier also is diuided into 24 grains, and the graine into foure quarters.

Item, 100 Markes in Antwerpe, Troy waight, maketh at Lyons 103 Marks, two ½ ounces, and 20 graines 23 l. At Noremburge 103 markes 2 ½ ounces, 2 quints, 3 deniers, at Frankford 105 markes, at Ausburge 104 markes, three ounces, 1 quint At Venice 103 markes, 1 ounce, 7 deniers 18 graines. At London 66 pounds.

The marke of Gold or siluer at Antwerpe,

Troy

Troy waight, which is 8 ounces, maketh 7 ounces common waight, with which all other merchandise is weighed. So that the Troy waight is greater then the common waight by 6 ¼ in the C. By this waight of Troy, they also weigh Muske, Amber, Pearle &c.

All silkes are bought at Antwerpe, by the Burges elle, which is greater than the common measure, by which they retaile by two in the hundred. Their common elle is ¼ of our yard, and ⅞ of our elle.

Lyons.

At Lions is vsed 3 sorts of waight, whereof the first is the common town waight, with which they weigh all kinde of Spicery, and diuers other merchandise. The second is called Geneua waight, which is 8 in the C greater then the cõmon weight, with which they weigh silkes, &c. The third is French weight, called commonly the Marke weight, and 100 pounds thereof maketh 106 ¼ pound Geneua, & 114 ¼ of their common weight: with which french weight is weighed al things that paied custome or tole.

At Lions is also vsed two sorts of elles or Aulnes. The one wherewith they measure grosse clothes, as Canuas, and such like. The other is called the french Elle or Aulne, with which they measure all other kinde of merchandise, whereof seuen common town elles maketh

maketh 11 ordinarily French Elles.

Roan.

At Roan 6 ½ Muydes of salt being the measure of the place, make an hundred at Armuiden in Zeland, & the C. of Bronage measure of Armuiden make that Roan 11 Muydes, 30 Pines make a last of Cotne, and 16 a Last of Dates, 100 pound weight there, maketh at London 114 ½, and 190 ½ at Antwerpe. And 100 Elles make at London 115 ½.

Noremburge.

A 100 pound weight at Noremburge maketh at London 111 ½, at Antwerpe 97 ½, And 110 Elles at Noremburge make at London 74 ½, at Antwerpe 95 ½, &c.

Lisbone.

The 100 weight at Lisbone maketh foure Roues, every Roue 32 pounds, so that their C. weight is 128 pounds, and their pound containeth 14 ounces, and a 100 pounds of their weight maketh at London 113 ½.

Their silke, cloth of Gold, and Woollen is measured with a measure which they call a Cubite, containing abount ½ of a Varre of Castile. Howbeit, their common measure is called a Varre, which maketh 5 Palmes, and containeth 1 ½ of Varre of Castile, our Elle of London is equal with the Varre of Lisbone.

All

All kind of merchandise brought from Flanders, Roan, or Britaine payeth at Lisbone, as a duety or custome to the king, 20 in the 100, which they call the tenth in merchandise, and the other tenth in money.

Note also that all kind of merchandise comming to Lisbone by land, payeth lesse in custome then that that commeth by water.

Ciuill.

The Roue of Ciuill is 30 pound, 4 Roues make their C weight, which is 120 pounds. The 100 pounds of Ciuil maketh at London 102 pounds. Their other common measure is a Varre, wherof 100 maketh at London 74 Elles and at Rome 40 Canes, &c.

Venice

At Venice be two sorts of waights, the one called La Grosse, the other La Suttle, with the grosse is weied all kind of greart wares, and with the small all kinde of spicerie, and such like: 96 poūds of grosse waight there maketh at London 100 pounds, and 100 pounds of spicerie there without any tare or allowance, make at London 64, and with tare 65.

Their owne common measure are Braces whereof 100 make at London 55 $\frac{1}{2}$ Elles, at Antwerpe 92 $\frac{1}{2}$, &c.

Florence.

At Florence the 100l waight maketh at Aqui-

Aquila, for Saffron 110, & 145 pounds of Florence make at Roane but 100 pounds, the weight of Florence and that of Luke is al one.

Their other measures are braces, whereof 100 maketh at Antwerp Burges measure, 81 ½ elles. 100 Braces there make at London 49 elles, &c.

Lucque.

The Lucque Sattens are commonly sold at Lions by weight, & 133 ⅓ pounds maketh at Lions 100 pound, so that 1 pound ⅓ maketh at Lions but one pound.

Their other measures are braces, whereof 100 of them make at London 50. Elles at Antwerpe 83 ⅓ Elles, &c.

Aquila.

At Aquila their 100 pounds maketh at London 71 ¼, their 136 ½ pounds of Saffron maketh at Geneua but 100, and 11 l' of Geneua maketh 15 l' at Aquila.

Valentia.

At Valentia be 2 sorts of weights, a great and a small. The 100 weight of great weight containeth 4 Roues, the Roue 46 l' so the 100 great weight is 144 l, & the 100 weight small cõtaineth but 120 pounds, and is also parted into 4 Roues, which is 30 pounds to a Roue. By the small is sold the scarlet groin with all other kind of spicery, and by the great is sold woolll, with all such like grosse wares: The ½ pounds

pounds of silk at Valentia, maketh at Lyons 1 pound Geneua waight. The charge of great merchandise at Valentia containeth 432 l. and in small wares 360 pounds. The weight here at and at Barsellone is all one. Their 100 l. weight maketh at London 78 l. and at Antwerpe 75.

Dansicke.

At Dansick or Spruce-land the rule is that whosoeuer buyeth any Merchandise there, buyeth it by the ship-pound, which is 3201, 20 Lispounds make a Ship-pound and 9 Lispound containeth 161, which ship-pound of Dansicke maketh at Antwerpe 256½ l. Their 100 l weight maketh at London 86 ¼ &c.

Their other common measures are elles, whereof 100 make at London 72 ¼, and at Antwerpe 220¼ Elles.

Toulhouse.

At Toulhouse 6 Cabes of Mead maketh a Charge, two Cesternes of Corne, and al kind of graine maketh a Charge, the Cesterne weigheth 160 l weight of that place. Their C l in weight, maketh at London but 91 ¼ l.

Geanes.

At Genua of Geanes, C. l of their weight maketh at London 71 ¼, and at Antwerpe 67 ½, a 100 l weight at Genua maketh at Venice, to wit, Suttle 106 l.

Their other common measures are Palms, whereof 100 make at London 20½ Elles, and at Antwerpe 34½.

The eighteenth Chapter treateth of Sports, and Pastimes done by Number.

IF you would know the number that any man doth think or imagine in his mind, as though you could diuine, bid them triple it, or put twice so much more to it as it is, which done, aske him whether it be euen or odde: if he say od, bid him take one to it, to make it euen, and for that one kéep one in your minde. Now after he hath taken one to it, to make it euen, bid him giue away halfe, and kéep the other halfe for himselfe, which when hee hath done, bid him triple that halfe, and againe after he hath tripled it, aske him whether it bee euen or od: if he say od, then bid him take one to make it euen againe, and for that last one kéepe two in your minde, now after he hath made his number euen, bid him cast away the one halfe, and kéep the other still, from which halfe that he kéepeth, cause him subtilly to put away or giue you 9 out of his number, & for each 9 that he giueth you, keep 4 in your mind and thereunto ioine the 3 which I bad you kéepe, and you shall haue your desire.

Example.

Imagine he thought 7, the triple whereof is 21

is 21, and because it is odde, he is to take 1. to make it euen, which first, 1 giuen is one for you to keepe in minde. Then the halfe of his 22 beeing cast away, hee reserueth still 11, which after you haue bid him triple, it maketh 33, then in giuing of him one againe to make it euen: vpon that last 1 reserueth 2 in your minde, then his halfe of 34 maketh 17, from whence he can giue you 9 but once. Therfore that yelding to you 4, and the 3 that you keep make 7, your desire.

Another kinde of Diuination, to tell your friend how many pēce or single peeces, reckoning them one with another, he hath in his purse, or should thinke in his minde.

Which to do, first bid him double the pēces hee hath in his purse, or the number hee thinketh, if hee participate his number or secrecy vnto some one friend that sitteth by him that can but multiply, and adde neuer so little: if their number be great, then shall they worke as you bid them so much the surer.

Now after hee hath doubled his number, bid him adde thereunto 5 more, which done, bid him multiply that his number by 5 also: which done, bid him tell you the iust sum of of his last Multiplication, which summe the giuer thinking it nothing auaileable, because it is so great aboue his pretended imagination; yet thereby shall you presently with the

helpe of Subtraction tell his proposed number.

The Rule is this.

Imagine hee thought 17. double 17, and it maketh 34, whereunto if you ad 5, it maketh 39: which multiplied by 5, as here is practiced in the margent, it yeldeth 195, which 195 is the summe deliuered you in the worke: then for a generall rule, you shall euermore cut off the last figure towards your right hand, with a dash of your pen, as here is performed, as a figure nothing auaileable vnto your worke, and then rebate 2 from your first figure, after 5 is cut off, and the rest shall euermore be your desire, as by this example doth appeare.

17	
2	
34	
5	
39	
5	
195	
2	
17	

If in any company you are disposed to make them merry by maner of diuining, in deliuering a ring vnto any one of them, which after you haue deliuered it vnto them, that you will absent your selfe frō them & they to deuise after you are gone, which of them shall haue the keping thereof, and that you at your returne will tel them what person hath it, vpon what hand, vpon what finger, and what ioint. Which to doe, cause the persons to sit downe all on a rowe, and to kep likewise an order of their fingers now, after ye are gone out from them to some other place, say vnto one of the

lokers

lowers on, that he doble the numbers of him that hath the Ring, and vnto the double, bid him adde 5, & then cause him to multiply the Addition by 5, and vnto the product, bid him adde the number of the finger of the person that hath the Ring. And lastly, to end the worke, beyond that number toward his right hand, let him set downe a figure signifying vpon which of the ioints he hath the Ring, as if it be vpon the second ioint, let him put downe 2. Then demand of him what number he kæpeth, from the which you shal abate 250 And you shall haue three figures remaining at the least. The first towards your left hand shall signifie the number of the person which hath the Ring, the second or middle number shall declare the number of the finger, and the last figure towards your right hand shall betoken the number of the ioint.

Example.

Imagine the fourth person is determined to kæp the Ring vpon the fift finger, and the third ioint: first double 7, it maketh 14, thereto adde 5, it maketh 19, which multiplied by 5, yeldeth 95, vnto which 95, adde the number of the finger, and it maketh 100: and beyond 100 toward the right hand, set downe 3 the number of the ioint, all maketh 1003, which is the number that is to bee deliuered you, from which 25, there resteth 750 there

753, which prefigureth vnto you the seuenth person, the first finger, and the third ioint.

But note, that when you haue made your subtraction, if there do remain ot in the place of tennes, that is to say, in the second place, you must thē abate 1, from that figure which is in the place of hūdreds, that is to wit, from the figure which is next your left hand, and that shall be worth 10 tenths, signifying the tenth figure, as if there should remaine 803, you must say, that the seuenth person vpon his tēth finger, and vpon his third ioint, hath the King.

And after the same manner, if a man doe cast 3 Dice, you may know the points of euerye one of them. For if you cause him to double the points of one Die, and to the double to adde 5, and the same sum to multiply by 5, & vnto the prodnct adde the points of one of the other Dice. And behind the number towards the right hand, to put the figure which signifieth the points of the last Die, and then to aske what number he kepeth, from which abate 250, and there will remaine 3 figures, which doe note vnto you the points of euery Die.

Another.

If three diuers things are to be hidden of three diuers persons, & you to diuine, which of the three persons hath the three diuers things,

things, doe thus: imagine the three things to be represented by ABC. Then secondly keepe well in your minde which of the persons you meane to be the first, second and third. Then take 24 counters or stones, and your three things, and giue A to the party whom you imagine to be your first man, and therwithall giue him one of your 24 counters in his hand And B vnto your second man, and therewithall 2 counters. And C vnto your third man, and therewithall 3 counters: and leaue the rest which are 18 still among them, which done, separate your selfe from them, and afterwards bid them change the things among them as they shall think good: which done, after they are agreed, bid him that hath such a thing, as before you haue represented by A, for euery counter that he hath in his hand, to take vp as many more. And for him that hath B, for euery one in his hand to take vp two. And for him that hath C, for euery one in his hand to take vp 4, and the rest of them to leaue still vpon the boord. These three things and the three persons being fully printed in your minde, come to the Table, and you shall euermore finde one of these six numbers, 1, 2, 3, 5, 6, or 7. If therefore one remaine still vpon the boord, the haue they made no exchange, but kept them still, as they were deliuered vnto them. So that the first man hath A, the second B, and the third C. But if 2 remain, then

the first man hath B. your second man A, and your third man C. The rest of the work and the order thereof are here apparant by the Table following.

	1	A		1	B
1	2	B	5	2	C
	3	C		3	A
	1	B		1	C
2	2	A	6	2	A
	3	C		3	B
	1	A		1	C
3	2	C	7	2	B
	3	B		3	A

Another Diuination of a number vpon the casting of two Dice.

First let the caster cast both the Dice, and marke well the number: then let him take vp one of them, it maketh no matter which, and looke what number it hath in the bottome, & adde all together: then cast the dye againe, kéep in his minde what all together maketh: then let the Dice stand: hauing seuen with you and thereunto adde the rest of the pits that you sée vpon the vpper side of the side of the dice, and so many did the caster cast in all.

The

The new additions here followeth by M. R. N. both plaine, pithy and profitable:

1. *The Art and application of decimal Arithmeticke.* 2. A table of board and timber measure exactly calculated & the vse thereof. 3. The extracting and demonstrations of the square and cubick rootes. 4. Necessary Questions and Tables for interest after 10 for 100.

Scholar.

SIr I haue heard of a new application of Arithmetick, which is called decimal Arithmetick, I desire you to shew me what it is and the vse and operation thereof; because I haue heard it much commended for auoiding of all fractions.

Master. Decimal Arithmeticke, may wel be called Fractions in whol numbers, because it keepeth the like progression of places vnder the vnity (namely in the parts of any vnity) as the multitudes of whol numbers do aboue the vnity: that is, it keepeth alwaies the tenth progression. For, as the tenth part of 100 is 10 and $\frac{1}{10}$ of 10 is 1; so one tenth of one is a prime, which wee marke with one stroake thus,

thus ', and $\frac{1}{10}$ of a prime is a second marked thus '': and $\frac{1}{10}$ of a second is a third noted thus ''' and so forth infinitly vnder the vnity, as wel as aboue the same, in a continued tenth progression: whereby there ariseth a double benefit. First it bringeth vs nearest to perfection for exactnesse, and leadeth vs nigheſt to expedition for quicknes and easines: requiring only ỹ vnity, be it of any thing whatsoeuer to be first diuided into 10 parts, or primes and each prime into ten seconds &c. Which in all mensurations produceth most exquisit exactnes, and aptnes to be expressed both numerally and lineally: wherin this only obseruation will be sufficient. Namely, that each principal part or measure be diuided, and subdiuided into tens of equal parts. As in measuring of land, the principall measure being the perch, shall first be diuided into 10 equall parts or primes, and againe each prime shall be subdiuided into tenne equall parts or seconds, &c. The like must be done where the yard is the principall, and also where any other measure is chefe.

As for tinber and board-measure, the foote which ordinarily is diuided into 12 ynches, shall for this perfect practice be diuided into tenne equall parts or primes, and each of them againe into 10 equal parts, or seconds &c. And the like is to be done, where the inch or any other measure is principall.

Scholer.

Scholer. I pray you shew the difference betweene the addition of such tenthes, and the vulgar addition of whole numbers.

Master. There can be no difference shewed therein. As the example following will make plaine. For suppose you are to Adde 31 perches 5'7'2'' vnto 91 perches 4'3'8'' you shall finde by setting them in this manner and having added them according to the ordinary fashion of Addition of whole numbers that the totall some will amount vnto

	315'7'2''
	91438
	123010

123 perches 0' 1' 0'' wherein you are onely to obserue that the last signe of the Totall which herein are thirds marked 3 must be equal to the last signe of the numbers to be added, and that if one number hath any signe that the other wanteth, then the place thereof must be supplied with a cypher retaining still the succession of the signes. As if 42 ynches 3' 2' 9'' (wherein there wants the signe of seconds) where to be added to 724 ynches 9' 3' 5'' wherein also there wanteth fowrthes, their disposition will be thus. And their Totall wil be

	423'0''2'9''
	7249340
	7672309

767 ynches 2' 3' 6' 9'' which is named 767. 9 prymes 3 seconds, 6 thirds and 9 fowerths because the last signe is fouths.

Scho-

Scholar. All this I plainely perceiue to ſtand with reaſon but is there no difference in ſubſtraction neither.

Maſter. None but that you ſhall ſubſtract the number which is to be ſubſtracted out of the number which you are to make your ſubſtraction from, and ſet down the ſuperior laſt ſigne for the laſt ſigne of the reſt or remainder continuing the ſucceſſion of the other ſignes in their proper order, as if you were to ſubſtract 864'3''5''' out of 987'5''8'''' the diſpoſition and working thereof in the common order to ſubtract whole numbers there will reſt 123'2''3''' which is 12 and 3 primes 2 ſeconds and thirds.

987'5''8''''
864'3''5'''
Reſt 12323

Multiplication.

Scholar. But how is multiplication performed: I ſuppoſe that, and diuiſion ſhould be more difficult, for the former are exceeding plaine?

Maſter. There is no more hardneſſe in them then in the former, for you ſhall ſet downe all the parts or ſubdiuiſions of primes ſeconds, &c. as if they were whole numbers and worke them accordingly, onely this you muſt obſerue that hauing finiſhed your

your multiplication, you shall adde the last signe of the multiplication to the last signe of the Multiplicant and the totall thereof shall bee the last signe of the product, as if $72^1\ 3^2\ 4^3$ were to bee multiplied by $23^1\ 2^2$ they shall be disposed thus.

And their product will be 16, & 7^1 or primes 6 seconds 2 thirds, 8 forths, as you see here.

$$\begin{array}{r} 72^1\,3^2\,4^3 \\ 23^1\,2^2 \\ \hline 14468 \\ 21502 \\ 14468 \\ \hline 167^1\,6^1\,2^3\,8^4\,8^5 \end{array}$$

Diuision.

But in diuision you hauing also made your diuision in the wonted fashion for diuiding of whole numbers, you shall subtract the last signe of the diuisor from the last signe of the diuident, and retaine the succession of the signes in their order. As if you should diuide $72^1\ 3^2\ 4^3$ by $23^1\ 2^2$ the quotient will be 3 and 1 prime with the rest of $\frac{42}{232}$ wherein seconds the last signe of 232^2 takē from thirds the last signe of the diuident, there resteth primes for the last of the quotient,

$$\begin{array}{ll} \cancel{2}4 & \\ \cancel{2}\cancel{3}\cancel{7}\,(2 & \\ \cancel{7}\cancel{2}\cancel{3}\cancel{4} & (3121 \\ \cancel{2}\cancel{3}\cancel{2}\cancel{2} & 116 \\ \cancel{2}\cancel{3} & \end{array}$$

Of

Of figured numbers.

Scholer. I haue heard of figured numbers & their rootes. I pray you Sir shew me what they are, and to what vse they chiefely serue.

Master. My leasure will not permit me to speake of them at large, therefore in briefe, A figured number is a number made by the multiplying of one number by another, and the roote of a figured number, is the side or number by whose multiplication it was made. A figured number also is two-fold, a plaine and a solid number. A plaine number is either made of two equall numbers, as a Square, or of two vnequall numbers, as an Oblong or long square. A square number is made by a number multiplied by it selfe, as 10 times 10 are 100. Therefore the side or square roote of 100 is 10.

Scholer. But how shall I finde out the square rote of a greater number.

Master. It is necessary to haue this table, or to beare in your memory the squares and cubes of the nine digits: and for the finding of the rootes of greater numbers the two Theorems following, and the demonstrations and examples ensured

Rootes.	Squares.	Cubes.
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	502
9	81	729

ered shall make it sufficiently plaine.

The first Theoreme, for the extracting of the square roote.

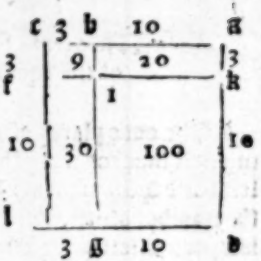
(The odde places of a square number being first marked from the right toward the left hand with points.) If you subtract from the number giuen, the perticular square of the last period, setting the side thereof alone by it selfe. And then if you diuide the remainder (if any be) with the figure going before it, by the double of the said side, set by it selfe, and also multiplie the quotient so found (being placed both by the said first found side, and also before the doubled number towards the right hand) by both those numbers counted as one deuisor, subtracting the products from the number giuen, and renew this last worke of diuision so many times as there are pricks or points remaining. The side of the square shall be found out thereby.

Scholer, Whereupon is this Theoreme grounded?

Master. This artificiall is borrowed out of the 4 pro. of the second of Euclid where it is proued by demonstration that if a right line be cut into two segments howsoeuer. That the square of the whole line is equall to the squares of the segments, and to the two oblongs made of the segments. As in the

figure

figure here annexed the square a c l d of the whole line a c 13 is equall to the squars of 10 which is k g 100 and of 3, which is b f 9, and to the two oblongs made of 3 and 10 namely a i and l i, each being 30, all



which parts amount vnto 169 equall to the square of 13. Therefore as the square of 13 being 169, is made by adding together of the two square of 10 which is 100, and 3 which is 9 being multiplied in themselues, and of the two oblongs made of 3 multiplied by 10 which is 30, and 10 multiplied by 3 which is also 30: so by subtracting them severally it is resolued by the precedent Theoreme. As for example, the first and third place being pointed, you shall first subtract the square of 1 which is 1 from that vnity and there remaineth nothing, this vnity set alone by it selfe, and doubled maketh 2 by which dividing 6 (the number before it there being no remainder) the quotient will be 3 which placed by the said first found side 1 and also before the said doubled number 2

169	
23	
	(13

and

and multiplying the said quotient by both those 2 numbers, and with their products as with one diuisor diuiding the figures ouer them, there remaining nothing, it sheweth that 109 is a square number and that 13 is the roote thereof required.

Scholar. I pray you shew me how to extract the square roote of a greater number, as of 15129.

Master: The odde places being pointed I subtract as before the square of 1 which I place alone from 1 and there resteth nothing then I double the 1 and set that double forward vnder the 5 betwéene the points, and diuiding the 5 I finde the quotient to be 2 which I both set behind the lunar line, and also next to the said doubled number, namely vnder 1, then I multiply that quotient by both the said numbers which maketh 44, and then with both their products, as with one common diuisor, I diuide the number ouer them, and there will remaine 7 which placed as you see. I againe double the whole quotient which makes 24, setting the 4 thereof vnder the 2 betwéene the prickes, and the

```
 15129
 2      (1
```

```
   1
  17
 15129
  22
   43
  44     (123
   29
   2
```

2 I set vnder the 7, and then I seeke how often 2 will be found in 7, which is, 3 times and 1 remaine, therefore I set 3 in the quotient, and also vnder the 9: by which 3 I multiply the 4 and 3, and they make 129, and thereby as by a common diuisor, I diuide the number ouer it, and there rests nothing whereby I finde that 15129 is a square number, and that 123 is the square roote thereof.

Scholar. But what shall I doe if the number vnderneth when it is in place of a diuisor be greater then the number of that period which is to be diuided by it.

Master. Then shall you set a cypher in the quotient, as in diuision. And the double of your whole quotient shall be remoued one place more towards the right hand, and thenceforth you shall obserue the former generall order.

Scholar. Now if the number giuen should be some other number which is not a square number, what shall I doe to finde the nereſt square in whole numbers and such decimall fractions as may not onely be expressed by measure but also multiplying it by the ordinary manner of multiplying of whole numbers, I may againe come nereſt to the making of the said giuen number.

Master. If the number be not a square number, it is impossible to finde a perfect square

rote thereof, yet hath euery Author almost set downe their conceits for the finding of a number nearest, wherein they haue runne into a labyrinth of errors which I will shew you hereafter, and how you shall not misse aboue the $\frac{1}{1000}$ part of an vnity, and also how to expresse that neere rote, by measure as well as by number, which in many practises will be found exceeding necessary. But first I will teach you to extract the Cubicke rote of any Cubicke number, or the nearest Cubicke rote in any number giuen, and then making but one rote for the remainders, I will performe the former promise, both for the square and Cubicke remainders.

Scholer. I pray you sir vse such methods as shall seeme best vnto you.

Master. If you desire to finde the Cubicke rote of a number Cubicke, or the nearest Cubicke rote in any number giuen, you shall doe it by the Theoreme following.

The second Theoreme is for the extracting of the Cubicke roote.

The first, fourth, seuenth, &c. places being pricked alwayes, leauing two figures betweene euery two pricks: if you subtract the particular Cube of the last period from the number giuen, setting the side thereof in the quotient, and then set the triple of the quo-

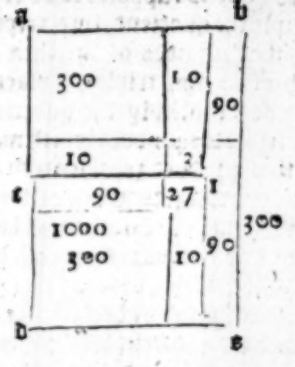
tient vnder the figure next following the former point on the right hand, and the square of the quotient being triple beneath it one degree more toward the left hand, and afterward diuide the number aboue written by the triple of the square, setting the quotient by it selfe, and then multiply the diuisor by the quotient found out, and the tripled square by the square of the quotient, and the quotient cubically, subtracting the products (so orderly being added together that each figure may answere the number whereof it was multiplied) from the number giuen, and renew this last manner of diuision, so many times as there are points remaining, the side of the Cube shall be found?

Scholer. Whereupon is this Theoreme grounded?

Master. This deuise is drawne out of that Theoreme which Ramus made, imitating that of Euclide concerning square numbers thus. If a right line be cut into two segments, the Cube of the whole line shall be equall to the Cubes of the Segments, and the two solide figures comprehended three times vnder the square of his segment and the segment remaining. As the line ci which is 13 is cut in two segments, 10 and 3: the Cube of the whole line, namely 2197 is equall to the Cubes of the segments, namely to 1000 the Cube of 10, and to 27 the cube of 3 and

also

and also to the two solides of Paralellipipedons thrice taken whereof there are 3 lesser of like solidity each being 90 made of 3 times 10 30, & 3 times 30, and there are three of them greater of like soliditie also, made of 10 times 10 100, and of 3 times 100 which are each of them 300. Now the same method is also to be observed in resoluing of the Cube. As for example, having marked the first & fourth places with prickes in this manner 2·197·. Then I subduct the particular Cube of the number set vnder the last prick, but because that number is no cubicke number, I take the nerest Cube vnto it, namely an vnitie which also I set in the quotient: this vnity in the number giuen is 1000, but in the quotient it is but 10, the vnity subducted from 2, the remainer is 1, which must be set ouer the number giuen. So that hereby the greater



2 1 9 7 · (1
1
1 3

Cube 1000 is supposed to be extracted. Then I triple the quotient, this triple 3 representeth the three sides of the three lesser solides, each of 90, this triple 3 I place vnder the 9. Againe I multiply the quotient squarewise and triple the product which maketh likewise 3, this product representeth the three squared sides taken together of the three greater solids, each of 300, then I place the product 3, one degree more to the left hand vnder the 1, with which I diuide 11 that standeth aboue it and the quotient wil be 3. This segment or quotient 3 being multiplied by 3, the diuisor maketh 9, which in respect of the place where it standeth is 900, and representeth the three greater solides taken iointly together. Then the same quotient 3 being multiplied squarewise maketh 9, and multiplied also by the tripled number that standeth vnder 9, it maketh 27 which in respect of the place where it standeth is 270 and representeth the three lesser solids taken together. Lastly the said quotient being multiplied cubically bringeth forth the lesser cube of 3, namely 27. These three products therefore being added together make 119, and the totall subtracted from the numbers ouer them,

1	
2197	
93	
3	(1
9	
27	
27	

there

there remaineth 'nothing, which sheweth that 2197, the number giuen is a cubicke number and that the cubicke roote therof is 13 the thing sought.

1000 The greater ☾
900 } the 6 Para.
270 }
27 the lesser ☾.
2197

Scholar. Sir is there none other manner of working.

Master. Yes many men haue varied this ground-worke as they haue thought readiest for their owne purposes, yet depend vpon the same principles whereof one is thus, hauing found in the former table or by memory the figure representing the cubicke root or side contained in the number standing vnder the first pricke on the left hand, and set the same in the quotient, and subtracted the particular cube of that figure as you did before. Then the square of the product shall be the diuisor, whose first figure shall be set vnder that figure which is on the right hand, next of all to the said first point. Secondly, see how many times the diuisor is contained in the number written ouer it, multiply the diuisor by the quotient, and subtract the product from the diuident: yet here you must beware you choose not a greater quotient then that the products made afterwards, thereby may be subtracted from the number giuen. Third

ly, when the subtraction is done, triple the first figure which was set in the quotient, and place the last number which was set in the quotient vpon the right hand of the product. And lastly multiply all that number by the square of the quotient last found out, and set downe the product so that the first figure thereof towards the right hand, may stand right vnder the point next going before the same hand, and then subduct it from the number giuen: as in 804'357 the particular cube of 9, namely 729 being taken from 804 the number standing vnder the first prick on the left hand, there will remaine 75357: the side of the said particular cube 9, I set in the quotient, which squared maketh 81, & that square tripled produceth 243 for my diuisor which I set vnder the number giuen, so that the 3 may stand vnder the 3 of the number giuen with this diuisor, I diuide the number standing ouer it, and finding 2 to be contained in 7, three times I set therefore 3 in the quotient and multiply the diuisor by it, which produceth 729 that being subtracted from 753 resteth 24, which I set ouer it: then I triple the quotient 9, the product is 27, by which I

24	
753'57	
804'357	
243	
729	(93
273	
2457	

set

set 3, the last found quotient which maketh 273, this number I multiply by 9 the square of 3 the quotient last found, and the product is 2457 which being subtracted from the number standing over it, ther remaineth nothing whereby I see that 804357 is a cubicke number & that 93 is the Cubicke root thereof.

Some others extract the cubick root in the manner following. The figure in the quotient, being found out by deducting the particular cube, and also the second figure in the Quotient being found by division, they triple the whole quotient, and multiply the tripled number by the former figure of the quotiets. Then multiply the prooduct againe by the latter figure, found out and set a Cypher on the right hand of that product. And last of all, they adde the cube of the saide latter figure to product, and subtract the whole summe from the num given. As in 373·248. The cube of 7 is 343 which deducted from the number, vnder the first prick there resteth 30 Therefore I set 7 in the quotient: Then I square the side 7, and it maketh 49. and triple that square which maketh 147 for the diuisor, by this I diuide 302, the quotient will bee 2. Then I triple the whole quotient 72, which maketh 216 and multiply that triple by 7 the first figure of the quoti-

	30
373	248
343	
49	(72
147	
34240	

ent

ent (which produceth 1512. I multiply also that triple by 2, the latter figure of the quotient, & set a Cypher which maketh it 30248, vnto this I adde 8 the cube of the latter figure, and subtract it from the vpper number and there resteth nothing, which as before sheweth it to be cubick number, and the cubick roote or side thereof to be 72 as was required.

Scholer. But you promised to shew, if the number were not a square nor cubick number: Yet if the roots were required how to come a necessary nearenesse, and what others haue taught therein.

Mast. Cardan appointeth to multiply the root squarely and again by 3, and that number shall bee the Denominator vnto the remainder.

Scheubelius biddeth triple the roote and the square of it also, and adde both these numbers together and 1 more, so haue you a denominator to the numerator: which is the remainder: These are for the cubick rootes.

For the square rote they double the rote and adde thereunto 1, for the denominator, and the remainder they take for the numerator, but these waies misse much of the truth.

Therefore, to come as neare as may bee, you shal adde paires of Cyphers vnto the remainder for the square roote, and threes vnto the remainders of the cubick roots, & pro-

ceed

cēd by the former instructions, and so shall the first figure of the new quotiēt be primes, or tenths of an vnity; the second shall be seconds, or hundreths; the third shal be thirds or thousands of an vnity, &c. So that thereby, if you will, you may come within lesse then $\frac{1}{10000000}$ part of a vnity. And also it may in primes, and seconds be lineally taken from a scale of decimall subdiuisions, when need shal require: One example will make the matter sufficiently plaine.

As if the square roote should be extracted out of 123, the nereſt in whole numbers will bee 14, and there will remaine 27; vnto which adding two paires of Cyphers, and continuing the former order, you shall finde in the quotient 8 primes, and 9 seconds with the remainder 12879; which comes to lesse then $\frac{1}{100}$ part of an vnity.

Also, suppose the cubick roote of 1739 were to be extracted, the neereſt cubick roote in whole numbers wil be 12, and the remainder will be 11, vnto which adding two thrées of Cyphers, and continuing the former direction of the cubick extractions, there will bee only two seconds, or hundred parts of an vnity, 2654408 remaining, all which lacks much of $\frac{1}{100}$ part of an vnity.

Thus adding more paires for the square, or thrées of cyphers for the cubicke, you may come to thirds, fowrths, and fifths: namely,

ly, to lesse then $\frac{1}{10000}$ part of an vnity, which shall be also to be expressed vpon a decimall scale in measure, as I promised, and this shal at this time suffice for extracting of rootes. Now for the proofe, if you multiply the root in it selfe squarely, for the square roote, and cubickly for the cubick rootes, and vnto the product adde the remainder, if it make the first giuen number, you haue done rightly else not.

A briefe addition of Interest.

Definitions of Interest.

1 PRincipal, is the summe from which the interest is reckoned.

2 Interest is the summe reckoned for the lending or forbearance of the Principal for any termes or time.

3 The proportion betweene the Interest, and the Principall is the reason of Interest.

4 Interest simple, is that which is counted from the Principall only.

5 Interest compound is that which is coũted for the Principall, together with the Arriage of Interest, and Principall:

6 Inte-

6 Interest profitable, is that which is added to the Principall.

7 Interest domageable, is that which is to be subtracted from the Principall.

Scholar. Sir, I haue bene much troubled with resolving questions of Interest: I pray you shewe me some way to ease the tediousnesse, which I find bredeth errors also.

Master. Although vsury ought not to bee found amongst Christians: yet seeing our hearts are growne to that hardnesse, that Charity shal haue nothing but what she paies for; therefore most Common-weales haue, though not allowed, yet, winked at this vsury, only curbing the horrible excesses, which else would grow, and tolerating the vse therof for auoiding worse extreamities.

And because the compound interest, yea and the simple interest also in many questions of profit and losse; otherwise aske much labour, and so becomes subiect (as you say) too much error. I haue thought good to set downe this Table containing fower Columnes: The first whereof is for Interest simple domageable, where losse commeth, by receiuing before the day due. The second is, for the number of yéres, quarters, moneths, or other equall tearmes mentioned in each question. The third is for compound Interest, when an yérely, quarterly, and continued

the payment is not in the question. The last is for yeares continued. As the questions following shall these vnto you, wherein I would haue you to marke diligently the reason of them, that you may apply any other question, for that this will serue for any number of yéres of equall termes, whatsoeuer vnder 21, and for any summe of money propounded.

Scholar. I pray you proceed as plainely, and briefly as you can for my memories sake, and I shall bee euer bound to you for your paines.

Master. There is 324 l. to be paid in fiue yéres: namely, 54 l. each yeere: the question is, what it is worth in ready money, abating simple interest after 10 for a 100.

ent (which produceth 1512. I multiply also that triple by 12, the latter figure of the quotient, & set a Cypher which maketh it 12048, vnto this I adde 8 the cube of the latter figure, and subtract it from the vpper number and there resteth nothing, which as before sheweth it to be cubick number, and the cubick roote of 8 is 2 as was required.

Scholer. But you promised to shew, if the number were not a square nor cubick number: yet if the roots were required how to come a necessary nereneile, and what others haue taught therein.

Mast. Cardan appointeth to multiply the root squarely and again by 3, and that number shall bee the Denominator vnto the remainder.

Scheubelius biddeth triple the roote and the square of it also, and adde both these numbers together and 1 more, so haue you a denominator to the numerator: which is the remainder: These are for the cubick rootes.

For the square rote they double the rote and adde thereunto 1, for the denominator, and the remainder they take for the numerator, but these waies misse much of the truth.

Therefore, to come as nere as may bee, you shal adde paires of Cyphers vnto the remainder for the square roote, and thirds vnto the remainders of the cubick roote, & pro-

ceed

ted by the former instructions, and so shall the first figure of the new quotient be primes, or tenths of an unity, the second shall be seconds, or hundreths, the third shal be thirds or thousands of an unity, &c. So that thereby, if you will, you may come within lesse then $\frac{1}{1000000}$ part of a unity. And also it may in primes, and seconds be lineally taken from a scale of decimall subdivisions, when need shal require: One example will make the matter sufficiently plaine.

As if the square roote should be extracted out of 223, the neerest in whole numbers will bee 14, and there will remaine 27; unto which adding two paires of Cyphers, and continuing the same order, you shall finde in the quotient 8 primes, and 9 seconds with the remainder 12870; which comes to lesse then $\frac{1}{1000}$ part of an unity.

Also, suppose the cubick roote of 1739 were to be extracted, the neerest cubick roote in whole numbers will be 12, and the remainder will be 11, unto which adding two threes of Cyphers, and continuing the former direction of the cubick extractions, there will arise only two seconds, or hundred parts of an unity, 2654408 remaining, all which lack much of $\frac{1}{1000}$ part of an unity.

Thus adding more paires for the square or threes of cyphers for the cubicke, you may come to thirds, fowrths, and fifths: name-

ly

ly, to lesse then $\frac{1}{10000000}$ part of an vnity, which shall be also to be expressed vpon a decimall scale in measure, as I promised, and this shal at this time suffice for extracting of rootes. Now for the proofe, if you multiply the root in it selfe squarely, for the square roote, and cubickly for the cubick rootes, and vnto the product adde the remainder, if it make the first giuen number, you haue done rightly else not.

A briefe addition of Interest.

Definitions of Interest.

1 PRincipal, is the summe from which the interest is reckoned.

2 Interest is the summe reckoned for the lending or forbearance of the Principal for any terme of time.

3 The proportion betweene the Interest, and the Principall is the reason of Interest.

4 Interest simple, is that which is counted from the Principall only.

5 Interest compound is that which is coũted for the Principall, together with the Arrages of Interest, and Principall.

6 Inte-

6 Interest profitable, is that which is added to the Principall.

7 Interest domageable, is that which is to be subtracted from the Principall.

Scholar. Sir, I haue béene much troubled with resolving questions of Interest: I pray you shewe me some way to ease the tediousnesse, which I find bréedeth errors also.

Master. Although vsury ought not to bee found amongst Christians: yet séeing our hearts are growne to that hardnesse, that Charity shal hane nothing but what the paies for; therefore most Common-weales haue, though not allowed, yet, winked at this vsury, only curbing the horrible excesses, which else would grow, and tolerating the vse therof for auoiding worse extreamities.

And because the compound interest, yea and the simple interest also in many questions of profit and losse; otherwise aske much labour, and so becomes subiect (as you say) too much error. I haue thought good to set downe this Table containing fower Columnes: The first whereof is for Interest simple domageable, where losse commeth, by receiuing before the day due. The second is, for the number of yéeres, quarters, moneths, or other equall tearmes mentioned in each question. The third is for compound Interest, when an yéerely, quarterly, and continued

ed payment is not in the question. The last is for yeeres continued. As the questions following shall shewe vnto you, wherein I would haue you to marke diligently the reason of them, that you may apply any other question, for that this will serue for any number of yéres of equall termes, whatsoeuer vnder 21, and for any summe of money propounded.

Scholar. I pray you proceed as plainely, and briefly as you can for my memories sake, and I shall bee euer bound to you for your paines.

Master. There is 324 l. to be paid in fiue yéres: namely, 54 l. each yeere: the question is, what it is worth in ready money, abating simple interest after 10 for a 100.

The Table for intereſt, after 10 for 100 *taken out of* SYMON STEVENS Arithmetick.

Inter. ſimp. dom.	*yeeres.*	*Compound*	*Intereſt.*
9090909	1	9090909	9090909
17424242	2	8264463	17355372
25116549	3	7513148	24868520
32259406	4	6830135	31698655
38926072	5	6209214	37907869
45176072	6	5644740	43552609
51058425	7	5131582	48684191
56613980	8	4665075	53449266
62058424	9	4240977	57590143
67058414	10	3855434	61445677
71820328	11	3504940	64950617
76365782	12	3186309	68136926
80713608	13	2896645	71033571
84880274	14	2633314	73666885
88880274	15	2393922	76060807
92726428	16	2176293	78237100
96430131	17	1978448	80215548
	18	1798589	82014137
	19	1635081	83649218
	20	1486437	85135655
	21	1351306	86486961

The roote of theſe three Tables is alwaies 10000000.

Herein

Herein you may perceiue it will come vnto lesse mondy, and therefore it must bée domageable and belongeth vnto the first columne, whereby you shall finde the solution. Thus, Multiply the roote of these Tables which is 10000000 by the yéeres of the question 6, and it will produce 60000000. And then say if 60000000 giue 45176072 (which is the number of the first collumn answering to the sixt yéere) how much will 324 l' giue, which if you worke by the golden Rule will answere the question.

Scholar. This can I soone doe, and finde it giueth 243 l' and $\frac{9347328}{10000000}$ parts of a pound, which I also can reduce into shillings and d, by your former instructions. But I pray you procéed to some questions of other kinds of Interest.

Master. A man oweth 224 l' to be paide by 56 l' a yéere for 4 yéeres, what shall he pay by simple interest 10 pro 100. if hée pay all the parts together at the end of the 4th yéere. To answere this and such like you must consider what hee holdeth (which by the first hee ought not) part of the principal and séeke the interest of each princi-

$$\left\{\begin{matrix}100\\100\\100\\100\end{matrix}\right\}\text{giueth,}\left\{\begin{matrix}0\\10\\20\\30\end{matrix}\right\}$$

pall

pall for each yeere.
Or thus, 400 giues 400l. 60l.
60l. what wil 224l. giue.

Answer. 33l. 12s. 224l. 257l. 12s.
which added to the
whole debt 224l. maketh 257l. 12s.

Another Question is, if a man be to pay 324l: namely 55l. per An? How much shall hee pay ready money, abating compound interest, after 10 for 100. For this seeke in the fowrth columne of the Table which is for continued paiments of compound interest) for the number against the 6. yeare, which is 43552610, and multiply the rate of the Table 10000000 by 6, and it will produce 60000000. And then say if 60000000 giue 43552610, what shall 324 giue.

Scholar. That I find by the rule of three direct to giue 235l. $\frac{114564}{600000}$ part of a pound: yet I pray you propound a question or two, and proue if I can worke it right.

Master. What commeth 380l. with his compound interest profitable for 8 yeares after 10 for 100.

Scholer. The question being of compound interest, profitable and not for continued paiments, I must seeke in the third columne for the number against the 8 yeare, where I finde 4665075: Therefore I say

if that number gine the 1000000, the rote of the table what wil ,80 giue 814 pound $\frac{120000}{1000000}$ parts of a pound.

Master. You conceiue the reason well, I will trie you with one question more, and so leaue you to your practise. There is one oweth 700l to be paid at the end of 10 years, the question is what it is worth in readie money, abating interest compound damageable.

Scholer. Now ye had almost spoil'd me but that I considered that it was no paiment continued, and being domageable, it must decrease the summe, I therefore know the question belongeth vnto the third column of the table, and must herein say if the rote of the Table 1000000 giue 2855434 the number therin against the 10 yeare or terme what shall 700 l gi e. So that by multiplying and diuiding I finde 267 l, and $\frac{800000}{1000000}$ parts of a pound.

Master. If your question be to seeke the reason of your interest. Multiply the rote of the Table by the yeares or termes of the question, and the product shall be in such reason to the said rote either as the principall to his interest, or as the interest vnto the principal according to the nature of the question, which reason it selfe will sufficiently direct, and if you desire to haue it after 12. 16 20 or any other sum for 100 if you multiplie

tiply the roote of the Table, by 100 the Principall, and diuide the product by the Principall and his Interest, the quotient will be the number answering the same reason required. And thus much shall be sufficient to haue spoken of the calculation of Tables and solutions of Questions of Interest. And so I will leaue you to your owne practice, or my further leasure.

Scholar Sir, I much thank you for your louing paines, and desire I may be able to requite this your courtesie: And least I should be troublesome to your greater businesse, I will take my leaue at this time, and attend your better leasure.

FINIS.

I. D. To the earneſt Arithmetician.

MY louing friend to Science bent,
Thou haſt by this booke ſome ſmal gains
But if thou wilt be excellent,
You muſt endeuour further paines.

Yet few ſupplies doe thereto need,
And none but ſuch as in our phraſe,
(By *Records* pen) thou maiſt well reade,
Proceede therefore without amaſe.

The ground moſt ſure, whereon this race
With ſpeedie courage muſt be runne,
Of late hath turn'd his Greekiſh face,
By Engliſh tilth, as is begunne.

The famous Greeke of *Platoes* lore,
Euclide I meane Geometer:
So true, ſo plaine, ſo fraught with ſtore,
(As in our ſpeech) is yet no where.

A treaſure ſtrange, that booke wil proue,
With numbers skil matcht in due ſort.
This I thee warne of ſincere loue,
And to proceed do thee exhort.

Within Northumberland Place, at the lower end of Fanne-Church ſtreete, neere Algate, is taught the principles of the arts Mathematicall as followeth: by N. Phyſhe.

Arithmetike.

Vulgar, with fraction, and the rule of proportion.
Extraction of the ſquare roote cubicke, and ſquare cubicke roor, at one worke, according to Vieta.
Equation, with Coſſe or Algeba
The qualities reſulting, of medicines compound, and the finding or the Dose, or quantity of Compoſitions pure tiue by their ingredience.

Geometrie.

Principles thereof with practicall demonſtration,
Surueying, with the vſe of Topographical inſtruments,
Reduction of Plots, or Maps to any proportion deſired,
Meaſuring of ſolid contents, as Timber, ſtone & changing of veſſels, with the vſe of inſtruments thereto belonging.
The art of Dialling in generall.
The Doctrine of triangles, with the manifold vſe of the Sinus, Tagents, and Secants.

Astronomy.

The calculation of the true place of the Planet, & fixed ſtars, according to the moſt worthy Tycho Brahe, for any time, or place
The vſe of the Sphere & Globes.

The erection of the figure Aſtralogicall, for any time, or place.
The vſe of the *primum Mobile*, with the effects of the Planets and fixed ſtars in euery particular figure, and Manſion. Alſo their direction to any point of the Heauens aſsigned, with their profection, Reuolutions, & Tranſirions.

☿ ☽

Principles thereof, with inſtruments for that purpoſe.
Direction to finde the diſtance of places, by Longitude, and Latitude. And to finde the Longitude, and Latitude by inſtruments, or planetical obſeruations.

Scire meum nihil eſt, niſi me ſcire hoc ſciat alter.

by knowledge mine I haue no name,
till others know, I know the ſame.

Date Issued			